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by

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May 2000

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Abstract: We study the quantitative effects of German income tax reform proposals in a general equilibrium model with elastic labor supply. Agents are heterogenous with regard to their productivity and their assets. Two tax reform proposals are considered: i) a flat-rate tax and ii) a consumption tax. Total government expenditures are assumed to remain constant. The model is calibrated with regard to the German economy in 1996. In particular, the endogenous labor income distribution as computed from our model is equal to the empirical labor income distribution in Germany. As our first main result, both reform proposals are shown to have only negligible effects on labor income distribution. Second, both tax reform proposals results in a strong increase of both aggregate employment and aggregate savings. And third, both reform proposals imply significant steady-state welfare gains equivalent to a rise of total consumption of 7.3% and 14.9%, respectively.

Keywords: Income Taxation, Consumption Taxation, Income Distribution, Flat-Rate Tax

JEL classification: D33, E62, H23, H24

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1 Introduction

This paper studies the efficiency and distribution effects of two tax reform proposals which have been discussed prominently in recent German policy debates: 1) a flat-rate income tax reform and 2) a switch from the income tax to a consumption tax. In Germany, the income tax is progressive even though the recent tax reforms in the years 1986, 1988, 1990, and 1998 have helped to reduce marginal tax rates.[1] The effects of a progressive income tax on efficiency and welfare are not straightforward, however, and critically depend on the labor market structure. In competitive labor markets, which will be the focus of the present paper, progressive income taxation reduces the labor supply of the very productive worker on the one hand and the savings of the wealth-rich on the other hand which will result in smaller aggregate labor supply, capital accumulation, and income.[2] A flat-rate tax, however, increases income inequality and, hence, decreases welfare. The net effect on welfare can only be evaluated numerically. Our second focus is the analysis of a shift from income taxation to consumption taxation. This proposal has also received considerable attention in recent fiscal policy studies.[3] Contrary to the income tax, a consumption tax is neutral with regard to the intertemporal allocation as it does not tax interest income.

We develop an intertemporal general equilibrium model which is calibrated with regard to the characteristics of the German economy. In particular, we closely represent the present German personal income tax schedule in our benchmark calibration. Contrary to previous studies of German tax reform proposals,[4] we also develop a model of income inequality and income mobility. In particular, agents face uninsurable idiosyncratic productivity shocks in our economy and, between periods, individual productivity may change. As a consequence, individual labor income also changes. Our general equilibrium model is able to account for both the observed heterogeneity in wage rates and the observed labor income mobility. In addition, we model the household's labor supply decision. As a consequence, the labor income distribution is endogenous. As one major implication of our modelling framework, we are able replicate the German labor income distribution quite closely.

In our model, labor adjusts only along the intensive margin as agents change their supply

[1]As a second major impact, the recent tax reforms have also relieved the tax burden on the low-income families.

[2]In the presence of non-Walrasian labor markets, however, a progressive income tax does not necessarily increase employment. See, e.g., Lockwood/Manning (1993) for both theoretical and empirical evidence.

[3]See, e.g., Rose et al. (1988) and Sinn (1987).

[4]Previous work in applied general equilibrium analysis includes Conrad/Henseler-Unger (1988), Conrad (1990), and Keuschnigg (1994). As one exemption, Fehr (1999) also considers distributional issues of tax reform proposals. In his overlapping generations model, however, he does not consider income mobility and households do not change their productivity type during their finite lifes.

of working hours. Recent computable general equilibrium studies with heterogeneous-agent economies have also emphasized the effects of public policy on aggregate employment. E.g., Heer (1999) shows in a model of search unemployment that an increase of unemployment benefits results in a rise of the unemployment rate. We do not analyze the adjustment of labor along the extensive margin in our model, which we consider to be a fruitful area for further research, but rather take the unemployment rate as given. However, contrary to similar studies of the US economy such as Castañeda et al. (1998) or Ventura (1999), we consider the risk of unemployment in our model. Agents face an exogenous probability both to become and to remain unemployed. During unemployment, agents receive unemployment benefits and loose their skills.

We will only consider revenue-neutral tax reforms and keep government consumption constant. In our first tax experiment, we compute the flat-rate income tax rate which implies the same tax revenues as in our benchmark economy keeping the consumption tax rate constant. Welfare is measured by the average life-time utility of all households in the economy. As our first main result, we show that a flat-rate tax increases aggregate employment and savings as well as average welfare. Moreover, both the unemployed worker and the low-productivity worker benefit from the introduction of a flate-rate income tax. In our second tax experiment, we set the income taxes to zero and compute the consumption tax rate which implies the same tax revenues as in the benchmark model with the present German tax structure. As our second main result, we find that an increase of the consumption tax which is offset by a reduction of the income tax rate mainly boosts savings, but also increases aggregate employment. The distribution of wealth becomes much more unequal. Again, aggregate welfare increases and both the unemployed workers and the low-productivity worker of each decentile of the wealth distribution benefit from a tax reform.

The organisation of the paper is as follows. Section 2 introduces the model. In section 3, the model is calibrated with regard to characteristics of the German economy. Furthermore, the computational procedure is described. In section 4, our numerical results are presented. Section 5 concludes.

2 The model

The model is based on the stochastic neoclassical growth model with elastic labor supply and idiosyncratic risk, augmented by a government sector. Agents are subject to idiosyncratic productivity and employment shocks which they are not able to insure against; however, there is no aggregate uncertainty in the economy. Three different sectors are depicted: households, firms, and the government. Households maximize discounted life-time utility with regard to their intertemporal consumption and labor supply. Firms maximize their profits and produce with constant returns to scale using labor and capital as inputs. The government taxes income and spends the revenues on government consumption, unemployment compensation, and transfers.

Households

Households are of measure one and infinitely-lived. Households are heterogeneous with regard to their employment status, their productivity ϵ^j, and their wealth k^j, $j \in [0,1]$.[5] We assume productivity ϵ to take a value from the finite set $\mathcal{E} = \{\epsilon^1, \epsilon^2, \ldots, \epsilon^{n\epsilon}\}$, where $\epsilon^1 = 0$ describes the state of unemployment. We assume that productivity follows a first order finite state Markov chain with conditional transition probabilities given by:

$$\pi(\epsilon'|\epsilon) = Pr\{\epsilon_{t+1} = \epsilon'|\epsilon_t = \epsilon\}, \tag{1}$$

where $\epsilon, \epsilon' \in \mathcal{E}$. Although the dynamics of productivity may be modelled slightly better by a second order Markov chain (Shorrocks, 1976) the improvement in accuracy is rather small and does not justify the considerable increase in the model's complexity.

Household j, which is characterized by productivity ϵ_t^j and wealth k_t^j in period t, maximizes his intertemporal utility with regard to consumption c_t^j and labor supply n_t^j:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^j, 1 - n_t^j), \tag{2}$$

where $\beta < 1$ is a discount factor and expectations are conditioned on the information set of the household at time 0. Instantaneous utility $u(c_t, 1 - n_t)$ is assumed to be addititively separable in the utility from consumption and the utility from leisure as given by:[6]

$$u(c_t, 1 - n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \gamma_0 \frac{(1-n_t)^{1-\gamma_1}}{1-\gamma_1}. \tag{3}$$

Agents are not allowed to borrow, $k^j \geq 0$. In addition, the household faces a budget constraint. He receives income from labor n_t and capital k_t as well as government transfers tr_t which are spent on consumption c_t and next-period wealth k_{t+1}:

$$k_{t+1}^j = (1+r)k_t^j + w_t n_t^j \epsilon_t^j - (1+\tau_c)c_t^j - \tau(y_t^j) + tr_t + 1_{\epsilon=\epsilon^1} b_t, \tag{4}$$

[5] As we only consider one type of asset, we will refer to k as capital, wealth, and asset interchangeably.

[6] Our choice of the functional form for utility follows Castañeda et al. (1998). Most quantitative studies of general equilibrium model specify a Cobb-Douglas functional form of utility. In this case, however, the elasticity of individual labor supply with regard to wealth is larger than for the utility function (3) and, consequently, the distribution of both labor income and wealth are even more homogenous than for our choice of the utility function (3).

where r_t, w_t, τ_c, and $\tau(y)$ denote the interest rate, the wage rate, the consumption tax rate, and the taxes on income y, respectively. $1_{\epsilon=\epsilon^1}$ is an index function which takes the value one if the household is unemployed ($\epsilon = \epsilon^1$) and zero otherwise. If the agent is unemployed, he receives unemployment compensation b_t. Taxable income is composed of interest income and labor income:

$$y_t^j = y_t^j(\epsilon_t^j, k_t^j) = rk_t^j + w_t n_t^j \epsilon_t^j. \tag{5}$$

Production

Firms are owned by the households and maximize profits with respect to their labor and capital demand. Production $F(K_t, N_t)$ is characterized by constant returns to scale using capital K_t and labor N_t as inputs:

$$F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}. \tag{6}$$

In a market equilibrium, factors are compensated according to their marginal products and profits are zero:

$$r_t = \alpha \left(\frac{N_t}{K_t}\right)^{1-\alpha} - \delta, \tag{7}$$

$$w_t = (1-\alpha) \left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{8}$$

where δ denotes the depreciation rate of capital.

Government

Government expenditures consists of government consumption G_t, government lump-sum transfers Tr_t to households, and unemployment compensation B_t.[7] In our benchmark case, government expenditures are financed by an income tax and a consumption tax. The latter is proportional to consumption. The income tax structure is chosen to match the current income tax structure in Germany most closely. In particular, the income tax is comprised of M different tax brackets $y_t \in (Y_{m-1}, Y_m]$ with corresponding marginal tax rates τ_m, $m = 1, 2, \ldots, M$. An agent with income $y_t \in (Y_{m-1}, Y_m]$ pays the amount of taxes (**tax policy i**):

$$\tau(y_t) = \tau_1(Y_1 - Y_0) + \tau_2(Y_2 - Y_1) + \ldots + \tau_m(y_t - Y_{m-1}). \tag{9}$$

[7]Government consumption does not have any effect on either utility nor production. In this paper, we hold government consumption fixed and only analyze revenue-neutral tax reforms. The reason to include government consumption is to get a realistic value of lump-sum transfers Tr. Without government consumption, wealth inequality would be lower.

The marginal tax rates τ_i and the thresholds Y_i will be computed below from the current income tax structure. In particular, the German income tax structure is progressive with $\tau_i < \tau_{i+1}$ for all $i = 1, \ldots, M-1$.

We will compare the employment, distribution, and welfare effects of the current tax structure with the effects of two other tax structures, (i) a flat-rate income tax structure and (ii) a consumption tax. In the former case, taxes are a proportional function of income (**tax policy ii**):

$$\tau(y_t) = \tau_y y_t, \tag{10}$$

In the second case, a consumption tax $\tau_c c_t$ is imposed on individuals, while $\tau(y_t) = 0$ (**tax policy iii**). The two alternative tax policy regimes are calibrated in order to yield the same stationary tax revenues T_t as the current income tax structure.

The government budget is assumed to balance in every period so that government expenditures are financed by tax revenues T_t in every period t:

$$G_t + Tr_t + B_t = T_t. \tag{11}$$

Stationary Equilibrium

The concept of equilibrium used in this paper uses a recursive representation of the consumer's problem following Stokey et al. (1989). In the following, we concentrate on the study of a stationary equilibrium and drop time subscripts. The household's state variable is denoted by $x = (\epsilon, k) \in \mathcal{X}$. Let $V(\epsilon, k)$ be the value of the objective function of a household characterized by productivity ϵ and wealth k. $V(\epsilon, k)$ for the benchmark tax policy is defined as the solution to the dynamic program:

$$V(\epsilon, k) = \max_{c,n,k'} \left[u(c, 1-n) + \beta E \left\{ V(\epsilon', k') \right\} \right], \tag{12}$$

where ϵ' and k' denote next periods productivity and wealth, subject to the budget constraint (4), the tax policy (9) and the stochastic mechanism determining the productivity level (see below).

Let $(\mathcal{X}, \mathcal{B}, \psi)$ be a probability space where $\mathcal{B}$ is a suitable σ-algebra on $\mathcal{X}$ and ψ a probability measure. We will define a stationary equilibrium for given government tax policy and stationary measure ψ.

Definition

A stationary equilibrium for a given set of government policy parameters is a value function $V(\epsilon, k)$, individual policy rules $c(\epsilon, k)$, $n(\epsilon, k)$, and $k'(\epsilon, k)$ for consumption, labor supply, and next-period capital, respectively, a time-invariant distribution of the state variable $x = (\epsilon, k) \in \mathcal{X}$, time-invariant relative prices of labor and capital $\{w, r\}$, and a vector of aggregates K, N, Tr, B such that:

1. Factor inputs, consumption, tax revenues, transfers, and unemployment compensation are obtained aggregating over households:

$$\begin{aligned} K &= \int_{\mathcal{X}} k d\psi &(13)\\ N &= \int_{\mathcal{X}} \epsilon n(\epsilon, k) d\psi &(14)\\ C &= \int_{\mathcal{X}} c(\epsilon, k) d\psi &(15)\\ T &= \int_{\mathcal{X}} \tau\left(y(\epsilon, k)\right) d\psi + \tau_c C &(16)\\ Tr &= tr &(17)\\ B &= \int_{\mathcal{X}} 1_{\epsilon=0} b d\psi, &(18) \end{aligned}$$

2. $c(\epsilon, k)$, $n(\epsilon, k)$, and $k'(\epsilon, k)$ are optimal decision rules and solve the household decision problem described in (12).

3. Factor prices (7) and (8) are equal to the factors' marginal productivities, respectively.

4. The goods market clears:

$$F(K, L) + (1 - \delta)K = C + K' + G = C + K + G. \tag{19}$$

5. The government budget (11) is balanced.

6. The measure of households is stationary:

$$\psi(B) = \int_{\mathcal{X}} 1_{(\epsilon', k'(\epsilon, k)) \in B} \pi(\epsilon' | \epsilon) d\psi \tag{20}$$

for all $B \in \mathcal{B}$.

Since the household's decision problem is a finite-state, discounted dynamic program, an optimal stationary Markov solution to this problem always exists.

3 Calibration and computation

3.1 Calibration

In order to compute the quantitative effects of the different fiscal policy regimes on output, employment, distribution, and welfare, the model has to be calibrated. The model parameters are chosen with respect to the characteristics of the German economy. Model periods correspond to years. Data are mainly from the quarterly national account statistics of the German Institute for Economic Research (DIW), Berlin. The annual data on the unemployment rate is taken from the yearbooks of the *German Statistical Office (Statistisches Bundesamt).* Data on the wage and income distribution is taken from the Socio-Economic Panel for Germany (GSOEP).

Utility

For the utility function parameters, we chose the usual discount rate of $\beta = 0.96$ and set σ equal to 2. The parameters γ_0 and γ_1 are chosen in order to imply (i) an average working time of approximately 1/3 and (ii) a coefficient of variation of workers' labor supply equal to the empirical value. Using data from the Socio-Economic Panel, we estimated a coefficient of variation of 0.385 during 1995-96. For our benchmark case with the utility parameters $\gamma_0 = 0.13$ and $\gamma_1 = 10$, the average labor supply of the workers is equal to 0.32, while the coefficient of variation amounts to 0.36. Our calibration is summarized in table 1.

Productivity

The productivities $\epsilon \in \mathcal{E} = \{\epsilon^1, \ldots, \epsilon^{n\epsilon}\}$ are chosen to replicate the discretized distribution of hourly wage rates which according to (4) are proportional to productivity. The number of productivities is set equal to $n\epsilon = 5$. ϵ^1 characterizes the state of unemployment and is set equal to zero. The productivities $\{\epsilon^2, \epsilon^3, \epsilon^4, \epsilon^5\}$ are estimated from the empirical distribution of hourly wages in Germany (1995). The productivity ϵ^i corresponds to the average hourly wage rate of earners in the $(i-1)$-th quartile. Normalizing the average of the four nonzero productivities to unity we arrive at

$$\{\epsilon^2, \epsilon^3, \epsilon^4, \epsilon^5\} = \{0.4476, 0.7851, 1.0544, 1.7129\}. \tag{21}$$

The transition probability into and out of unemployment, $\pi(\epsilon' = 0|\epsilon > 0)$ and $\pi(\epsilon' > 0|\epsilon = 0)$ where ϵ' represents next period's productivity, are chosen in order to imply an average unemployment rate of 10.95% and an average duration of unemployment equal to slightly more than one year (we assume that the average transition takes place in the middle of the year). Further, we assume that the probability to loose one's job does not depend on the individual productivity. During unemployment, the worker's human capital depreciates or, equivalenty, his productivity decreases. We assume that the worker can only reach productivity ϵ^2 after

Table 1: Calibration of parameter values for benchmark case

Description	Function	Parameter
utility function	$u_t = \frac{c_t^{1-\sigma}}{1-\sigma} + \gamma_0 \frac{(1-n)^{1-\gamma_1}}{1-\gamma_1}$	$\sigma = 2$, $\gamma_0 = 0.13$, $\gamma_1 = 10$
discount factor	β	$\beta = 0.96$
production function	$F(K, N) = K^\alpha N^{1-\alpha}$	$\alpha = 0.36$
depreciation	δ	$\delta = 0.04$
government consumption	$\bar{G} = \gamma_g F(\bar{K}, N)$	$\gamma_g = 19.6\%$
unemployment compensation	b	$b = 0.50\epsilon^2 w\bar{n}^2$
consumption tax rate	τ_c	$\tau_c = 15\%$

unemployment and set $\pi(\epsilon' = \epsilon^2|\epsilon = 0) = 1 - \pi(\epsilon' = 0|\epsilon = 0)$ and $\pi(\epsilon' > \epsilon^2|\epsilon = 0) = 0$.[8] The remaining $(n\epsilon - 1)^2 = 16$ transition probabilities are calibrated such that (i) each row in the Markov transition matrix sums to one, (ii) the model economy matches the observed quartile transition probabilities of the hourly wage rate from 1995 to 1996 as given by the GSOEP data. Our approach to the issue of mobility is hence markedly different from Castañeda et al. (1998) who calibrate the transition matrix is order to replicate the U.S. earnings and wealth distribution as closely as possible. As a consequence, the diagonal elements of the transition matrix calibrated by Castañeda et al. (1998) are far larger than the empirical counterparts.

Our transition matrix is given by:

$$\pi(\epsilon'|\epsilon) = \begin{pmatrix} 0.3500 & 0.6500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0800 & 0.6751 & 0.1702 & 0.0364 & 0.0383 \\ 0.0800 & 0.1651 & 0.5162 & 0.2003 & 0.0384 \\ 0.0800 & 0.0422 & 0.1995 & 0.5224 & 0.1559 \\ 0.0800 & 0.0371 & 0.0345 & 0.1606 & 0.6879 \end{pmatrix} \tag{22}$$

Tax function

The benchmark case is calibrated in order to approximate the features of the income tax code in Germany in 1996. For this reason, tax income thresholds Y_i, $i = 1, \ldots, M$, relative to the

[8] Alternatively, we could have assumed that the worker's productivity does not decrease during unemployment. In this case, however, we had to introduce an additional state variable into the model which makes the computation and calibration even more cumbersome.

average income are chosen to match the values of their empirical counterparts. The marginal tax rates τ_i, $i = 1, \ldots, M-1$, are chosen to reproduce the effective average tax payments of every tax income bracket in Germany, considering that there exists many complex exemptions and deductions in the German tax code. The tax code of our model economy is described in table 2.

Table 2: Tax code

Taxable income relative to average income	Marginal income tax rate τ_i
$[0, 0.20]$	0%
$]0.20, 0.33]$	20%
$]0.33, 0.50]$	30%
$]0.50, 1.00]$	32%
$]1.00, 1.25]$	36%
$]1.25, 1.50]$	42%
$]1.50, 2.00]$	48%
> 2.00	54%

The consumption tax rate τ_c is set equal to 15%.

Production

The production elasticity of capital, $\alpha = 0.36$, and the annual rate of capital depreciation, $\delta = 0.04$, are taken from Heer and Linnemann (1998).

Government Expenditures

Government consumption G is calibrated in order to imply a government consumption share in output $G/F(K, N)$ equal to 19.6%.[9] Unemployment compensation b is set equal to 50% of the average wage of the lowest productivity workers, $\epsilon^2 \bar{n}^2 w$, net of income taxes. The replacement ratio of 50% is taken from Steiner (1997).[10] Transfers Tr are calculated from the government budget (11) and amount to approximately 10.4% of total income.

[9] This value is taken from the monthly report of the *Deutsche Bundesbank*.

[10] Alternatively, we could have assumed that unemployment compensation depends on the previous labor income of the unemployed worker. The main reason for our simplifying assumption is to keep the number of state variables in our model to a minimum.

3.2 Computational method

The model has no analytical solution. Algorithms to solve heterogenous-agent models with an endogenous distribution have only recently been introduced into the economic literature. Notable studies in this area are Aiyagari (1994,1995), den Haan (1996), Huggett (1993), İmrohoroğlu et al. (1995), and Krussell/Smith (1998). Like most of these studies, we will only focus on the steady state of the model.[11] The solution algorithm for the benchmark case with progressive income taxation is described by the following steps:

1. Make initial guesses of the aggregate capital stock K, aggregate employment N, mean income $\bar{y}$, transfers tr, and the value function $V(\epsilon, k)$.
2. Compute the wage rate w, the interest rate r, and unemployment compensation b.
3. Compute the household's decision functions by value function iteration.
4. Compute the steady-state distribution of assets, labor supply, labor income, and household income.[12]
5. Compute K, N, $\bar{y}$, and taxes T that solve the aggregate consistency conditions.
6. Compute the transfers that solve the government budget.
7. Update K, N, $\bar{y}$, and tr, and return to step 2 if necessary.

In step 3, the optimization problem of the household is solved with value function iteration. For this reason, the value function is discretized using an equispaced grid of 1000 points on the interval $[0, k^{max}]$. The upper bound on capital $k^{max} = 12$ is found to never be binding. The asset decision is first obtained by bracketing the maximum over the asset grid by iterating over the grid. We then apply the Golden Search method[13] to solve for the optimal next-period capital stock. As the optimal next-period capital stock may not be a grid point, we interpolate linearly between the neighboring points of the discretized value function.

[11]den Haan (1996) and Krusell/Smith (1998) also compute the transition function of the capital stock distribution. For this reason, den Haan uses a specific class of function for the cross-sectional distribution of assets. Choosing the exponential family, he is able to characterize the distribution by a finite number of parameters. This procedure allows him to model the transition function of the distribution with a dynamic equation in a few parameters. Similarly, Krussell/Smith (1998) characterize the distribution by a finite number of moments. In the present analysis, however, the distribution is calculated without any assumptions on its functional form.

[12]Our numerical method for the computation of the stationary distribution follows Hugget (1993).

[13]For a description of the algorithm, see Press et al. (1992).

Each time, the optimal labor supply has to be computed. In the case of a progressive income tax, the utility function is not differentiable everywhere with respect to labor hours. For tax regime (i), we proceeded as follows. We computed the marginal tax rate τ_j if the individual does not supply labor, $n = 0$. If $\frac{u_{1-n}(c,1-\kappa)}{u_c(c,1-\kappa)} < \frac{u_{1-n}(c,1)}{u_c(c,1)}$, where u_x denotes the first derivative of the utility function with respect to the argument $x = c, 1-n$ and with $\kappa = 0.001$ being a small constant, the optimal labor supply is set equal to zero. Otherwise, we computed the optimal labor supply for the tax rate τ_j from the condition

$$\frac{u_{1-n}(c, 1-n)}{u_c(c, 1-n)} = (1-\tau_j)\epsilon^i w. \tag{23}$$

If the income lies in the open interval (Y_{j-1}, Y_j), we have found the optimal solution. If the implied income is larger than Y_j, we set the marginal income tax rate to τ_{j+1} and compute the optimal labor supply for the new tax rate. If the optimal income associated with τ_{j+1} lies in the interval $(Y_{j-1}, Y_j]$, the optimal labor supply is given by $n = (Y_j - rk)/(1-\tau_j)\epsilon^i w$ and the solution for the optimal labor supply corresponds to the non-differentiable point.

4 Results

In this section, the quantitative effects of a tax reform on employment, savings, distribution, and welfare are studied. First, equilibrium properties of the benchmark case with a progressive income tax are presented. In the following two subsections, the two tax reform proposals, the flate-rate income tax and the increase of consumption taxation, are considered in turn.

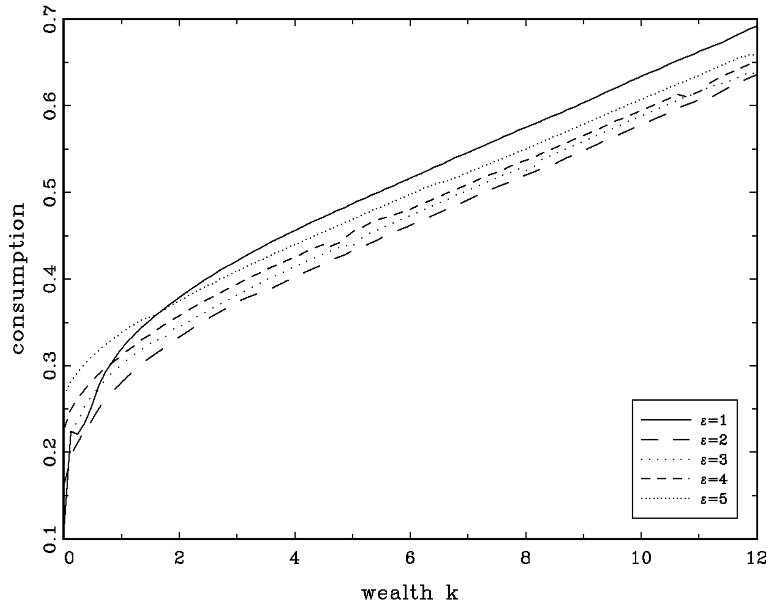
4.1 Equilibrium properties

In this subsection, we study the properties of the benchmark equilibrium which is characterized by the parameterization as presented in table 1. The household maximizes utility by the choice of labor supply and consumption. The policy functions depend on wealth k and productivity ϵ. Optimal consumption is graphed in figure 1. Consumption of the employed worker increases with both productivity and wealth. Notice that wealth-poor agents are liquidity-constrained. While the liquidity-constrained unemployed worker consumes less than the employed workers, the rich unemployed worker consumes more than the employed worker with equal wealth.

Figure 2 illustrates the optimal labor supply of the employed workers with $\epsilon \in \{\epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5\}$. First, except for very low levels of wealth k, agents with higher productivity work longer.[14]

[14]Notice, however, that workers with low wealths level have different marginal income tax rates depending on their productivity level.

Abbildung 1: Optimal consumption



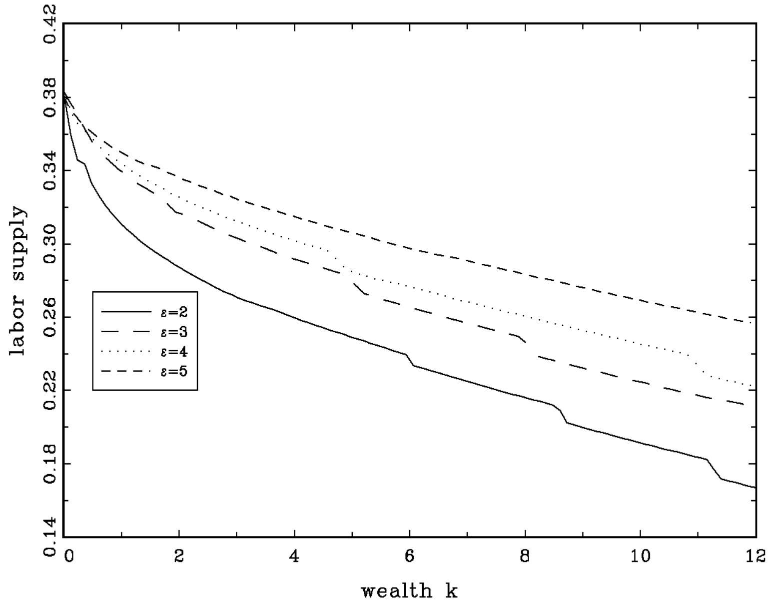
Second, labor supply is a decreasing function of wealth. From the inspection of figure 2, the marginal propensity to supply labor changes over the wealth range considered. For example, for productivity $\epsilon = 3$ (broken line), labor supply decreases more rapidly in the wealth interval $k \in [4.8, 5.2]$. In this interval, the worker reduces his labor supply in order to remain in the same tax income bracket as his wealth and, hence, his interest income changes. At the upper bound of this interval, the optimal labor supply fulfills the optimality condition of the household with regard to the marginal tax rate of the next tax bracket, τ_{j+1}, with equality, whereas in the interval the following condition holds:

$$(1-\tau_j)\epsilon^i w > \frac{u_{1-n}(c, 1-n)}{u_c(c, 1-n)} > (1-\tau_{j+1})\epsilon^i w. \tag{24}$$

The wage rate of each worker is simply his productivity ϵ^i times the aggregate wage level w. The net effect of productivity heterogeneity on the distribution of gross labor income, $n\epsilon w$, relative to the distribution of wage income, $w\epsilon$, is not straightforward in our model. On the one hand, high-productivity agents supply more labor for given wealth (above a certain threshold value). On the other hand, the high-productivity agents are also richer than the low-productivity agents on average, which tends to reduce the supply of average working hours of high-productivity agents. For our calibration, we find that gross labor income is more unequally distributed than the wage rate. In fact, the Gini coefficient of gross labor income (wage rate) is equal to 0.305 (0.254) and compares favorably with the empirical Gini coefficient which amounts to 0.317[15] (0.2748). The Lorenz curve of the labor income distribution (broken

[15]We computed the empirical Gini coefficient of gross wage income using the GSOEP data on annual

Abbildung 2: Optimal labor supply



line) is graphed in figure 3 and compared with the empirical distribution (solid line). In our model, labor income is a little less concentrated than observed empirically as the very productive agents receive a smaller labor income share and the least productive agents receive a higher labor income share. The difference is rather small as can be seen from the comparison of the Gini coefficients of the theoretical and the empirical distribution of gross labor income.

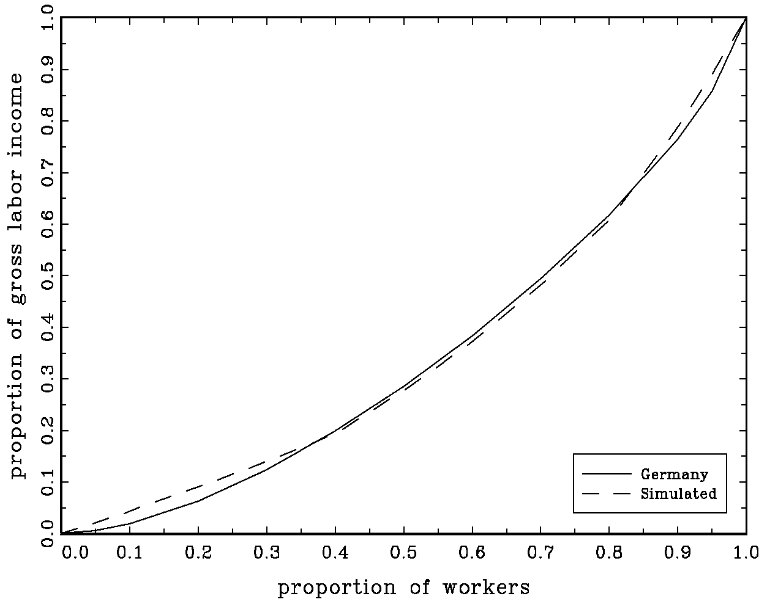
In equilibrium, the unemployment rate is equal to 10.95%. Aggregate effective labor supply amounts to $N = 0.235$ (compare table 2) with an average working time approximately equal to $\bar{n} = 0.325$. The aggregate capital stock amounts to $K = 2.070$ which is associated with a capital-output coefficient equal to $K/Y = 3.28$. During 1991-97, the empirical value of K/Y was equal to 5.0 (2.6) in Germany for the total economy (producing sector). The distribution of wealth, however, is not modelled in a satisfactory manner. In our model, the concentration of wealth is too low with a Gini coefficient equal to $\text{GINI}_{wealth} = 33.8\%$. Empirically, wealth is distributed much more unequally and characterized by a Gini coefficient in the range 0.59-0.89.[16] The most important reasons why our model fails to replicate the empirical wealth concentration are the neglect of life-cycle savings and business ownership.[17]

individual labor income. For the computation, we deleted individuals with implausibly low or high implied hourly wage rates. We chose 7 DM as the lower limit and 200 DM as the upper limit. The number of deletions is small (about 0.17% at the top and about 6.5% at the bottom of the distribution).

[16]Bomsdorf (1989) analyzes Gini coefficients of the wealth distribution for different kinds of assets in the periods 1973, 1978, and 1983 for West Germany. Within each asset group, Gini coefficients are remarkably stable. The distribution of savings, securities, and real estate in 1983 are characterized by Gini coefficients equal to 0.59, 0.89, and 0.74, respectively.

[17]Quadrini/Ríos-Rull (1997) present a review of recent studies of wealth heterogeneity in computable general

Abbildung 3: Lorenz curve of gross labor income



4.2 A Flate-Rate Tax reform

In our first tax experiment, we replace the progressive German income tax schedule with a flat-rate income tax structure. In order to keep the government expenditures and tax revenues constant, we have to set the income tax rate equal to $\tau = 25.0\%$. Moving from a progressive to a flat-rate income tax, workers change their behavior. As all agents face the same marginal income tax rate τ, the labor supply of the high-productive (low-productive) workers increases (decreases) as the marginal tax rates become lower (higher). However, in addition to this substitution effect, high-productivite (low productive) worker also earn higher (lower) income and reduce (increase) labor supply because of the wealth effect. The net effect on aggregate labor N is positive and N increases by almost 5% from 0.235 to 0.244 (see the second row of table 2). In addition, the labor supply curves of the agents with different productivities do not intersect any more in the case of a flat-rate tax (not presented) and the first derivative of the optimal labor supply is continuous rather than discontinuous as in the case of a progressive income tax (compare figure 2).

The change in the savings behavior of households is similar to the one in the labor supply.

equilibrium models with uninsurable idiosyncratic exogenous shocks to earnings, including business ownership, higher rates of return on high asset levels, and changes in health and marital status, among others. A more ad hoc approach is provided by Krusell/Smith (1998) who simply introduce preference heterogeneity in the stochastic Ramsey model. In particular, they assume that the discount factor β can take three different values and follows a Markov process with average duration of 50 years at the highest and lowest value of β.

Table 2: Effects of tax policy reforms

Tax policy	K	N	r	GINI_{labor}	GINI_{wealth}	W	Δ_c
(i) progressive income tax	2.070	0.235	4.94%	30.5%	33.8%	-94.67	0%
(ii) flat-rate income tax	2.589	0.244	3.94%	31.7%	38.0%	-88.75	7.31%
(iii) consumption tax	3.270	0.248	2.91%	31.2%	38.4%	-81.50	14.93%

Households characterized by high productivity and wealth increase their savings as they face a lower tax rate on interest income under a flat-rate tax policy regime, while the opposite holds for wealth-poor and low-productivity households. The net effect on savings is positive and the aggregate capital stock K rises from 2.070 to 2.589 following a switch from tax policy (i) to policy (ii). The rise of the capital stock is even more pronounced than the increase of aggregate employment so that the interest rate r falls from 4.94% to 3.94%.

The change associated with the switch in tax policy in both income and wealth distribution is rather modest. The distribution of gross labor income of the employed workers ($\epsilon \geq \epsilon^2$) as measured by the gini coefficient GINI_{labor} increases by one percentage point only as the low-productivity workers reduce their labor supply compared to the one of the high-productivity workers.[18] Notice that the non-interest income of the unemployed workers even remains unchanged as both transfers tr and unemployment benefits b are kept constant.[19] The distrbution of wealth becomes more concentrated, too, and the wealth gini coefficient GINI_{wealth} rises from 34.5% to 38.0%. The more marked rise in the wealth gini compared to the one of the labor income gini simply reflects the relative increases of savings and labor supply.

In order to compare the welfare effects of the different tax systems, we need to specify a welfare measure. In the following, welfare W is measured by the average life-time utility in the economy:

$$W(\Omega) = \int_{\mathcal{X}} V(\epsilon, k; \Omega) d\psi(\Omega), \tag{25}$$

where Ω denotes the tax policy. Note that the measure ψ also depends on Ω since it describes the distribution of both state variables k and ϵ. The welfare effect of a change in the tax policy from Ω to Ω' is measured by the consumption equivalent increase Δ_c as suggested by

[18]This result is in accordance with the result obtained by Castañeda et al. (1998) for the US economy.

[19]In our computation of the GINI_{labor}, we only accounted for the labor income of employed workers. The gini coefficient of labor earnings, the latter defined as wage income plus unemployment benefits, is higher, both in our model and empirically (not presented).

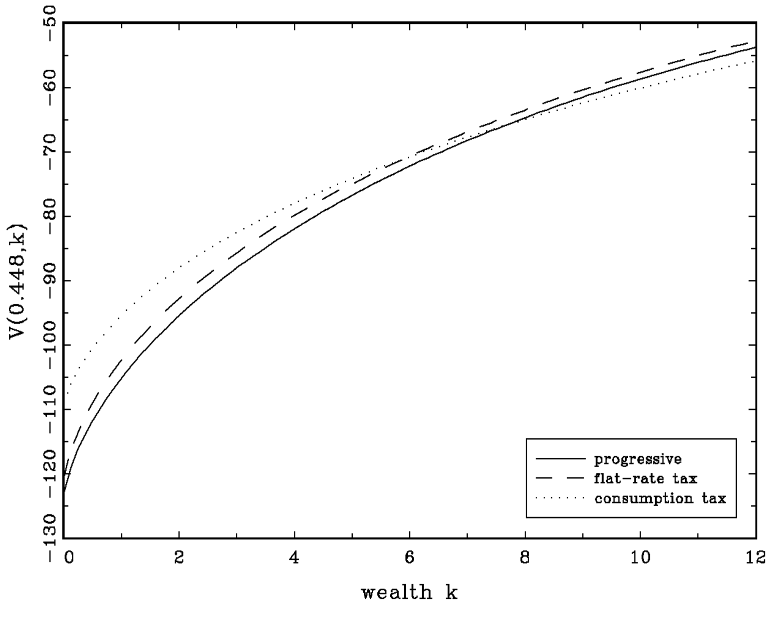
McGrattan (1994). For our choice of the utility function, Δ_c can be computed from:

$$1-(1+\Delta_c)^{1-\sigma} = \frac{W(\Omega')-W(\Omega)}{\int_{\mathcal{X}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}\right] d\psi(\Omega)}, \tag{26}$$

where expectations E_0 are taken conditional on expectations at the beginning of period 0.

The change in welfare following a switch to the flat-rate income tax is rather strong. In fact, the welfare gain amounts to an increase of total consumption equal to 7.31%. More interestingly, not only aggregate welfare increases but also individual welfare at all levels of both wealth and productivity increases as well. In figure 4, we illustrate the value functions of the low productivity agent with $\epsilon^2 = 0.448$. For given wealth, life-time utility of all agents increases as the progressive income tax rate (solid line) is replaced by the flat-rate tax (broken line). The behavior of the value functions for the agents with the other four productivities is qualitatively the same.[20] Even though their marginal tax rate increases, the value of the low-productivity workers increases for two reasons: 1) the aggregate wage rate increases and 2) expected utility in future periods rises. The second effect, of course, crucially depends on our realistic assumption that workers are mobile and may change their productivity between periods.

Abbildung 4: Value function of the low-productivity employed worker



[20]More specifically, 1) the value function in the case of the flat-rate income tax rate always lies above the one in the case of the progressive income tax rate, and 2) the value function in the case of the consumption tax intersects the other two value functions for tax policies (i) and (ii) at wealth levels which are close to twice the average wealth K.

The value functions illustrated in figure 4 are only an indicator that all workers might benefit from a flat-rate tax reform. However, following a change in the tax policy, the wealth distribution changes. As pointed out above, wealth gets more concentrated so that we might end up with more wealth-poor agent under the tax regime (ii). A meaningful welfare comparison, therefore, should analyze the different percentiles of the wealth distribution for each productivity type (due to our assumptions of constant transition probabilities, the measure of each productivity class remains the same under each tax regime). In figure 5, we graph the average wealth of each decintile of the wealth distribution for the low productivity type $\epsilon = \epsilon^2$.[21] Obviously, each decentile of the wealth distribution gains from a flat-rate tax reform in steady state and we observe something close to a pareto-improvement[22].[23]

At this point, one word of caution is warranted. We would be rather careful to draw firm policy implications from our welfare analysis because we only considered steady-state behavior of our economy. The neglect of transitional dynamics following a change in tax policy can have significant effects on welfare results. For example, Lucas (1990) analyzes the abolition of capital income taxes in an endogenous growth model with human capital accumulation. In steady state, the change in welfare amounts to a 3% consumption equivalent gain. As demonstrated by Grüner/Heer (2000), also considering the transition from the old to the new steady state reduces the welfare gain of such a policy to 1% of total consumption. We carefully conjecture that switching from tax policy (i) to tax policy (ii) also implies welfare losses during the initial phase of transition. In the new steady state, average employment and average consumption are higher. Labor supply n is a jump variable and adjusts instantaneously after a change in policy. Therefore, disutility from labor increases during transition. The capital stock is a sluggish variable and builds up slowly. Agents save a higher proportion of their income and consumption is lower during transition than in the new steady state and so is utility from consumption. As a result, instantaneous utility during transition is likely to be below the one in the new steady state on average.

4.3 Consumption Tax

In our second tax experiment, we set the income tax rate to zero and increase the consumption tax rate in order to generate the same tax revenues as in the benchmark case. The new steady-state consumption tax amounts to $\tau_c = 40.57\%$. As interest income is not taxed any

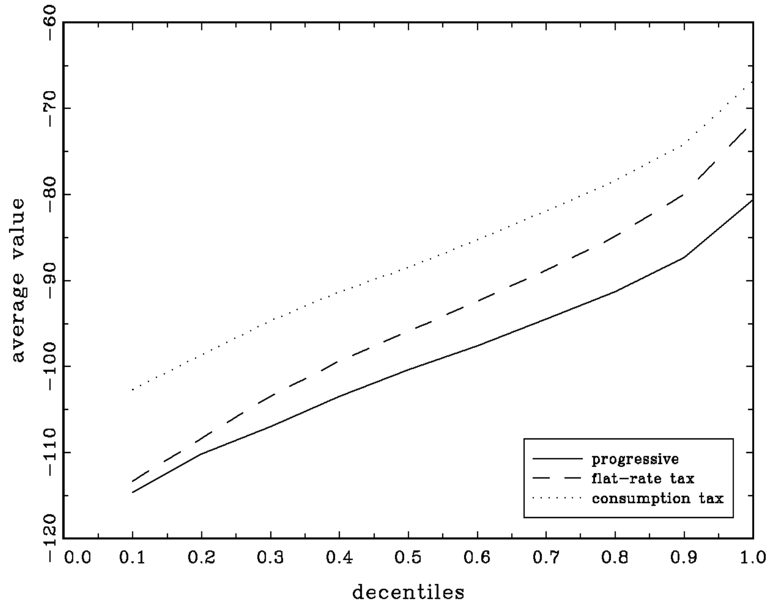
[21] Again, the result is qualitatively the same for all productivity types $\epsilon \in \mathcal{E}$.

[22] In fact, every percentile of all households gains from the introduction of a flat-rate tax (not illustrated).

[23] This finding is different from Ventura (1999) who studies the effects of a flat-rate tax reform for the US economy. Different from his study, we also consider the state of unemployment and unemployment compensation. Furthermore, Ventura (1998) analyzes an OLG model rather than a Ramsey model.

more, households increase their savings. Accordingly, the aggregate capital stock K rises from 2.070 in the benchmark case to 3.270. Associated with this 60% rise in the capital stock is an increase of the marginal product of labor and, hence, labor remuneration. Consequently, labor supply increases, even though to a much smaller extent than the capital stock, and aggregate employment goes up by 5%. Associated with these changes of the input factors is a decline of the interest rate r by almost two percentage points.

Abbildung 5: Welfare decomposition for low-productivity workers



The distribution effect of the tax reform, again, is rather modest and even smaller than in the case of the flat-rate tax reform. The Gini coefficient of gross labor income increases to 31.2%, merely half a percentage point above the benchmark case with progressive income taxation. Wealth, however, gets more concentrated and the Gini coefficient GINI_{wealth} even increases by 4.6 percentage points. As the tax on interest income is abolished, wealth-rich and high-productive workers increase their savings by a higher proportion than wealth-poor workers. Unemployed workers, in particular, suffer the most as their unemployment benefits can buy less consumption. All workers are liquidity-constrained over an increased range of low wealth compared to the benchmark case (not illustrated).

Again, the policy reform leads to substantial welfare gains in steady state. The welfare change is equivalent to a total rise of consumption equal to 14.9%. As can be seen from figure 4, welfare of the workers increases for given level of wealth up to a wealth level of approximately twice the average wealth. Furthermore, for every productivity class, every decentile of the wealth distribution gains (compare figure 5). From this steady-state comparison with our benchmark case, it is obvious that a consumption tax reform improves welfare in steady state.

5 Conclusion

This paper studies the general equilibrium effects of a flat-rate tax and a consumption tax reform for Germany. We develop a model that accounts for observed labor income mobility and distribution. Both reform proposals improve both efficiency and welfare compared to the allocation under the present German tax structure with a progressive income tax. Employment rises by approximately 5% under each proposed alternative tax regime, while savings even increase by more than 20% and 60% for the flat-rate tax and consumption tax regime, respectively. Welfare gains in steady state are substantial and amount to several percentage of total consumption for both reform proposals. Importantly, the wealth-poor and low-productivity workers which are the most likely to suffer from such a policy reform benefit as well. While the distribution of gross labor income is hardly affected, the distribution of wealth changes significantly and becomes much more concentrated.

We would like to reiterate that our results should be interpreted carefully. First, we have not computed the welfare losses which are likely to arise during transition after the tax policy switch to the new steady state. Second, we only studied labor adjustment along the intensive margin. The addition of non-competitive labor markets, e.g. in the form of search unemployment or union bargaining, which allow for the simultaneous study of labor adjustment along both the intensive and extensive margin is likely to have a profound effect on our results. Third, we assumed that the worker's productivity follows an exogenous stochastic process which is independent of the tax policy regime. In our view, the endogenization of productivity constitutes another promising extension of our model, in addition to the consideration of non-Walrasian labor markets. Productivity of the households, for example, could depend on the households' expenditures on education like in the model of Lucas (1988) which has been recently modelled by Heckman et al. (1998) in a heterogenous-agent economy in order to explain rising US wage inequality as a consequence of skill-biased technological change. As the tax policy regime is likely to affect individuals' expenditures on education and (on-the-job) skill formation, the distribution of productivity would be determined endogenously in such an extended model.

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