The Effect of Infrequent Trading on Detecting Price Jumps*

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4th Version

Abstract

Subject of the present study is to analyze how accurately an elaborated price jump detection methodology by Barndorff-Nielsen and Shephard (2004a, 2006) applies to financial time series characterized by less frequent trading. In this context, it is of primary interest to understand the impact of infrequent trading on two test statistics, applicable to disentangle contributions from price jumps to realized variance. In a simulation study, evidence is found that infrequent trading induces a sizable distortion of the test statistics towards overrejection. A new empirical investigation using high frequency information of the most heavily traded electricity forward contract of the Nord Pool Energy Exchange corroborates the evidence of the simulation. In line with the theory, a "zero-return-adjusted estimation" is introduced to reduce the bias in the test statistics, both illustrated in the simulation study and empirical case.

Keywords: Realized Variance, Bipower Variation, Zero-Returns, Infrequent Trading, High Frequency Data, Electricity Forward Contract.

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1 Introduction

The concept of realized variance rendered a remarkable progress both in theory and economic applications. Realized variance is a nonparametric estimator for the unobservable notional variance of typically a financial asset over an interval $[0, t]$. Previous approaches derived this variability measure from a theoretical diffusion log-price process and a set of reasonable assumptions. Empirically, it was questioned whether the assumption of a pure diffusion log-price process is realistic for any financial asset. Extensive empirical examination of financial high frequency time series gave an important reason to the consideration of potential discontinuities or jumps within the analyzed timeframe, e.g. Andersen, Benzoni and Lund (2002), Chernov, Gallant, Ghysels and Tauchen (2003), Eraker (2004) and Eraker, Johannes and Polsen (2003). This impulse led to the question in which manner the estimator for the notional variance changes if we assume a jump diffusion log-price process. Based on an adjusted set of assumptions, two important goals were attained. First, price jumps are explicitly incorporated in the notional variance process. Second, it was shown that realized variance is not qualified to separately measure the contributions emanating from the diffusion and jump part of the assumed log-price process. Theoretical work of Barndorff-Nielsen and Shephard (2004a, 2006) proposed a methodology to detect the contribution of price jumps to realized variance. Alternative nonparametric approaches are for instance by Aït-Sahalia and Jacod (2009), Andersen, Dobrev and Schaumburg (2008), Christensen, Oomen and Podolskij (2009), Corsi, Pirino and Renò (2009), Jiang and Oomen (2008), Lee and Mykland (2008) and Mancini (2009). Basically, Barndorff-Nielsen and Shephard (2004a, 2006) dealt with the challenge by defining a consistent estimator for the continuous variation of realized variance which is robust against a finite number of jumps over a finite period of time in the log-price process. This estimator is called realized bipower variation. Hence, a resulting jump measure is the difference of realized variance and realized bipower variation. As measurement errors in both estimators can produce false conclusions, Barndorff-Nielsen and Shephard (2004a, 2006, 2005) derived an asymptotic distribution theory for the difference and proposed differently modified jump detection test statistics, applicable on a day-to-day basis.

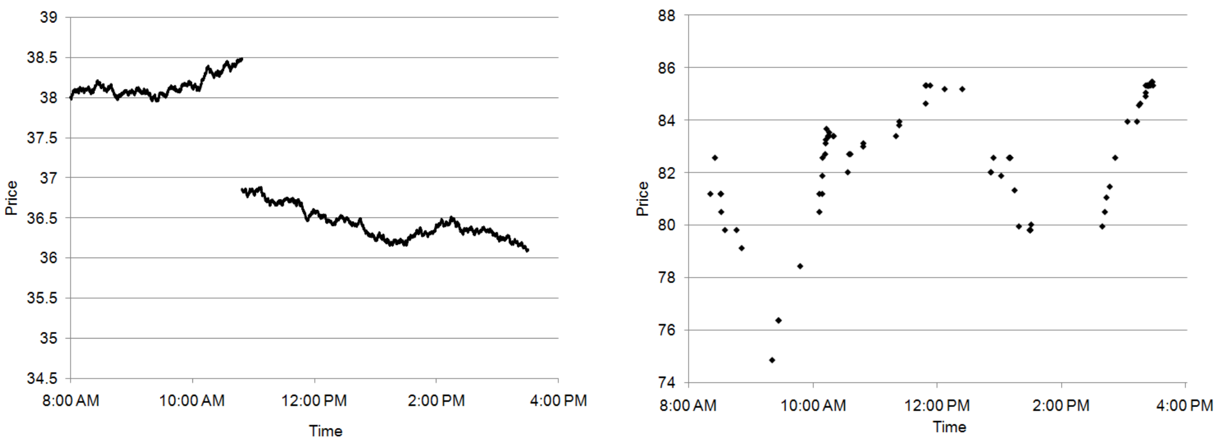
In an extensive simulation study, Huang and Tauchen (2005) analyzed these jump detection test statistics for specific parametric continuous time (jump) diffusion processes. Besides applications to simulated price processes, the jump detection methodology was tested in several empirical implementations. Examples are Barndorff-Nielsen and Shephard (2004a, 2006), who applied their methodology to time series of the foreign exchange spot market, i.e. German DM/U.S. dollar and Japanese Yen/U.S. dollar. Andersen, Bollerslev and Diebold (2007) proceeded the sole jump analysis of the foreign exchange spot market of German DM/U.S. dollar, equity futures market of U.S. S&P 500 index, and interest rate futures market of thirty-year

U.S. Treasury yield, by using the separately measured components of realized variance in a time series model to improve forecasts for realized variance.

From an application point of view it is worth mentioning that the empirical implementations essentially employed time series characterized by very frequent trading activities. As the theory of realized variance is derived from the crucial assumption of a continuous price process, the employed discrete and highly frequent price paths seem to be a sound approximation. Simulated time series with extraordinary high trading activities have also been implemented by Huang and Tauchen (2005), validating extensively the theoretical results in the absence of microstructure noise.

An interesting question arising straightaway is how accurately the jump detection methodology by Barndorff-Nielsen and Shephard (2004a, 2006) applies to financial time series characterized by less frequent trading. This is an important issue as the violation of assuming a continuous price process is likely more severe for such time series. The scenario of both a frequent and an infrequent trading[1] day (7.5 trading hours) shall be illustrated in figure 1. The left panel shows a (simulated) discretized trajectory of a continuous time jump diffusion price path (up-to-seconds) and the right panel an empirical intraday price path of an electricity forward contract of the Nord Pool Energy Exchange on December 4$^{\text{th}}$, 2002. One pivotal disparity

Figure 1: Different patterns of intraday price paths



Remarks: Simulated price path with parametric specification outlined in Section 3 (left panel) and an empirical intraday price path of an electricity forward contract of the Nord Pool Energy Exchange on December 4$^{\text{th}}$, 2002 (right panel). Both price paths represent an active trading day of 7.5 trading hours.

between both price patterns is the quantity of price observations. The simulated time series, which assigns a price to each second on the Euler clock, counts 25,000 prices. However, the empirical time series only counts 81 price observations within the trading day.

In order to circumvent such an application problem, a naïve approach would be to abstain

[1]Infrequent trading has two meanings. First, it can be understood as a microstructure noise in form of either longer data breaks, or flat prices (i.e. prices remaining constant do not represent efficient prices). Second, it can simply mean that either an asset is illiquid, i.e. nobody buys or sells for long times, or he/she buys or sells it at a price which remains constant.

from using high frequency information. But relaxing the use of high frequency data increases the corresponding measurement errors in realized variance, discussed in detail by Andersen and Bollerslev (1998). Besides, the detection of contributions from price jumps to realized variance complicates, outlined by Aït-Sahalia (2004). A more sophisticated approach for infrequent trading has been developed by Barndorff-Nielsen and Shephard (2002, 2004b) in their empirical implementation. They propose to build Brownian bridges between data points with longer intertrade duration or sequences of very small price changes in order to improve the approximation of a continuous price process. An important note to their empirical study is that the premise for employing Brownian Bridges was seldom fulfilled due to a high level of trading activity. However, it is questionable whether their approach can be applied without any concern to a time series characterized by infrequent trading, like in the right panel of figure 1. In this case, we would have to implement numerous Brownian bridges, nescient about this effect on realized variance.

In this paper, we construct a simulation study accounting for various patterns of infrequent trading in financial time series. For the simulation, basic parametric specifications of Andersen, Benzoni and Lund (2002) are utilized. The main objectives of the simulation study are of threefold nature. First, we are interested in the finite sample behavior of the jump detection test statistic, i.e. we question whether the test statistic under the null diverges from a standard normal distribution, with respect to increasing infrequent trading. Our Monte Carlo results suggest that the test statistic is quite sensitive to a small increase in the fraction of "zero-returns".[2] Second, we investigate the accuracy of the jump detection test statistic with regard to different fractions of "zero-returns" and variations of parameter settings in the simulation using the classical confusion matrix as an analysis tool. The evaluations show that the jump detection rate is negatively influenced by longer sampling frequencies and decreasing fraction of "zero-returns". Third, we propose and implement a "zero-return-adjusted estimation", henceforth "zero-adjusted estimation", based on the theory of Barndorff-Nielsen and Shephard (2004a, 2006) to improve the validity of the jump detection test statistic in case of infrequent trading. In realistic market scenarios, our more conservative estimation yields sound improvements, especially for short sampling intervals. Adjacent to the Monte Carlo results, we present a new empirical investigation. We employ high frequency information of the most heavily traded electricity forward contract of the Nord Pool Energy Exchange, corroborating the evidence of the simulation.

In the next section, we initially outline the theoretical framework, i.e. the concept of realized variance assuming a jump diffusion log-price process, realized bipower variation, and a selection of jump detection test statistics. Thereafter we address issues caused by infrequent trading

[2] To our knowledge, Corsi, Pirino and Renò (2009) are the first who explicitly address the impact of "zero-returns" on the computation of realized bipower variation. A similar discussion concerning this impact on bipower variation can be found in Andersen, Dobrev and Schaumburg (2008).

more closely and discuss a useful zero-adjusted estimation for such circumstances. Section 3 describes the simulation study and elaborately analyzes the results. The empirical investigation of an electricity forward contract is presented in section 4. Finally, section 5 concludes.

2 Theoretical Framework

2.1 General Theoretical Background, and Realized Variance

Initially, we brief on the notations and frame of assumptions required to derive the concept of realized variance, based on more detailed elaborations of Andersen, Bollerslev and Diebold (2002), Cont and Tankov (2004, pp. 247-267), and Barndorff-Nielsen and Shephard (2004a, 2006). In order to model uncertainty of a logarithmic price process $X(t)$ of any financial asset, we define a filtered probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t \geq 0}, P)$. Furthermore, we assume a frictionless and continuous setting with no arbitrage opportunities. Sufficient for this, the log-price process $X(t)$ is meant to constitute a semimartingale with $X_0 = 0$, i.e. a nonanticipating right-continuous process with left limits (càdlàg), implicating in turn convenient properties for the quadratic variation process of $X(t)$. A widely used specification of the semimartingale $X(t)$ is a continuous-time stochastic volatility jump diffusion process. The expansion of the log-price process is presented in form of a stochastic differential equation

$$dX(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t) \ , \qquad t \in [0,1] \ , \tag{1}$$

where $\mu(t)$ is the drift term, $\sigma(t)$ is a strictly positive stochastic càdlàg process and $W(t)$ is a standard Brownian motion. $\kappa(t)$ is the size of the corresponding discrete jump in time t in the log price process and $q(t)$ is a counting process with finite activity and (possibly) time-varying intensity $\lambda(t)$. An ex-post measurement which captures the price variation for equation (1) is the quadratic variation of $X(t)$ over a discrete interval. The **quadratic variation process** of $X(t)$ with $t \in [0,1]$ is a nonanticipating càdlàg process (see e.g. Cont and Tankov, 2004 p. 263):

$$[X,X]_t = X(t)^2 - 2\int_0^t X(u_-)dX(u) \ . \tag{2}$$

One advantage using quadratic variation for capturing the price variability is that it is a well-defined quantity for all semimartingales. But why does the literature focus a priori on an ex-post measure over a discrete time interval instead of one point-in-time? The reason is that in empiricism microstructure effects prevent us from observing a continuous price process. According to this, it seems much more difficult to define an appropriate estimator for the instantaneous variance than for the (average) variance over a discrete time interval. Furthermore, we can subsequently link quadratic variation with an ex-post variance measure using an important property.

Proposition 1:
If $\pi^M = \{t_i \mid t_0 = 0 < t_1 < \cdots < t_{M+1} = 1\}$ is a sequence of partitions of $[0,1]$ such that $\left|\pi^M\right| = sup_k \left|t_k - t_{k-1}\right| \to 0$ as $M \to \infty$, then

$$\sum_{t_i \in \pi^M \cap [0,t[} \{X(t_{i+1}) - X(t_i)\}^2 \quad \xrightarrow{p} \quad [X,X]_t \ ,$$

where the convergence is uniform in t (Cont and Tankov 2004, pp. 263-264).

The explicit quadratic variation, or here **notional variance**, for the process in (1) can be derived over $[t\text{-}h, t], 0 < h \leq t \leq 1$, as follows:

$$NV_{t,t-h} \equiv \underbrace{\int_{t-h}^{t} \sigma^2(s)ds}_{\text{continuous variation}} \quad + \quad \underbrace{\sum_{q_{t-h}<s\leq q_t} \kappa^2(s)}_{\text{jump part}} \ , \tag{3}$$

where $\sigma^2(s)$ is the instantaneous return variation, $\kappa^2(s)$ is the squared size of the corresponding discrete jump at time s. h is typically set to one, representing one trading day. We can state that the first part, i.e. **continuous variation** or **integrated variance**, of equation (3) is the quadratic variation of the standard Brownian motion. The second part of equation (3) stands for the quadratic variation of the Poisson process.
Bearing *Proposition 1* in mind, the estimator for the notional variance, called **realized variance**, for one trading day t now can be defined as:

$$RV_t \equiv \sum_{j=1}^{M} r_j^2 \ , \qquad \text{with} \quad r_j := r_{j,t,M} := X\left(\frac{j\,t}{M}\right) - X\left(\frac{(j-1)t}{M}\right). \tag{4}$$

The integer M is the amount of sufficiently small equidistant intraday sampling intervals and r_j is a continuously compounded interval return. Realized variance is converging for $M \to \infty$ in probability limit to the notional variance in equation (3). Formally,

$$RV_t \quad \xrightarrow{p} \quad \int_{t-1}^{t} \sigma^2(s)ds \quad + \quad \sum_{q_{t-1}<s\leq q_t} \kappa^2(s) \ , \tag{5}$$

i.e. we can approximate the notional variance with accumulating squared returns, sampled at a fairly high frequency. In other words, realized variance is a consistent estimator for daily increments of the quadratic variation process. Obviously now from equation (3) and (4), it is not possible to separately measure the contribution emanating from the continuous variation and jump part.

2.2 Realized Bipower Variation

As already pointed out in the introduction, one general approach to isolate the contribution of price jumps to realized variance is to define an estimator for the continuous variation in order to infer on the jump part. Barndorff-Nielsen and Shephard (2004a) derive theoretical results for such an estimator, called **realized bipower variation**. A general form of this estimator can be found in Barndorff-Nielsen and Shephard (2004b, p. 10) and Huang and Tauchen (2005, p. 486):

$$BP_{t,i} \equiv \mu_1^{-2} \left(\frac{M}{M-1-i} \right) \sum_{j=2+i}^{M} |r_{j-(1+i)}||r_j| \,, \qquad i \geq 0 \,, \tag{6}$$

where $\mu_1 \equiv \sqrt{2/\pi}$. $BP_{t,i}$ is defined by accumulated cross-products of absolute adjacent intraday returns. Barndorff-Nielsen and Shephard (2004a) show for the case $i = 0$ that for $M \rightarrow \infty$,

$$BP_{t,0} \quad \xrightarrow{p} \quad \int_{t-1}^{t} \sigma^2(s) ds \,, \tag{7}$$

meaning realized bipower variation is robust against a finite number of jumps over a finite period of time in the log price process in (1).

Especially thereinafter important for time series characterized by infrequent trading and our zero-adjusted estimation is that Barndorff-Nielsen and Shephard (2004b) further mention that the convergence in probability of realized bipower variation is not limited to computing the cross-product of only directly adjacent absolute returns, i.e. for $i = 0$. It even holds for $i \geq 0$ (but fixed over h) and is obviously constrained by a finite choice of M in empirical applications. For applications to financial time series, Andersen, Bollerslev and Diebold (2007, pp. 710-711) suggest using $i = 1$ to avoid potential serial correlation between directly adjacent returns sampled at a high frequency, distorting the realized bipower variation measure. Similar results are shown both analytically and in a simulation study by Huang and Tauchen (2005, pp. 483-492) for a noisy price process. Choosing even longer staggering periods, e.g. $i = 2$, is also addressed by Huang and Tauchen (2005, pp. 483-492). Interestingly, they demur that a higher i might introduce finite-sample bias, worsening the asymptotic approximation. The bias might be attributed to the fact that each cross-product of the staggered returns covers a longer interval.

2.3 Selection of Jump Statistics

Adapted from the theoretical results presented in the previous sections, we can specify a jump measure J_t over $[t\text{-}h, t]$ as the difference between RV_t and $BP_{t,i}$, according to Barndorff-Nielsen and Shephard (2004a, 2006). The limit in probability for J_t as $M \to \infty$ is

$$J_t = RV_t - BP_{t,i} \quad \xrightarrow{p} \quad \sum_{q_{t-1} < s \leq q_t} \kappa^2(s) \ , \tag{8}$$

i.e. the term J_t converges in probability to the theoretical jump part in equation (3). Andersen, Bollerslev and Diebold (2007) argue that employing this jump measure to an empirical time series most likely produces finite sample problems, like theoretically infeasible negative differences and an unreasonable large number of small positive "jumps", subject to measurement errors. In order to circumvent the finite sample problems, Barndorff-Nielsen and Shephard (2004a, 2006) provide a jump detection test statistic for $RV_t - BP_{t,i}$, applicable to test for price jumps:

$$\tilde{Z}_{t,i} = M^{\frac{1}{2}} \frac{RV_t - BP_{t,i}}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5) \int_{t-1}^{t} \sigma^4(s)ds}} \stackrel{\text{d}}{\Longrightarrow} N(0,1) \ . \tag{9}$$

The null hypothesis reads as follows: No jumps are present in the underlying price process. Under the null, the test statistic converges in distribution to a standard normal distribution. Obviously, the test statistic is infeasible as it includes $\int_{t-1}^{t} \sigma^4(s)ds$, termed integrated quarticity. This term, factored with $\frac{1}{M}(\mu_1^{-4} + 2\mu_1^{-2} - 5)$, can be interpreted as the asymptotic variance of the discrepancy between RV_t and $BP_{t,i}$. Andersen, Bollerslev and Diebold (2007) suggest a consistent estimator to be employed for the integrated quarticity, called **realized tripower quarticity** ($TriP_{t,i}$). For $i \geq 0$,

$$TriP_{t,i} \equiv \vartheta_i \sum_{j=1+2(1+i)}^{M} |r_{j-2(1+i)}|^{4/3} |r_{j-(1+i)}|^{4/3} |r_j|^{4/3} \ , \tag{10}$$

where $\vartheta_i = M\mu_{4/3}^{-3}\left(\frac{M}{M-2(1+i)}\right)$, and $\mu_{4/3} = 2^{2/3} \cdot \Gamma(\frac{7}{6}) \cdot \Gamma(\frac{1}{2})^{-1} = E(|z|^{4/3})$, with $z \stackrel{\text{iid}}{\sim} N(0,1)$. For $M \to \infty$,

$$TriP_{t,i} \quad \xrightarrow{p} \quad \int_{t-1}^{t} \sigma^4(s)ds \ , \tag{11}$$

meaning that $TriP_{t,i}$ is converging in probability limit to the integrated quarticity.

Concerning the test statistic in equation (9), Huang and Tauchen (2005) notice that it tends to exhibit a positive bias. Some extensions of this basic test statistic with improved finite-sample performance are suggested in Barndorff-Nielsen and Shephard (2004a, 2006, 2005). Their approach is to reasonably transform $RV_t - BP_{t,i}$ in order to receive a more stable variance for the asymptotic distribution of realized variance and bipower variation. One considered transformation is the relative jump measure $(RV_t - BP_{t,i})/RV_t$ and another one the log differences, i.e. $\log(RV_t) - \log(BP_{t,i})$. The corresponding test statistics are termed $Z1_{t,i}$ and $Z2_{t,i}$, with respectively adjusted asymptotic variances in the denominator:

$$Z1_{t,i} = M^{\frac{1}{2}} \frac{(RV_t - BP_{t,i}) \cdot RV_t^{-1}}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5) \max\left\{1, \frac{TriP_{t,i}}{(BP_{t,i})^2}\right\}}} \overset{\mathrm{d}}{\Longrightarrow} N(0,1) \;, \tag{12}$$

$$Z2_{t,i} = M^{\frac{1}{2}} \frac{\log(RV_t) - \log(BP_{t,i})}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5) \max\left\{1, \frac{TriP_{t,i}}{(BP_{t,i})^2}\right\}}} \overset{\mathrm{d}}{\Longrightarrow} N(0,1) \;. \tag{13}$$

The null reads the same as for equation (9). At this point, we would like to mention that in our following simulation study the sensitivity of the convergence result in distribution for time series characterized by infrequent trading is investigated upfront. There we solely focus on these two extensions and motivate our choice with the simulation results of Huang and Tauchen (2005). They find that these two extended versions have good power and are quite robust against parametric changes of the simulated continuous time jump diffusion process. The final step towards deciding on a daily basis the portion of realized variance attributable to jump part $J_{t,i,\alpha}$ and to continuous variation $C_{t,i,\alpha}$ is a straightforward task:

$$J_{t,i,\alpha} \equiv I[Z_{t,i} > \Phi_{1-\alpha}] \cdot [RV_t - BP_{t,i}]^+ \quad \text{and} \quad C_{t,i,\alpha} \equiv RV_t - J_{t,i,\alpha} \;, \tag{14}$$

where $Z_{t,i} \in \{Z1_{t,i}, Z2_{t,i}\}$. I is an indicator function equaling one if the condition $Z_{t,i} > \Phi_{1-\alpha}$ is true, and zero else. $\Phi_{1-\alpha}$ is the corresponding value of a standard normal distribution function. In the corresponding literature, the level of significance (α) is usually set to a value in the range of 0.1% to 1%.

2.4 Zero-Adjusted Estimation

Let us now turn to one key consideration of this paper, namely to understand the effect of infrequent trading on detecting jumps by means of extreme cases and contemplate a solution. In the introduction, we briefly addressed the aspect of infrequent trading (see figure 1), that is a rather small amount of prices observable for a financial asset over a trading day. This sparse set of information is meant to represent the basis for computing intraday returns over equidistant and sufficiently small intervals. In consequence of a small number of observed intraday prices, we likely receive several intraday returns equaling to zero, called zero-returns. Intuitively, this effect is reinforced if a higher sampling frequency is chosen. Due to the fact that intraday returns are directly used to compute realized variance, bipower variation and tripower quarticity, it is of great concern to comprehend the potential exposure resulting from zero-returns on these measures and later on the test statistics.[3] The effect of generic microstructure noise[4] on realized variance has already been well addressed in the literature. Anderson, Bollerslev, Diebold and Labys (1999) propose a variance signature plot to visualize the dimension of noise in realized variance for different sampling frequencies. To handle microstructure noise in realized variance Bandi and Russell (2008) even propose an analytical approach to mitigate noise effects by optimally choosing the sampling frequency M, which we will get to in more detail in the empirical part. The effect of microstructure noise on realized bipower variation is analytically employed to some extent by Huang and Tauchen (2005).[5] A graphical analysis is proposed by Andersen, Bollerslev, Frederiksen and Nielsen (2006). In their comment, they apply the signature plot methodology of Anderson, Bollerslev, Diebold and Labys (1999) to get an idea about microstructure effects on realized bipower variation and tripower quarticity.

Now, to better grasp the effect of infrequent trading in particular on bipower variation and tripower quarticity we illustrate a scenario in table 1. The information reported in table 1 is from intraday prices of an electricity forward contract, traded at the Nord Pool Energy Exchange on April 14$^{\text{th}}$, 2003. We chose a sampling frequency of 15 minutes, producing 30

Table 1: Illustration: effect of infrequent trading on $Z1_{t,i}$

			$i=1$		$i=3$	
Interval	r_t	RV_t	$BP_{t,1}$	$TriP_{t,1}$	$BP_{t,3}$	$TriP_{t,3}$
3	-0.0010	1.10E-06	0	0	1.10E-06	3.07E-12
6	-0.0010	1.10E-06	0	0	0	0
7	-0.0011	1.10E-06	0	0	2.21E-06	0
11	-0.0021	4.43E-06	0	0	0	0
19	0.0032	9.96E-06	0	0	6.68E-06	0
22	-0.0063	4.00E-05	0	0	2.15E-05	2.79E-10
23	-0.0021	4.48E-06	4.48E-07	0	0	-
25	0.0002	4.49E-08	0	0	0	-
26	-0.0034	1.15E-05	0	0	1.08E-05	-
30	0.0032	1.01E-05	-	-	-	-
Daily Value	-0.0105	8.39E-05	7.55E-07	0	7.66E-05	2.01E-08
$Z1_{t,i}$			6.96		0.33	
$\Phi_{1-\alpha=0.99}$			2.33		2.33	

Remarks: The upper part of the left column reports all intraday intervals of the trading day 04/14/2003 with $|r_j| > 0$ followed by the corresponding accumulated daily value. Accordingly, the following columns specify interval (daily) return, realized variance, bipower variation and tripower quarticity. The last two rows report the resulting $Z1_{t,i}$ test statistic and the respective value of a standard normal distribution at $\alpha = 1\%$.

[3]Further nonnegligible drawbacks of bipower variation, which shall not be of concern in this study, are discussed by Corsi, Pirino and Renò (2009) and Andersen, Dobrev and Schaumburg (2008).

[4]Microstructure noise: e.g. price discreteness, bid-ask spreads and measurement errors.

[5]Huang and Tauchen (2005) assume a simple noisy price process to conduct their analysis.

intervals for 7.5 trading hours per day.[6] We solely report interval returns which are in absolute value greater than zero and compute the respective increments required for the jump analysis. Following the proposition of Andersen, Bollerslev and Diebold (2007), we choose $i = 1$ to break potential serial correlation in adjacent returns used for the computation of realized bipower variation and tripower quarticity. Obviously striking is the fact that due to the alignment of returns unequal to zero only one increment of $BP_{t,1}$ is unequal to zero and none of $TriP_{t,1}$. Under the reported setting we would reject the null hypothesis despite the fact that there are no obvious indications for price jumps. Concerning this example, we analyze more closely the sensitivity of $Z1_{t,i}$ for cases where $BP_{t,i}$ is compared to RV_t relatively small due to the presence of zero-returns and not due to the reason of abnormal price movements. To ensure that the test statistic is defined, we assume that $k = \max\{1, TriP_{t,i}/(BP_{t,i})^2\}$ and $h = (RV_t - BP_{t,i}) \cdot RV_t^{-1}$, where $k \in [1, \infty[$ and $h \in]0, 1]$, and write

$$Z1_{t,i} = M^{\frac{1}{2}} \frac{h}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5)\, k}} \,. \tag{15}$$

In table 2, we report $Z1_{t,i}$ for varying k, h and M. For $k = 1$ and $M = 30$, we can observe that the test statistic gets artificially large by increasing the difference between $BP_{t,i}$ and RV_t.[7] Beyond $h = 0.33$ we would reject the null hypothesis for $\alpha = 1\%$. This effect is lagged by increasing k. $Z1_{t,i}$ converges to a maximum value of 7.02 for fix k and $h \to 1$. The distortion

Table 2: Sensitivity of $Z1_{t,i}$ for varying k, h and M

		M = 30					M = 90				
		k									
		1	1.25	1.5	2	2.5	1	1.25	1.5	2	2.5
h	0.1	0.70	0.63	0.57	0.50	0.44	1.22	1.09	0.99	0.86	0.77
	0.2	1.40	1.26	1.15	0.99	0.89	*2.43*	2.17	1.99	1.72	1.54
	0.3	2.11	1.88	1.72	1.49	1.33	*3.65*	*3.26*	*2.98*	*2.58*	2.31
	0.4	*2.81*	*2.51*	2.29	1.99	1.78	*4.86*	*4.35*	*3.97*	*3.44*	*3.08*
	0.5	*3.51*	*3.14*	*2.87*	*2.48*	2.22	*6.08*	*5.44*	*4.96*	*4.30*	*3.84*
	0.6	*4.21*	*3.77*	*3.44*	*2.98*	*2.66*	*7.29*	*6.52*	*5.96*	*5.16*	*4.61*

Remarks: Reported test statistic values of $Z1_{t,i}$ are computed with equation (15) for varying k, h and M. Emphasized values are greater than $\Phi_{1-\alpha} = 2.33$ (for $\alpha = 1\%$).

in $Z1_{t,i}$ is even more severe for $M = 90$. If $k = 1$ and $h > 0.19$, we would already reject the null hypothesis given $\alpha = 1\%$. Proceeding with a sensitivity analysis for $Z2_{t,i}$ produces directionally similar results. Therefore, it is not explicitly reported. The sole difference to the previous analysis is that $h = \log(RV_t) - \log(BP_{t,i})$, where $h \in]0, \infty[$. Additionally, $Z2_{t,i}$ converges to ∞ for fix k and $h \to 1$.

[6] We employed the optimal sampling methodology according to Bandi and Russell (2008). Further details to this can be found in Section 4.

[7] We choose $M = 30$ ($M = 90$) as this corresponds to the number of intraday interval returns for 15 (5) minute sampling frequency in the empirical case.

After having exhibited several scenarios caused by infrequent trading and given the theory discussed in the previous sections, we now want to present a zero-adjusted estimation to deal with the issue of zero-returns. Therefore, it is of importance to remember that the limits in probability and distribution for equation (6), (10), (12) and (13) remain true when $i \geq 0$. The only condition is that i has to be fixed over day t, and has to be the same for both bipower variation and tripower quarticity on day t, pointed out by Huang and Tauchen (2005). That means, deleting all zero-returns and then proceed with the analysis is not a proper option as we cannot ensure that i is fixed. Additionally, it was mentioned that it is common to use $i = 1$ instead of $i = 0$ in order to avoid serial correlation and that $i > 1$ is usually not chosen as it is not obvious whether with any extra lagging some finite-sample bias might be introduced. However, no concrete result exists which rules out $i > 1$. In the case of a considerable amount of zero-returns, it can be especially convenient to daily adjust the choice of i, before computing $BP_{t,i}$, $TriP_{t,i}$ and $Z_{t,i}$. One feasible strategy is to optimally choose i on a daily basis with the following approach:

1.) fix the number of intraday sampling intervals M (effective for the full-sample),

2.) $$\max_{\{i \in I\}} \frac{TriP_{t,i}}{(BP_{t,i})^2}, \qquad \text{where} \quad I = \left\{1, 2, ..., \left\lfloor \frac{M}{2} \right\rfloor \right\}. \tag{16}$$

The aim of this strategy is to maximize the number of increments in $BP_{t,i}$ and $TriP_{t,i}$ which are unequal to zero. Obviously, with the strategy in equation (16) we do not change the limit distribution of the test statistics. We solely formulate a method on how to fix i in a first step, which seems useful in case of zero-returns, before computing the test statistic and its components in a second step. This proceeding can be understood as a classical two-stage approach.

Applying the zero-adjusted estimation to our example in table 1 produces $i = 3$. Choosing $i = 3$ yields $BP_{t,3}$ and $TriP_{t,3}$ greater than zero, leading on to $Z1_{t,3}$ being defined. Under the assumed level of significance ($\alpha = 1\%$) we cannot reject the null hypothesis, a conclusion not hardly to believe as there is no abnormal price movement within this specific trading day.

The considerations and observations of this section give rise to further analyze to what extent infrequent trading causes a positive distortion of $Z_{t,i}$, i.e. leads to an overestimation of significant jumps. This will be discussed in detail in the simulation study in the next section. Furthermore, we are interested in the performance of the zero-adjusted estimation in a simulation setup.

3 Simulation Study

3.1 Setup

In this section we describe the assumed price processes, parameter settings and the algorithm for infrequent trading to conduct the Monte Carlo experiment. Due to the fact that we are both interested in the accuracy of the limit distribution of $Z_{t,i}$ and the accuracy of the test statistics to detect jumps, we simulate a Heston type model with and without jumps:

I. Basic Heston Type Model (BHM):

$$\begin{aligned} \frac{dp_t}{p_t} &= \mu dt + \sqrt{v_t} dW_{p,t} \ , \\ dv_t &= (\theta - \gamma v_t)dt + \eta\sqrt{v_t} dW_{v,t} \ , \end{aligned} \tag{17}$$

II. Heston Type Model with Jumps (HMJ):

$$\begin{aligned} \frac{dp_t}{p_t} &= \mu dt + \sqrt{v_t} dW_{p,t} + \kappa(t) dq_t \ , \\ dv_t &= (\theta - \gamma v_t)dt + \eta\sqrt{v_t} dW_{v,t} \ , \end{aligned} \tag{18}$$

where μ is the drift, $W_{(\cdot),t}$ are standard Brownian motions, $corr(dW_p, dW_v) = \rho$ is the leverage correlation, v_t is a stochastic volatility factor, $\kappa(t)dq_t$ is a compound Poisson process with a constant jump intensity λ_{jmp} and a random jump size distributed as $N(0, \sigma^2_{jmp})$.[8] To insure that the mean-reverting square-root diffusion process dv_t is positive, the condition $2\theta \geq \eta^2$ has to hold. To simulate realistic scenarios, we utilize basic parametric specifications estimated for the daily S&P 500 equity index by Andersen, Benzoni and Lund (2002, p. 1256). In

Table 3: Input parameters for the simulation

Parameter	*Specification*
μ	0.0304
θ	0.0064
γ	0.012
η	0.0711
ρ	-0.622
σ_{jmp}	$\{0.0134, 0.05, 0.1, 0.25\}$
λ_{jmp}	$\{0.058, 0.082, 0.118, 0.5\}$

Remarks: Parameters are expressed in percentage form and on daily basis.

order to analyze realistic scenarios for a greater range of financial assets, variations in standard deviation, frequency of jumps and trading activity are provided. Details about the parameters and variations can be found in table 3. Each simulated time series has a length of 30 years at 255 trading days a year and 7.5 trading hours per day. The discretized trajectory of the diffusion parts is simulated using the basic Euler scheme with an increment Δt of one second per tick

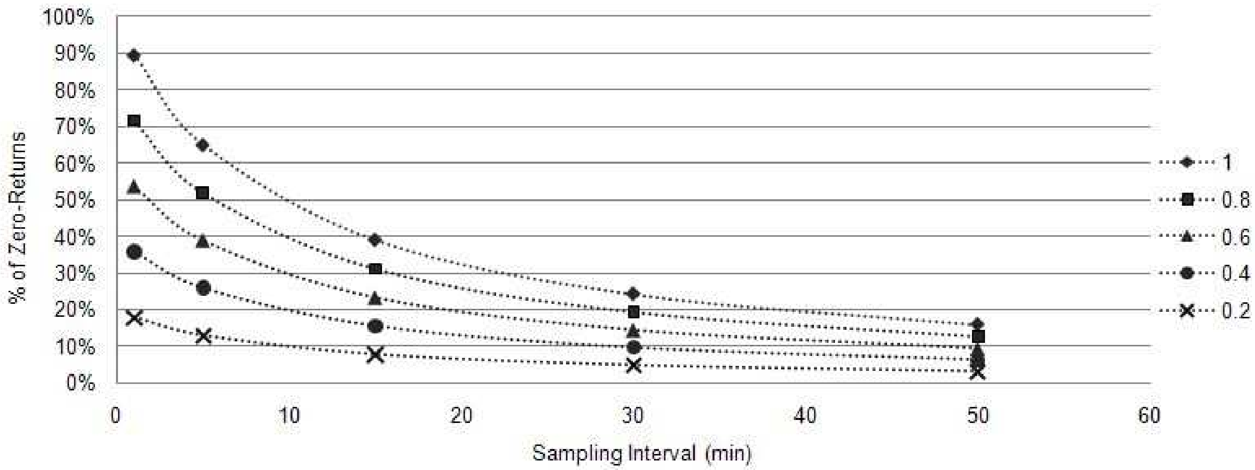
[8]The chosen price process is similar to the one used by Huang and Tauchen (2005, pp. 465-466).

on the Euler clock. We first simulated prices for *BHM* and *HMJ*, and use the log-transformed price series as a basis for different sampling intervals. The simulation of the compound Poisson process required in *HMJ* follows an algorithm of Cont and Tankov (2004, p. 174). That followed, we compute continuously compounded returns for 5, 15, and 30 minute sampling intervals, and receive a time series of "non-zero-returns". To simulate infrequent trading, we proceed with constructing a filter, skimming "non-zero-returns". The algorithm goes as follows:

- Simulate with a Poisson distribution a random variable N_r, which represents the total number of events of returns unequal to zero, with the frequency parameter $\lambda_{zr} = -\ln(\delta\ f_{\mathrm{zr},(\cdot)})$. $f_{\mathrm{zr},(\cdot)}$ is the average number of zero-returns for 5, 15, and 30 minute sampling intervals with respect to the corresponding total number of returns over the full-sample. In our case, the choice of $f_{\mathrm{zr},(\cdot)}$ is based on an empirical analysis of the electricity forward contract traded at the Nord Pool Energy Exchange. We varied the fraction of zero-returns by reducing the amount $f_{\mathrm{zr},(\cdot)}$ with the factor $\delta = \{1, 0.8, 0.6, 0.4, 0.2\}$ to conduct a sensitivity analysis. Figure 2 shows the original fraction of zero-returns ($\delta = 1$) for different sampling frequencies and also the factorized fractions.
- Uniformly distribute N_r events of returns unequal to zero across a null vector $(T \times 1)$ to accomplish the filter. Set one at the event time of a return unequal to zero, and zero else. T now represents the total number of sampling intervals over the simulation horizon. Control for keeping all sampling intervals which include a jump.
- Skim "non-zero-returns" by multiplying respective return series with the filter.

This algorithm allows for simulating a continuous price process with a varying latent part.

Figure 2: Empirical fraction of zero-returns $\delta\ f_{\mathrm{zr},(\cdot)}$



Remarks: Empirical full-sample averages of factored intraday fractions of zero-returns of the electricity forward contract, traded at the Nord Pool Energy Exchange 2002-2008, for different sampling frequencies.

3.2 Price Process without Jumps

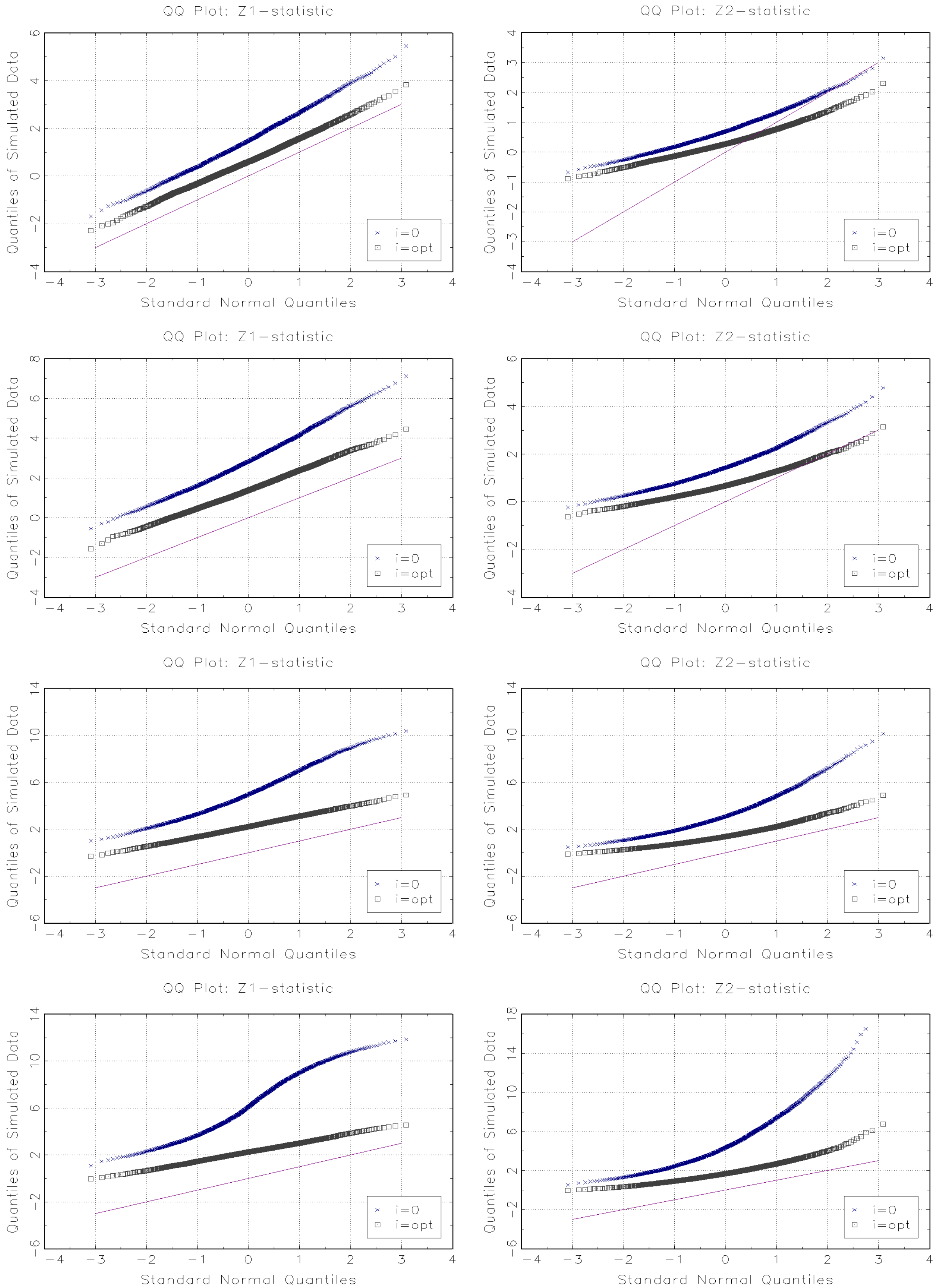
The focus of this section is to analyze the theoretical result of the convergence in distribution for $Z1_{t,i=0}$ and $Z2_{t,i=0}$ (conventional estimation), and $Z1_{t,i=opt}$ and $Z2_{t,i=opt}$ (zero-adjusted estimation), using a time series simulated with the *BHM* in equation (17) and the infrequent trading algorithm. We use QQ plots to conduct the evaluation of differently sampled return series, and various fractions of zero-returns specified with $\delta f_{\mathrm{zr},(\cdot)}$, where $\delta = \{0.2, 0.4, 0.8, 1\}$. In the interpretation, we mostly focus on the typically considered upper quantiles greater than 1.65, equivalent to a level of significance of α less than 5%.

For $\delta = 0.2$, representing a small fraction of zero-returns in the respective return series, we can clearly observe in the upper left panels of figure 3 to 5 a size distortion of $Z1_{t,0}$ towards overrejection. The return series with 5 minutes sampling intervals is affected at its most fierce with the problem in size, whereas longer sampling intervals are to a lesser extent. Increasing the fraction of zero-returns negatively affects the size, most intense for 5 minute and lesser for longer sampling frequencies, which can be clearly seen in the left top to bottom panels of figure 3 to 5. Additionally, we can observe in each left panel of figure 3 to 5 that the zero-adjusted estimation reduces the size distortion in the test statistic. The correction in size is most intense for large δ.

Turning now to $Z2_{t,0}$ and starting again with $\delta = 0.2$, the size effect in the upper quantiles is corrected for the return series with 5 minute sampling intervals (see upper right panel of figure 3). However, for the 15 and 30 minute return series the correction is even stronger causing a size distortion of $Z2_{t,0}$ towards underrejection (see upper right panel of figure 4 and 5). For the 5 minute return series, the correction vanishes dramatically fast with an increasing fraction of zero-returns, graphed in the right top to bottom panels of figure 3. A mitigated progress towards overrejection of $Z2_{t,0}$ with an increasing δ is produced for 15 and 30 minute sampling frequencies (see right top to bottom panels of figure 4 and 5). Positively distorted outputs, produced by the conventional estimation for typically higher fraction of zero-returns, are again reduced by the zero-adjusted estimation. However, for small fractions of zero-returns the reduction in size can cause a negative distortion.

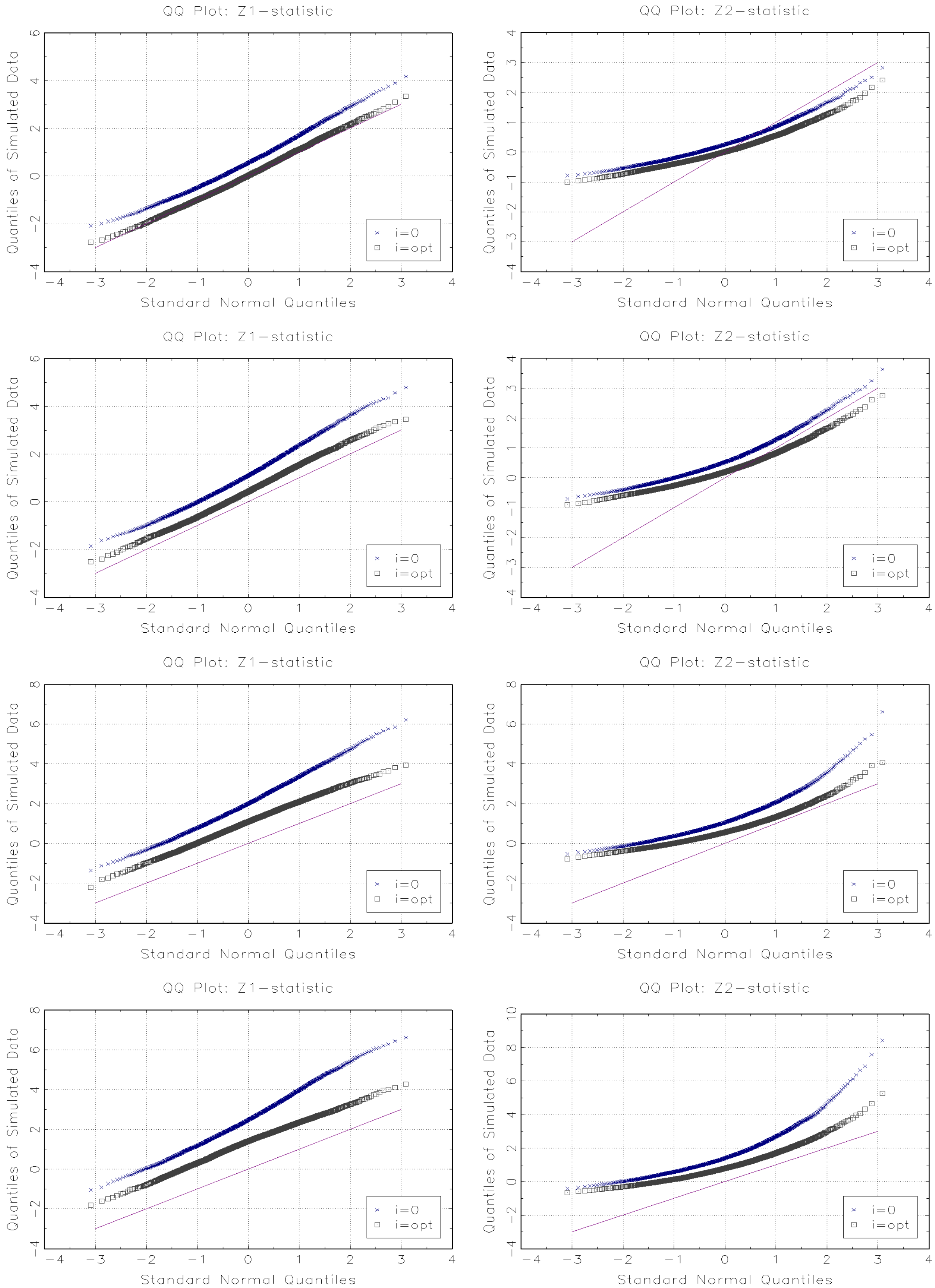
In summary, we can state for the conventional estimation that given a certain level of trading activity, higher sampling frequencies cause in both test statistics a more intense distortion towards overrejection than for longer sampling frequencies due to an increased fraction of zero-returns. This effect manifests for lessening trading activities. For $Z1_{t,i}$, the zero-adjusted estimation shows across all levels of trading activity better size than the conventional estimation, and for $Z2_{t,i}$ on days with lower trading activity.

Figure 3: QQ plots of daily $Z1_{t,i}$ and $Z2_{t,i}$ statistic for $i = \{0, opt\}$ and 5min sampling intervals



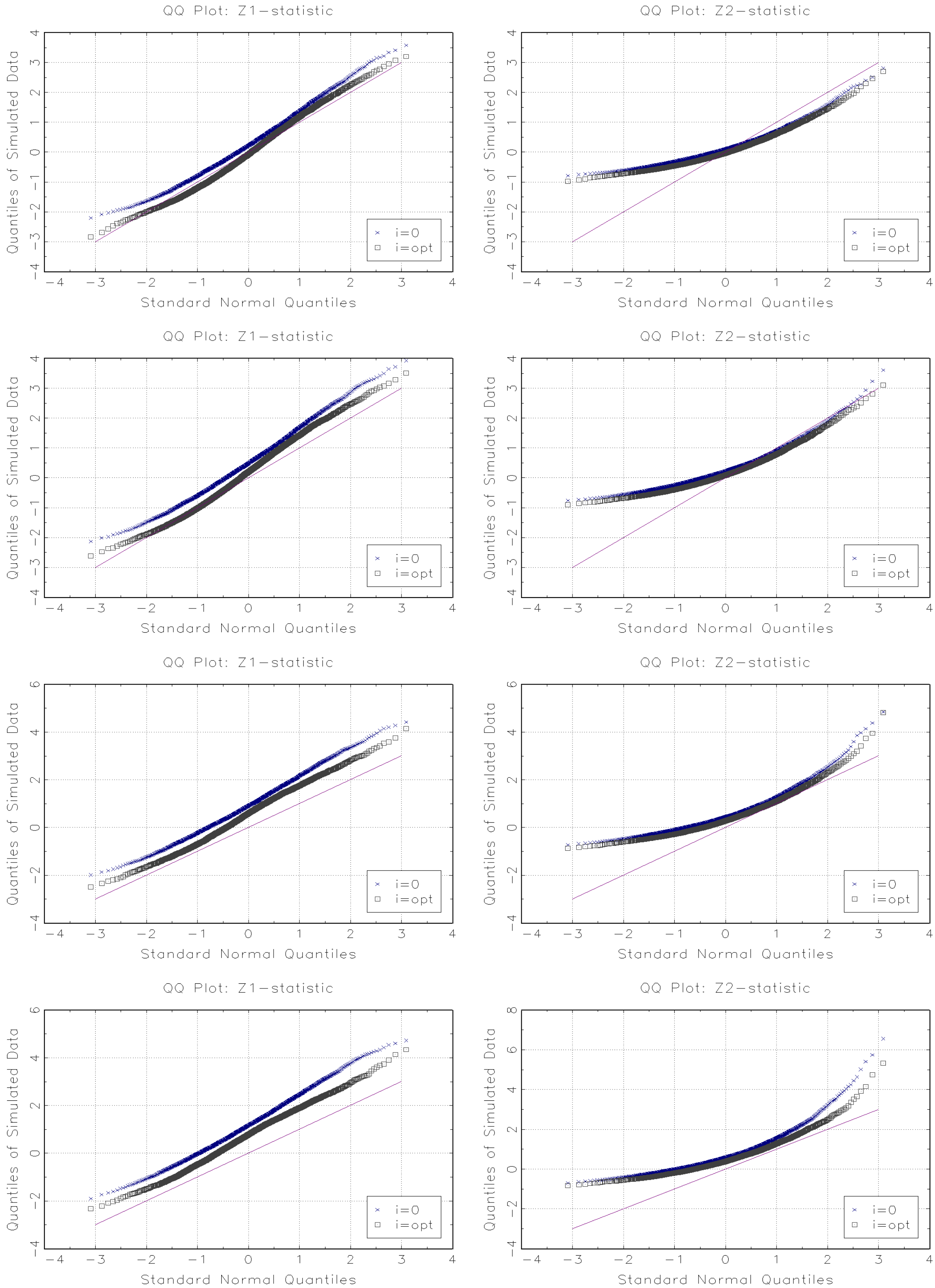
Remarks: Simulated realization of the BHM for 7650 days with parameter specifications, reported in table 3. Daily $Z1_{t,i}$ ($Z2_{t,i}$) statistic is graphed in the left (right) panels. From the top to bottom graph the fraction of zero-returns is factored by $\delta = \{0.2, 0.4, 0.8, 1\}$. The ordinate labels the quantiles of the simulated input sample, the abscissa the standard normal quantiles. The solid bisecting line graphs the theoretical result. The crosses (squares) represent the result for $i = 0$ ($i = opt$).

Figure 4: QQ plots of daily $Z1_{t,i}$ and $Z2_{t,i}$ statistic for $i = \{0, opt\}$ and 15min sampling intervals



<u>Remarks:</u> Simulated realization of the BHM for 7650 days with parameter specifications, reported in table 3. Daily $Z1_{t,i}$ ($Z2_{t,i}$) statistic is graphed in the left (right) panels. From the top to bottom graph the fraction of zero-returns is factored by $\delta = \{0.2, 0.4, 0.8, 1\}$. The ordinate labels the quantiles of the simulated input sample, the abscissa the standard normal quantiles. The solid bisecting line graphs the theoretical result. The crosses (squares) represent the result for $i = 0$ ($i = opt$).

Figure 5: QQ plots of daily $Z1_{t,i}$ and $Z2_{t,i}$ statistic for $i = \{0, opt\}$ and 30min sampling intervals



Remarks: Simulated realization of the BHM for 7650 days with parameter specifications, reported in table 3. Daily $Z1_{t,i}$ ($Z2_{t,i}$) statistic is graphed in the left (right) panels. From the top to bottom graph the fraction of zero-returns is factored by $\delta = \{0.2, 0.4, 0.8, 1\}$. The ordinate labels the quantiles of the simulated input sample, the abscissa the standard normal quantiles. The solid bisecting line graphs the theoretical result. The crosses (squares) represent the result for $i = 0$ ($i = opt$).

3.3 Price Process with Jumps

The primary matter of interest in this section is to analyze the accuracy of $Z1_{t,i}$ and $Z2_{t,i}$ using either the conventional or zero-adjusted estimation if the underlying data generating process equals to, with the infrequent trading algorithm transformed, *HMJ* in equation (18). More precisely, we are interested in the sensitivity of the results with respect to varying fractions of zero-returns, changing standard deviation of the jump size $\kappa(t)$ and frequency of jumps for fixed $\delta = 1$, and different sampling frequencies. The evaluation of the simulation results can be done quite intuitively with the classical confusion matrix, typically used in ROC analyses.[9] The setup of the confusion matrix given a predefined level of significance is the following. It is basically a (2×2)-matrix, where the upper left cell contains the proportion of days the test statistic correctly identified a no-jump day with respect to all simulated days with no jumps ($\widehat{=}$ *t-nj*). On the contrary, the upper right cell stands for the proportion of a false rejection of the test statistic among all simulated days with jumps, complying with the well known α-error. In the second row of the confusion matrix, we have in reverse to the first row in cell (2,1) the β-error, which is the proportion of falsely identified days with a jump among the total amount of simulated no-jump days. The lower right cell then represents the proportion of days the test statistic correctly identified a jump day with respect to all simulated days with jumps ($\widehat{=}$ *t-j*). By definition, the first and second column add up to one, respectively. Furthermore, the elements in the confusion matrix can be combined to compute the well known sensitivity index d' (from signal detection theory).[10] d' is quite convenient in determining the overall performance of the conventional estimation choosing $i = 0$ in contrast to our zero-adjusted estimation, $i = opt$. We decided on implementing this general performance measure, as we do not intend to advance a method with a high (no-)jump detection rate at any cost. In this setup, the larger d' the better the overall performance.[11]

3.3.1 Accuracy

The analysis of the accuracy of $Z_{t,i}$ for $i = 0$ and $i = opt$ is subdivided into four scenarios. The output of each simulation run is reported in table 4 to 7, respectively. To simplify matters, we only report *t-nj* and *t-j*. Generally, we choose a level of significance α of 1% as in Huang and Tauchen (2005).

Scenario 1: Small and rare jumps with changing fraction of zero-returns
In this scenario, we simulated a price process with the parameter settings utilized by Andersen,

[9] Huang and Tauchen (2005) report the transposed form of the classical confusion matrix.
[10] It is defined as: $d' = \Phi_{(t\text{-}nj)} - \Phi_{(1-t\text{-}j)}$.
[11] If $d' < 0$: method performs worse than guessing. If $d' = 0$: method performs as well as guessing. If $d' > 1$: method yields more strikes than false alarms. For details on d', the reader is referred to Wickens (2001).

Benzoni and Lund (2002). Leaving out the zero-returns, the basic setting seems representative for a financial asset with a rather low volatility, typical for country specific leading stock indices (e.g. S&P 500, DAX, FTSE, CAC40), and in principle also quite characteristic for the electricity forward contract more closely analyzed in Section 4. The simulated days with jumps count 435. The jumps reach values between roughly -0.04 to 0.04 in terms of returns. Starting with $Z1_{t,0}$, and the highest sampling frequency and fraction of zero-returns in table 4, we can state that the test statistic yields unsatisfactory results for the detection of no-jump days, whereas strong results for jump-days. In this context it means that almost every day, $Z1_{t,0}$ produced a value greater than the critical value of 2.33, yielding obviously strong results on detecting jump days and poor results on detecting no-jump days. The zero-adjusted estimation, however, greatly improves the detection rate of the large amount of days without jumps but loses accuracy in detecting the few days with jumps. This result is very much in line with our expectations from figure 3 to 5. Decreasing the factorized fraction of zero-returns from 1 to 0.8 for $i = 0$, we can observe a slow-moving improvement concerning the detection rate of no-jump days. The jump-detection rate rather slightly decreases but still remains on a high level. That means, the less $Z1_{t,0}$ is affected by the presence of zero-returns, the more is the test statistic affected in its precision by the chosen length of interval returns.[12] For $\delta = 0.4$, we already have a considerable

Table 4: Scenario 1: Small and rare jumps with varying zero-returns $\delta = \{1, 0.8, 0.4, 0.2\}$

		5min		15min		30min		5min		15min		30min	
	i	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*
		$\delta = 1$						$\delta = 0.8$					
$Z1_{t,i}$	t-nj	0.03	*0.53*	0.47	*0.84*	0.82	*0.92*	0.04	*0.54*	0.60	*0.90*	0.87	*0.94*
	t-j	1.00	*0.76*	0.88	*0.65*	0.68	*0.52*	0.99	*0.78*	0.86	*0.59*	0.67	*0.47*
$Z2_{t,i}$	t-nj	0.15	*0.74*	0.80	*0.95*	0.95	*0.97*	0.29	*0.86*	0.89	*0.98*	0.97	*0.98*
	t-j	0.98	*0.78*	0.78	*0.54*	0.53	*0.44*	0.95	*0.69*	0.69	*0.52*	0.50	*0.40*
		$\delta = 0.4$						$\delta = 0.2$					
$Z1_{t,i}$	t-nj	0.34	*0.84*	0.85	*0.97*	0.94	*0.97*	0.76	*0.96*	0.94	*0.99*	0.97	*0.98*
	t-j	0.96	*0.74*	0.79	*0.56*	0.60	*0.46*	0.91	*0.68*	0.77	*0.55*	0.61	*0.47*
$Z2_{t,i}$	t-nj	0.86	*0.99*	0.98	*1.00*	0.99	*0.99*	0.99	*1.00*	1.00	*1.00*	1.00	*1.00*
	t-j	0.86	*0.60*	0.63	*0.44*	0.46	*0.36*	0.79	*0.56*	0.61	*0.42*	0.43	*0.34*

Remarks: Simulated realization of the HMJ for 7650 days with basic parameter specifications, reported in table 3. Jumps are simulated with $\sigma_{jmp} = 0.0134$ and $\lambda_{jmp} = 0.058$. The level of significance α is set to 1%.

improvement of the detection rate of no-jump days, which further doubles for $\delta = 0.2$. Along the way, the jump detection rate looses accuracy but remains on a fairly high level. $Z1_{t,opt}$ likewise improves the detection rates of no-jump days and reduces the precision of detecting jump days for smaller δ. For longer sampling frequencies, the conventional estimation yields on the one hand much better detection rates of no-jump days than higher frequencies but on the

[12] Without any zero-returns, we obtain for $i = 0$ a jump-detection rate of 0.830 for 5 minute interval returns.

other hand worse detection rates of days with jumps out of time-averaging effects.[13] A similar effect can be observed for the zero-adjusted estimation.

From the right panels of figure 3 to 5 we know that $Z2_{t,0}$ differently "corrects" the test statistic for various δ and sampling frequency. As expected, $Z2_{t,0}$ produces better detection rates of no-jump days than $Z1_{t,0}$ for all sampling frequencies. However, due to the fact that $Z2_{t,0}$ is more conservative for especially small δ, the jump detection rate is smaller than for $Z1_{t,0}$. Also in line with the panels of $Z2_{t,opt}$ in figure 3 to 5 is that the zero-adjusted estimation only seems of interest for higher fractions of zero-returns. For smaller δ it turns out to be too conservative concerning the detection of jump days.

Scenario 2: Large and rare jumps with changing fraction of zero-returns

This scenario differs from the previous scenario with respect to the size of the jumps and number of jump days (here: 442 simulated jump days). For $\sigma_{jmp} = 0.1$ the simulated jumps reach values between roughly -0.3 to 0.3, speaking again in terms of continuously compounded interval returns. This is seven and a half times more than in the first scenario and is meant to be a rather extreme case scenario but still seems quite realistic for highly volatile single stocks. We would expect to receive an improvement of the correct jump detection rate for $Z_{t,0}$, notably for smaller δ's. This is true for all three sampling frequencies (see table 5). Similar in direction to *scenario 1* are the results of the no-jump detection rate as well as the overall results for $i = opt$.

Table 5: Scenario 2: Large and rare jumps with varying zero-returns $\delta = \{1, 0.8, 0.4, 0.2\}$

		5min		15min		30min		5min		15min		30min	
	i	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*
		$\delta = 1$						$\delta = 0.8$					
$Z1_{t,i}$	t-nj	0.02	*0.54*	0.47	*0.84*	0.81	*0.92*	0.04	*0.53*	0.60	*0.89*	0.86	*0.94*
	t-j	1.00	*0.87*	0.97	*0.86*	0.92	*0.86*	1.00	*0.93*	0.97	*0.89*	0.92	*0.85*
$Z2_{t,i}$	t-nj	0.14	*0.74*	0.80	*0.94*	0.94	*0.97*	0.28	*0.86*	0.89	*0.97*	0.97	*0.98*
	t-j	1.00	*0.95*	0.94	*0.90*	0.89	*0.87*	0.98	*0.94*	0.94	*0.90*	0.89	*0.87*
		$\delta = 0.4$						$\delta = 0.2$					
$Z1_{t,i}$	t-nj	0.35	*0.83*	0.85	*0.97*	0.94	*0.97*	0.77	*0.97*	0.94	*0.99*	0.97	*0.98*
	t-j	0.99	*0.90*	0.95	*0.89*	0.91	*0.86*	0.98	*0.92*	0.94	*0.88*	0.90	*0.87*
$Z2_{t,i}$	t-nj	0.87	*0.99*	0.98	*1.00*	0.99	*0.99*	0.99	*1.00*	1.00	*1.00*	1.00	*1.00*
	t-j	0.97	*0.90*	0.92	*0.88*	0.88	*0.85*	0.94	*0.90*	0.91	*0.88*	0.87	*0.86*

Remarks: Simulated realization of the HMJ for 7650 days with basic parameter specifications, reported in table 3. Jumps are simulated with $\sigma_{jmp} = 0.1$ and $\lambda_{jmp} = 0.058$. The level of significance α is set to 1%.

Scenario 3: High fraction of zero-returns with changing jump-variance

To be more comprehensive for cases of high fraction of zero-returns and differently distributed

[13] This result is in line with Aït-Sahalia (2004) who outlines the complication of detecting contributions from jumps to realized variance if the interval length of intraday returns increases. Further evidence can be found in the simulation study of Huang and Tauchen (2005).

jumps, we analyze scenarios with fix $\delta = 1$ and varying σ_{jmp} in more detail. Reading table 6 with the same systematic as for the previous scenarios, we can note that even for an extremely large and unrealistic σ_{jmp} of 0.25, the conclusions concerning the jump and no-jump detection rate for 5 minute returns do not systematically change in any direction for $i = 0$. For 15 minute

Table 6: Scenario 3: Variation of jump size $\sigma_{jmp} = \{0.0134, 0.05, 0.1, 0.25\}$

		5min		15min		30min		5min		15min		30min	
	i	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*
		$\sigma = 0.0134$						$\sigma = 0.05$					
$Z1_{t,i}$	*t-nj*	0.03	*0.54*	0.46	*0.84*	0.83	*0.92*	0.03	*0.53*	0.47	*0.85*	0.82	*0.92*
	t-j	1.00	*0.74*	0.90	*0.63*	0.71	*0.53*	1.00	*0.84*	0.96	*0.84*	0.90	*0.83*
$Z2_{t,i}$	*t-nj*	0.16	*0.74*	0.78	*0.94*	0.95	*0.97*	0.15	*0.73*	0.79	*0.94*	0.94	*0.97*
	t-j	0.97	*0.78*	0.78	*0.55*	0.54	*0.48*	0.99	*0.93*	0.93	*0.85*	0.87	*0.80*
		$\sigma = 0.1$						$\sigma = 0.25$					
$Z1_{t,i}$	*t-nj*	0.02	*0.53*	0.47	*0.85*	0.82	*0.92*	0.03	*0.54*	0.46	*0.85*	0.82	*0.92*
	t-j	1.00	*0.89*	0.98	*0.90*	0.96	*0.90*	1.00	*0.90*	0.99	*0.91*	0.98	*0.93*
$Z2_{t,i}$	*t-nj*	0.15	*0.73*	0.79	*0.94*	0.94	*0.97*	0.15	*0.74*	0.79	*0.95*	0.94	*0.97*
	t-j	1.00	*0.97*	0.96	*0.92*	0.91	*0.90*	1.00	*0.99*	0.99	*0.96*	0.97	*0.96*

Remarks: Simulated realization of the HMJ for 7650 days with basic parameter specifications, reported in table 3. Jumps are simulated with $\lambda_{jmp} = 0.058$ and changing standard deviation. Fraction of zero-returns is adjusted with $\delta = 1$. The level of significance α is set to 1%.

sampling intervals the simulation yields a stringent improvement of the jump detection rate with an increasing σ_{jmp} for $Z1_{t,0}$ and $Z2_{t,0}$. However, only $Z2_{t,0}$ yields for the no-jump detection rate a definite positive growth with higher σ_{jmp}. A non-restrictive increase of the jump detection rate with higher σ_{jmp} is also identifiable for the lowest sampling frequency. At the same time the no-jump detection rate decreases systematically with higher σ_{jmp} for both $Z1_{t,0}$ and $Z2_{t,0}$. However, by choosing $i = opt$ the increase of σ_{jmp} improves across all sampling frequencies the detection rate of jump days. At the same time *t-nj* remains on a stable level.

Scenario 4: Large jumps, high fraction of zero-returns with changing jump frequency
In the final scenario we experiment with changing jump frequency given large jumps and high fraction of zero-returns, i.e. $\sigma_{jmp} = 0.1$ and $\delta = 1$ (see table 7). Within the sample, a number of 462, 612, 854, 3007 days with jumps were simulated for the respective λ_{jmp}. Interestingly, the jump frequency does not really influence the overall picture of the jump and no-jump detection rate across either sampling frequency for both $i = 0$ and $i = opt$. Overall, we can summarize that the no-jump detection rate stays almost unchanged low for 5 minute, medium for 15 minute and quite high for 30 minute sampling intervals. For the jump detection rate, a reversed order applies.

So far, we have analyzed in several market scenarios how the conventional and zero-adjusted estimation perform concerning detecting days with and without jumps. Yet to be mentioned

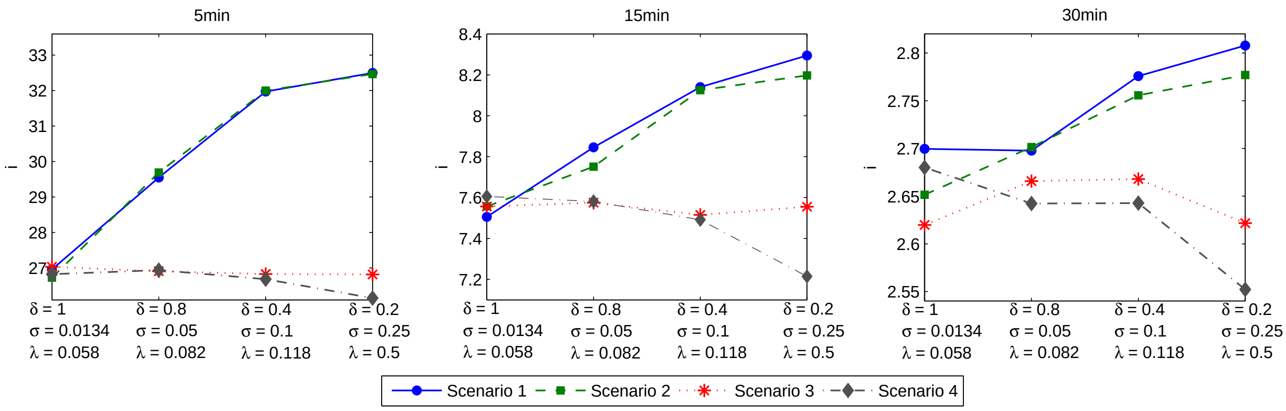
Table 7: Scenario 4: Variation of jump frequency $\lambda_{jmp} = \{0.058, 0.082, 0.118, 0.5\}$

		5min		15min		30min		5min		15min		30min	
	i	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*	0	*opt*
		$\lambda = 0.058$						$\lambda = 0.082$					
$Z1_{t,i}$	*t-nj*	0.02	*0.53*	0.47	*0.85*	0.82	*0.92*	0.03	*0.54*	0.47	*0.85*	0.82	*0.92*
	t-j	1.00	*0.89*	0.99	*0.91*	0.96	*0.88*	1.00	*0.87*	0.97	*0.87*	0.93	*0.88*
$Z2_{t,i}$	*t-nj*	0.16	*0.74*	0.79	*0.95*	0.94	*0.96*	0.15	*0.74*	0.79	*0.95*	0.94	*0.97*
	t-j	1.00	*0.98*	0.97	*0.95*	0.93	*0.92*	1.00	*0.95*	0.96	*0.91*	0.91	*0.91*
		$\lambda = 0.118$						$\lambda = 0.5$					
$Z1_{t,i}$	*t-nj*	0.02	*0.54*	0.47	*0.85*	0.82	*0.92*	0.02	*0.53*	0.47	*0.85*	0.82	*0.92*
	t-j	1.00	*0.89*	0.98	*0.91*	0.96	*0.88*	1.00	*0.86*	0.97	*0.86*	0.93	*0.83*
$Z2_{t,i}$	*t-nj*	0.15	*0.74*	0.79	*0.95*	0.94	*0.96*	0.15	*0.74*	0.78	*0.95*	0.94	*0.96*
	t-j	1.00	*0.97*	0.97	*0.94*	0.93	*0.92*	1.00	*0.94*	0.96	*0.90*	0.92	*0.87*

Remarks: Simulated realization of the HMJ for 7650 days with basic parameter specifications, reported in table 3. Jumps are simulated with $\sigma_{jmp} = 0.1$ and changing frequency. Fraction of zero-returns is adjusted with $\delta = 1$. The level of significance α is set to 1%.

is the actual choice of i by the zero-adjusted estimation, given a certain market scenario. For this, we graph in figure 6 the average of i for each scenario and sampling frequency. The left, middle and right panel in figure 6 represent 5, 15 and 30 minute sampling intervals. In *scenario 1* and *2* and across each sampling frequency we can observe that i increases for a higher trading activity. However, an increase in jump intensity negatively influences i, more for longer sampling intervals. Changes in jump size does not seem to have a clear impact on the choice of i. Relatively speaking, shorter sampling frequencies tend to choose on average larger i's than longer sampling frequencies. This becomes clear when we divide the average i by the corresponding M, which is 90/30/15 for 5/15/30 minutes sampling. Not explicitly reported in the graph is the minimum and maximum choice of i. For 5 minutes it amounts to 0 and 43, for 15 minutes 0 and 13, and for of 30 minutes 0 and 5.

Figure 6: Average i of zero-adjusted estimation



Remarks: In each panel, we graph for *scenario 1* to *4* the average of i, selected by the zero-adjusted estimation. The left, middle and right panel represent 5, 15 and 30 minute sampling intervals.

3.3.2 Performance

Based on the previously introduced market scenarios, we want to know to what extent the zero-adjusted estimation performs better or worse than the conventional one, by employing the overall performance measure d'. The findings are explicitly reported in table 8. Using 5 minute sampling intervals, the zero-adjusted estimation yields for $Z_{t,i}$ and across nearly all four scenarios a higher performance. One exception is for $\delta = 1$ in *scenario 1*. In some cases no d' could be computed, making a direct comparison impossible. A similar persistence in dominance can be confirmed for 15 minute sampling. Only for one simulated time series featuring less infrequent trading and small but rare jumps, the conventional estimation of $Z2_{t,0}$ superiorly separates jump and no-jump days. The dominance of the zero-adjusted estimation weakens for the lowest sampling frequency, as it yields performance measures for both approaches quite close to each other. In *scenario 1* it is not always of benefit to implement the zero-adjusted estimation, whereas in *scenario 2* it is. Overall, we can observe that in each scenario and across each sampling frequency $Z2_{t,i}$ outperforms $Z1_{t,i}$.

Our Monte Carlo experiment with the application of the zero-adjusted estimation on time series characterized by infrequent trading yields several interesting results. The approach is more conservative and works for both $Z1_{t,i}$ and $Z2_{t,i}$ using higher sampling frequency.

4 Case Study: Power Derivative

Supplementary to the results of the Monte Carlo experiment, we proceed in this section with a concrete empirical implementation. Our intent is to work with a time series which is economically substantial but is characterized by infrequent trading. Once more we are concerned with the question, how many price jumps are detected with the conventional estimation choosing $i = 1$ in relation to the zero-adjusted estimation, for different sampling frequencies.

4.1 Data

Before we come to the jump analysis, the setup of the dataset, and main issues concerning measurement procedures will be described in brevity. The dataset includes a unique time series of initial season and later on quarter electricity forward contracts traded at the Nord Pool Energy Exchange, covering a time period of more than six years.[14] The quarter contract is

[14] In short, an electricity forward contract tradable at the Nord Pool Energy Exchange is a standardized contract between two parties agreeing to "purchase/sell" electricity based on specific predetermined conditions (e.g. date, price, size). The contract only allows for cash settlement, i.e. the positive or negative difference between the forward price on the maturity day and the respective hourly Nord System spot electricity price in the delivery period will be credited either to the buyer or seller. The reader is referred to the Nord Pool Energy Exchange for further details.

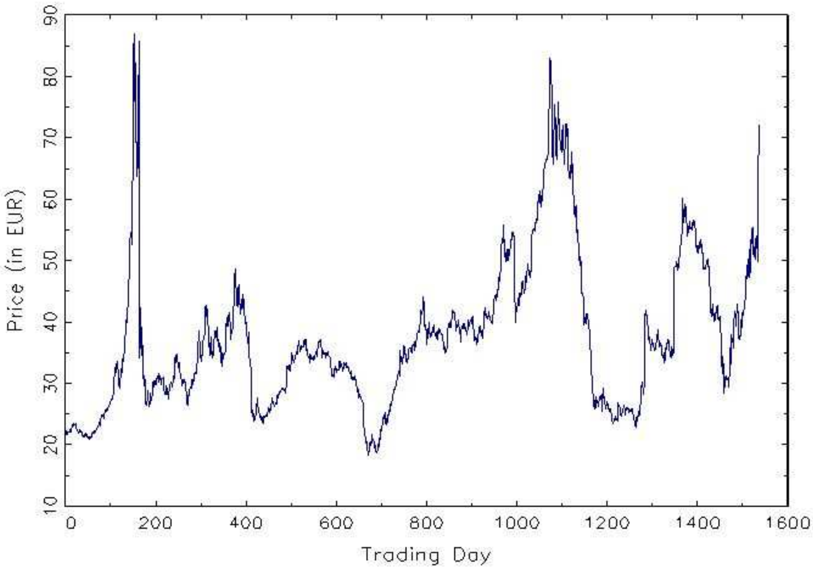
Table 8: Performance d' of the conventional and zero-adjusted estimation

	$Z1_{t,i}$						$Z2_{t,i}$					
	5min		15min		30min		5min		15min		30min	
	$i=0$	$i=opt$	$i=0$	$i=opt$	$i=0$	$i=opt$	$i=0$	$i=opt$	$i=0$	$i=opt$	$i=0$	$i=opt$
Scenario 1												
$\delta=1$	0.883	0.772	1.077	1.392	1.381	1.459	0.973	1.401	1.631	1.719	1.672	1.678
$\delta=0.8$	0.620	0.894	1.329	1.478	1.558	1.463	1.119	1.598	1.727	2.022	1.812	1.755
$\delta=0.6$	0.862	1.195	1.554	1.742	1.670	1.656	1.665	2.002	2.200	2.243	2.012	1.962
$\delta=0.4$	1.384	1.620	1.837	1.988	1.860	1.751	2.192	2.661	2.431	2.521	2.231	2.130
$\delta=0.2$	2.051	2.189	2.240	2.329	2.137	1.994	3.177	3.795	3.067	2.887	2.549	2.257
Scenario 2												
$\delta=1$	-	1.230	1.812	2.055	2.322	2.498	1.550	2.323	2.391	2.860	2.818	2.962
$\delta=0.8$	1.097	1.542	2.174	2.452	2.482	2.608	1.567	2.662	2.766	3.248	3.024	3.129
$\delta=0.6$	1.622	1.953	2.431	2.778	2.667	2.823	2.276	3.009	3.141	3.529	3.251	3.309
$\delta=0.4$	2.080	2.274	2.717	3.071	2.913	2.955	2.945	3.757	3.555	3.960	3.444	3.482
$\delta=0.2$	2.794	3.242	3.093	3.373	3.124	3.215	3.932	4.532	4.169	4.437	3.781	3.831
Scenario 3												
$\sigma_{jmp}=0.0134$	-	0.735	1.155	1.342	1.501	1.484	0.921	1.409	1.539	1.728	1.714	1.759
$\sigma_{jmp}=0.05$	0.897	1.081	1.623	2.019	2.188	2.396	1.197	2.075	2.281	2.617	2.699	2.670
$\sigma_{jmp}=0.1$	0.837	1.280	2.038	2.282	2.618	2.694	1.796	2.490	2.560	2.998	2.939	3.134
$\sigma_{jmp}=0.25$	-	1.363	2.373	2.386	2.911	2.862	1.803	2.945	3.028	3.368	3.430	3.621
Scenario 4												
$\lambda_{jmp}=0.058$	-	1.287	2.296	2.398	2.691	2.608	1.841	2.613	2.757	3.311	3.077	3.159
$\lambda_{jmp}=0.082$	0.767	1.216	1.858	2.165	2.401	2.590	1.915	2.252	2.559	2.932	2.927	3.147
$\lambda_{jmp}=0.118$	-	1.326	2.078	2.344	2.653	2.600	2.000	2.577	2.740	3.220	3.072	3.164
$\lambda_{jmp}=0.5$	0.847	1.154	1.863	2.101	2.385	2.380	1.550	2.224	2.540	2.915	2.907	2.916

Remarks: Simulated realization of the HMJ for 7650 days with basic parameter specifications, reported in table 3. The level of significance α is set to 1%.

designed for the replacement of the season contract. The first observation in the dataset is on May 3rd, 2002 and ends with the last observation on June 30th, 2008. In total, the time series contains 1536 active trading days with tick-by-tick transaction prices. The contracts are traded from 8:00am to 3:30pm only on weekdays. Its path is graphed in figure 7. We

Figure 7: Season-quarter electricity forward closing prices over the full-sample



Remarks: The solid line graphs closing prices (in €) for the season forward contract (realizations 1-841) and for the quarter forward contract (realizations 842-1534).

picked this time series due to several reasons. First, the time series is economically substantial. Speaking in terms of traded contract volume and terawatt hours (TWh), the quarter forward contract belongs to the most liquid category of derivative contracts. Among the offered variety of forward contracts, the quarter contract is the most heavily traded one. Furthermore, the contract is traded on a market place with favorable features: the Nord Pool Energy Market is the world's first international power exchange, the leading and most liquid power exchange in Europe, and the largest power derivatives exchange in the European Union.[15] The market offers both a physical and financial market. For further market specific information, the reader is referred to a discussion paper of Simonsen, Weron and Mo (2004) and the public appearance of the Nord Pool Energy Exchange. Besides, arguments concerning the advantages of the Nord Pool Market with respect to other European markets can be found in Amundsen and Bergman (2006).

Beyond its economic importance, the time series is still characterized by infrequent trading. Indicators for trading frequency are the previously introduced fraction of zero-returns for different sampling frequencies (see figure 2), number of trades per day, intertrade duration, and number of price changes per day. These features are reported in table 9. In table 9 we can

[15] An increasing importance can also be attributed to the most recent event as on October 22th 2008, NASDAQ OMX completed the acquisition of Nord Pool International AS.

ascertain that there is on average a considerable amount of trading activity over a trading day. But, if you compare the average intertrade duration of this time series with the ones of the individual stocks in the S&P100 for February 2006, reported in the paper of Bandi and Russell (2006, pp. 666-667), you can notice that the intertrade duration of the forward is roughly 3 to 100 times longer. Bearing this issue in mind, we now turn to determine realized

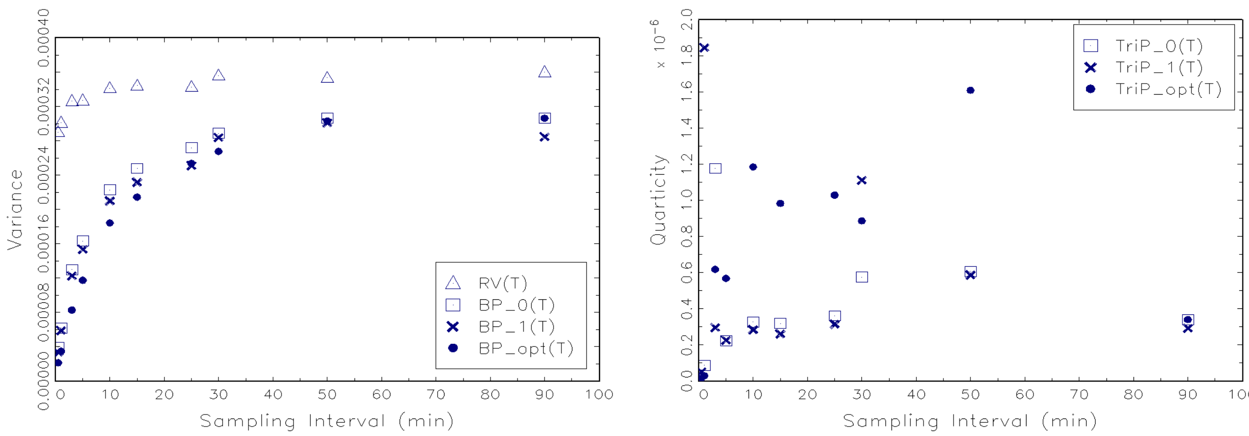
Table 9: Indicators for trading frequency

	Mean	Max	Min
Number of trades p.d.	169	868	7
Intertrade duration	3.16 min	3.64 h	1 s
Number of price changes p.d.	61	267	3

Remarks: Sample from May 2002 to June 2008. The original dataset separates each trade, even trades executed at the same time. The reported sample intertrade duration does not incorporate trades executed at the same trading time.

variance in equation (4) over a trading day by summing up squared returns, sampled at a sufficient small equidistant interval length. Problematic at this point is to determine the interval length because if you choose the sampling interval too small, you receive a highly distorted realized variance measure due to the dominant influence of microstructure noise. However, if you choose the sampling interval too long, you lose valuable information for realized variance but decrease the influence of microstructure noise. For this purpose, Bandi and Russell (2008) propose an analytical approach to identify an optimal equidistant interval length to compute log returns. This method determines the interval length by optimally balancing the continuous time arbitrage-free setup underlying the measure for realized variance and the troublesome effect coming from microstructure frictions. The application of this method to our time series produced an optimal sampling length of 15 minutes (or $M = 30$), conformable with the re-

Figure 8: Signature plot



Remarks: Left panel: each triangle-shaped realization in the graph represents the full-sample average of daily realized variance computed based on different return sampling intervals. Analogously, the quadratic/dagger/dot-shaped realizations graph the full-sample average of daily bipower variation computed according to equation (6) for $i = 0/1/opt$. Right panel: quadratic/dagger/dot-shaped realizations graph the full-sample average of daily tripower quarticity computed according to equation (10) for $i = 0/1/opt$.

sult of the variance signature plot in the left panel of figure 8.[16] The actual computation of interval returns follows in our case the previous tick method, theoretically discussed by Hansen and Lunde (2003, 2006). As the optimal sampling methodology of Bandi and Russell (2008) only applies to realized variance and not directly to realized bipower variation, we compute likewise a bipower variation plot, following the discussion of Andersen, Bollerslev, Frederiksen and Nielsen (2006). Additionally, we graphed a tripower quarticity signature plot in the right panel of figure 8. Based on the results of figure 8 we conducted the jump analysis additionally with 30 minute sampling intervals ($M = 15$). Out of sensitivity interests we varied also to 5 minute sampling intervals ($M = 90$).

4.2 Testing for Jumps

In this section we conduct the jump detection analysis using the conventional ($i = 1$) and the zero-adjusted estimation ($i = opt$) for $Z_{t,i}$, different levels of significance and sampling interval lengths. In the empirical analysis, we are confronted with the fact that the distribution of zero-returns is not uniformly distributed. There are trading days in the dataset with an extremely low trading activity causing, for specific choices of i, zero value for $BP_{t,i}$, thereby referring to the illustrative example in table 1. Obviously, if this happens, we have to exclude the day from the jump analysis. This kind of incident occurred 3 to 19 times, lowest for 30 minute sampling using the zero-adjusted estimation, to highest for 5 minute intervals using the conventional estimation. Furthermore, the average/minimum/maximum choice of $i = opt$ is 27.6/0/43 for 5 minute, 8.0/0/13 for 15 minute, and 2.9/0/5 for 30 minute sampling intervals. These results are very similar to our simulation. In table 10 we report the proportion of detected jump days with respect to all considered trading days. To interpret the empirical results, we compare them first with a simulation experiment most feasible. One appropriate scenario is

Table 10: Proportion of significant jump days for the conventional and zero-adjusted estimation

	$\alpha = 1\%$			$\alpha = 0.1\%$			$\alpha = 0.01\%$		
	5min	*15min*	*30min*	*5min*	*15min*	*30min*	*5min*	*15min*	*30min*
$\boldsymbol{Z1}_{t,i}$									
$i = 1$	0.887	0.436	0.182	0.757	0.248	0.065	0.626	0.156	0.020
$i = opt$	0.508	0.177	0.077	0.170	0.030	0.020	0.044	0.012	0.006
$\boldsymbol{Z2}_{t,i}$									
$i = 1$	0.604	0.170	0.058	0.438	0.088	0.020	0.343	0.058	0.011
$i = opt$	0.270	0.054	0.031	0.106	0.022	0.010	0.043	0.011	0.005

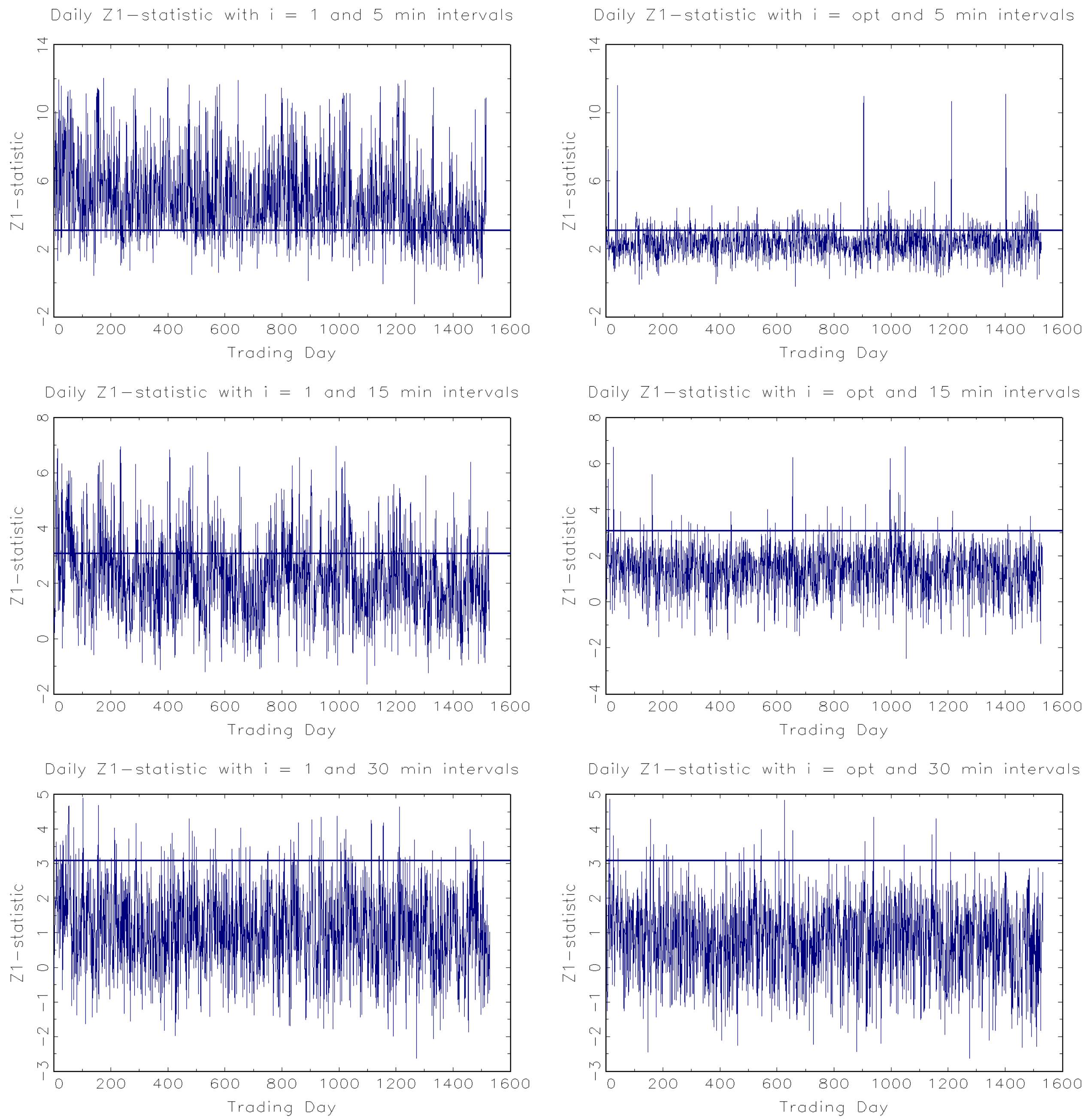
Remarks: Sample from May 2002 to June 2008.

[16] Referring to Andersen, Bollerslev, Diebold and Labys (1999), a rough indication for the optimal sampling frequency is the highest possible sampling frequency at which the corresponding total average of realized variance does not systematically differ from longer sampling frequencies. Besides they mention that the shape of the variance signature plot as of the right panel in figure 8 is typical for illiquid financial assets.

scenario 1 with the parameter specifications $\delta = 1$, $\sigma_{jmp} = 0.0134$ and $\lambda_{jmp} = 0.058$, due to its similarity in distribution of interval returns. Detailed, we compare the empirical ratio of detected jump days to all trading days in table 10 with the derivable simulated ones in table 4.[17] The empirical analysis likewise yields for $i = 1$ and $\alpha = 1\%$ a high ratio of jump days, highest for 5 minutes. Moreover conformable with the simulation, $Z2_{t,i}$ is emerging to be more conservative on detecting jump days than $Z1_{t,i}$. The application of the zero-adjusted estimation greatly reduces the ratio of jump days, in line with the simulation results in table 10. We graphically processed this effect in figure 9, which visualizes in the left panels the daily test statistic for $Z1_{t,i=1}$ and in the right panels $Z1_{t,i=opt}$ for 5, 15 and 30 minute sampling frequencies. Here we can clearly observe to what extent the zero-adjusted estimation is shifting the daily test statistic downwards over the full-sample. Beyond that, these results can be understood as an improvement in general precision of the test statistic, referring to the simulation result of table 8. Interesting is also the comparison of our empirical results with the ones of Andersen, Bollerslev and Diebold (2007) who analyzed the foreign exchange spot market of German DM/U.S. dollar, equity futures market of U.S. S&P 500 index, and interest rate futures market of thirty-year U.S. Treasury yield. In their extensive analysis using 5 minute sampling intervals, they obtained far smaller ratios. Despite similarities in stylized facts of the mentioned time series to the electricity forward, this result is not surprising as the electricity forward is highly exposed to zero-returns. To further grasp the advances of the zero-adjusted estimation, we briefly analyze the results more closely for the optimal sampling frequency of 15 minutes. Therefore, we initially analyze the characteristic of trading days with $Z1_{t,i=1} \geq 6$. Referring to the limit consideration of $Z1_{t,i}$ in Section 2.4 (equaling 7.02 for 15 minute interval returns), we want to know whether days with large $Z1_{t,i}$ values show a high fraction of zero-returns, and therefore likely yield non-plausible conclusions about jumps. Our analysis shows that these days are characterized by under average trading activities, i.e. above average fraction of zero-returns (here: 68.02%). Besides, there is mostly no clear indication of abnormal price movements, speaking in terms of unusually large interval returns. Second, we track these days after implementing the zero-adjusted estimation, i.e. we want to know whether they are still considered as days with jumps. We obtain that most of these days have test statistic values smaller than the quantile function at $\alpha = 0.1\%$ – a result clearly more plausible and militating in favor of our zero-adjusted estimation. Very similar are the results when we conduct the analysis for $Z2_{t,i} \geq 6$, as the trading days of $Z1_{t,i} \geq 6$ coincide with those of $Z2_{t,i} \geq 6$. A final note is that it remains difficult to decide on $Z2_{t,i}$ outperforming $Z1_{t,i}$, and vice versa.

[17]To derive the ratio of detected jump days to all trading days in table 4, compute the following equation: $(435 \times$ '*t-j*' $+ (7650 - 435) \times (1 -$ '*t-nj*'$))/7650$.

Figure 9: Daily $Z1_{t,i}$ statistic for 5, 15, 30 minute intervals and $i = 1, opt$



Remarks: Upper left and upper right panel are based on 5min sampling intervals, middle left and middle right are based on 15min sampling intervals, and lower left and lower right are based on 30min sampling intervals. Left (right) panels graph daily $Z1_{t,i}$ statistic for $i = 1$ ($i = opt$). The solid horizontal line graphs in each panel the level of significance for $\alpha = 0.1\%$.

5 Conclusion

In this article we studied in depth the effect of infrequent trading on an elaborated jump detection methodology for realized variance by Barndorff-Nielsen and Shephard (2004a, 2006). Analytically, we showed in detail that the considered test statistics are positively distorted if

a certain level of zero-returns causes daily realized bipower variation to be relatively small to realized variance, and tripower quarticity to squared realized bipower variation being less or equal than one. Besides this extreme case, we studied on the one hand how sensitive the limit results of the test statistics are, and on the other hand to what extent the jump and no-jump detection rate is influenced by infrequent trading. In a Monte Carlo experiment we proposed an algorithm to simulate infrequent return series with and without jumps on the basis of the Heston model in order to discuss the research questions for various market scenarios.
The simulation results show first and foremost that given a certain level of trading activity, higher sampling frequencies cause a more intense distortion towards overrejection than for longer sampling frequencies in both test statistics. This effect manifests for lessening trade activities. In other words, the theoretical limit results of the test statistics are quite sensitive with respect to a small increase in the fraction of zero-returns. Secondly, we find that the jump detection rate is primarily negatively influenced by longer sampling intervals and a decreasing fraction of zero-returns. The detection rate for days without jumps is, on the contrary, positively affected by longer sampling intervals and a decreasing fraction of zero-returns. Third, we proposed a more conservative zero-adjusted estimation, which in realistic market scenarios performs better than the conventional estimation, especially for short sampling intervals. The zero-adjusted estimation, which is based on the theory of Barndorff-Nielsen and Shephard (2004a, 2006), improves the validity of two considered jump detection test statistics in case of infrequent trading.
Adjacent to the Monte Carlo study we provide a new empirical investigation using an economically substantial time series, the most heavily traded electricity forward contract of the Nord Pool Energy Exchange. Herein, we can corroborate the evidence of a corresponding simulated market scenario - small and rare jumps - to a considerable extent. Furthermore, we refer to the reduction in bias using our zero-adjusted estimation.
Our simulation study and empirical analysis raise further interesting research questions. It would be of interest to work on additional approaches which are robust against zero-returns. This can be either done based on existing price jump detection methodologies or by introducing a new method. Moreover of interest is to establish additional plausibility checks for price jumps in empirical analyses.

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