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# Taxation of Labor and Capital Income in an OLG Model with Home Production and Endogenous Fertility

by

Burkhard Heer[1]
Mark Trede[2]

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**Abstract:** Many developing countries are characterized by a large share of home production. Households allocate their time on both market and non-market activities. The introduction of a tax on labor or capital income induces people to divert from market production to home production. Furthermore, children are often used as an input into home production. In this situation, a higher tax rate on capital income causes not only a decrease in the capital intensity but also an increase in the population growth rate. The effects of a wage tax on economic development are shown to depend on the opportunity costs of raising children. For reasonable parameter values of the tax rate, labor income taxation will reduce capital accumulation and cause a rise in home production. As a result fiscal policy should be combined with population policies.

[1]Staatswissenschaftliches Seminar, Universität zu Köln, Albertus–Magnus–Platz, 50923 Köln, Germany, Tel.: +49-221-470 2748, Fax: +49-221-470 5143, E-mail: ags08@rrz.uni-koeln.de

[2]Seminar für Wirtschafts- und Sozialstatistik, Universität zu Köln, Albertus–Magnus–Platz, 50923 Köln, Germany, Tel.: +49-221-470 2283, Fax: +49-221-470 5074, E-mail: trede@wiso.uni-koeln.de

# 1 Introduction

A central question of economic development addresses the transition from a low-income rural economy to a higher-income urban economy. Harris and Todaro (1970) study labor migration within a two sector-model and consider reasons and consequences of a rural-urban wage gap. Drazen and Eckstein (1988) study the organization of rural markets in order to explain the process of economic development. Capital accumulation is shown to be lowest when factor markets in agriculture, land as well as labor, are competitive. However, these models examine countries where households earn income from market activities only. In most developing countries, individuals also heavily engage in non-market activities: families often cultivate agricultural land at the subsistence level employing their children as a necessary input. In our analysis, households will allocate resources to both non-market and market activities. Further, non-market activities produce a non-tradable good and are not taxable. Consequently, fiscal policies affect the size of the market sector. In particular, income taxation from both capital and labor is likely to reduce capital accumulation.

The effects of income taxation on economic development received an increasing interest recently. Following the seminal article by Romer (1986), models of endogenous growth are trying to explain the different growth performance of countries and study how the growth rate depends on various parameters of the production and utility function as well as on governmental policies. In these models, capital does not exhibit diminishing returns. For this reason, an increase in the tax rate on capital income reduces growth in the models of Rebelo (1991), Barro (1990), Barro and Sala-I-Martin (1992). The effect of taxation of labor income is examined in models where human capital accumulation is the main engine of growth. If human capital is produced with human capital only, [cf. Lucas (1988) or Grüner and Heer (1995)] the steady state growth rate does not depend on the wage tax. If, however, learning also needs physical capital as an input, a wage tax reduces growth [cf. Caballé and Santos (1995)]. In our analysis, capital taxation is proven to be harmful to capital accumulation. Wage taxation might have different effects depending on the time and goods costs of children but is likely to have the same effects as capital income taxation.

Our analysis focuses on the households' allocation of time during economic development. Thus, our approach is similar to that of Becker (1965) and the dynamic analyses of Greenwood and Hercowitz (1991) and Benhabib et al. (1991). In our

Overlapping Generations model, households live two periods. In the first period, households work in the market-sector, engage in home production and raise children. Home production might best be interpreted as agricultural activities, presumably at the subsistence level, and children become a necessary input in order to take advantage of the division of labor. The household also decides upon the number of children; thus, fertility is endogenous and depends on the time and goods costs of children as well as on the productivity of the children in home production. In the second period, individuals do not work and just consume. The effects of fiscal and population policies on capital accumulation, the time spent on home production and the fertility rate are studied for the steady state and for the transition dynamics alike.

The paper is organized as follows. In section 2, we introduce the Overlapping Generations model with endogenous fertility and home production. The steady state capital intensity and the fertility rate are shown to be a function of the tax rate on capital income and labor income, respectively. The transition dynamics are analyzed in section 3. Section 4 concludes.

# 2 The Model

### Households

Consider a standard overlapping generations model à la Diamond (1965). Individuals live two periods. They work in the first period of their life and allocate their time to market and non-market activities producing goods $Y$ and $Z$, respectively. Utility is a function of consumption of both goods; children do not enter the utility function.

$$u_t = \frac{c_{t+1}^{1-\psi_c}}{1-\psi_c} + \phi \frac{z_t^{1-\psi_z}}{1-\psi_z}, \qquad 0 < \phi, \psi_c, \psi_z. \tag{1}$$

$c$ and $z$ denote per capita consumption of the goods $Y$ and $Z$, respectively. Note that for $\psi_c = \psi_z = 0$ there is perfect substitutability.

In developing countries children become a necessary input into home production, in particular for rural families. Cultivation of land might be very labor intensive. Looking after cattle, collecting water or fire wood etc. are very time-consuming

activities. This is well documented by Dasgupta (1992), Dasgupta and Mäler (1993) and Dasgupta (1994). Thus, the number of children $n_t$ is an endogenous variable of the household's decision problem. Therefore, population grows at the rate $n_t - 1$:

$$N_{t+1}/N_t = n_t. \tag{2}$$

The home good $Z$ is non-tradable. Further, it is assumed that home production does not use capital as an input, and that the good $Z$ is produced with a constant or decreasing returns to scale technology. Decreasing returns to scale are likely to prevail in the agricultural sector in developing countries. To increase production, land has to be used more intensively and households will have to cultivate marginal land as well. Therefore, productivity might decrease with increasing non-market activities. The qualitative results in this paper, however, do not change whether constant or decreasing marginal returns are assumed to prevail in the production of the home good $Z$. Let $l_t$ denote the time devoted to home production. Further assume the following simple home production function for analytical convenience:

$$z_t = \omega l_t^{\beta} n_t^{\gamma}, \quad \beta + \gamma \leq 1, \tag{3}$$

where $z_t$ is the net production of the home good after allowing for the children's consumption of good $z$. Due to the non-tradability of good $z$ households cannot accumulate savings via non-market activities. This assumption is justified in case of rural households where the institutional structure of extended families is likely to prevail. Furthermore, there might be a lack of financial intermediaries who transfer savings from the traditional, non-market sector to the modern, market sector.

In the market sector, individuals earn wage $w_t$ which is saved for consumption in the second period. The household faces the following budget constraint:

$$(1 - l_t - n_t\kappa)(1 - \tau_w)w_t = \frac{c_{t+1}}{1 + (1 - \tau_r)r_{t+1}} + n_t c_0. \tag{4}$$

$\tau_w$ and $\tau_r$ denote the tax rates on labor and capital income, respectively. Notice that in this simple model, $\tau_w$ can also be interpreted as a sales tax, which might be

the more relevant tax instrument in the case of developing countries. The budget constraint also accounts for the opportunity costs of children. On the one hand, there are fixed costs, $c_0$, per child like nutrition, clothing etc. In contrast to Barro and Becker (1989) we distinguish consumption of children during their childhood from that of their parents. We further use the simplifying assumption that the opportunity costs of children entirely arise at the time of birth in accordance with Barro and Sala-I-Martin (1995). On the other hand rearing children needs time $\kappa$ per child. Both $\kappa$ and $c_0$ are subject to governmental policies. The provision of child care services or child allowances by the state will cause a decrease of $\kappa$ and $c_0$, respectively.

The household takes the wage and interest rate as given and maximizes its utility (1) with respect to $c_t$, $l_t$, and $n_t$ subject to (3) and (4). The first order conditions are:

$$c_{t+1}^{-\psi_c} = \frac{\lambda}{1+(1-\tau_r)r_{t+1}}, \tag{5}$$

$$\phi\beta\frac{z_t^{1-\psi_z}}{l_t} = \lambda(1-\tau_w)w_t, \tag{6}$$

$$\phi\gamma\frac{z_t^{1-\psi_z}}{n_t} = \lambda(\kappa(1-\tau_w)w_t + c_0), \tag{7}$$

where $\lambda$ is the Lagrangian multiplier associated with the budget constraint.

**Production**

Firms act competitively and use effective labor, $L_t$, and capital, $K_t$, as inputs into production. Effective Labor, $L_t$, is simply the time individuals spend in production, $1 - l_t - n_t\kappa$, times the population, $N_t$. The production function is assumed to be Cobb-Douglas:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}. \tag{8}$$

Capital intensity is defined as capital stock per effective labor, $k_t = K_t/L_t$. The wage will equal the marginal product of effective labor and can be written as:

$$w_t = (1-\alpha)k_t^{\alpha}. \tag{9}$$

Likewise, the interest rate equals the marginal product of capital and is given by:

$$r_t = \alpha k_t^{\alpha-1}. \tag{10}$$

In the market equilibrium, profits are zero since production has constant returns to scale. In the capital market equilibrium, net savings equal investments:

$$s_t N_t = K_{t+1} = ((1-\tau_w)w_t(1-l_t-n_t\kappa)-n_t c_0)N_t. \tag{11}$$

The first order conditions (5) – (7), the wage and interest rate equations and the capital market equilibrium (11) yield the following system of nonlinear dynamic equations:

$$\begin{aligned}
1-l_t-n_t\kappa &= ((1-\tau_w)(1-\alpha)k_t^{\alpha})^{\frac{1}{\psi_c}-1}(1+(1-\tau_r)\alpha k_{t+1}^{\alpha-1})^{\frac{1}{\psi_c}-1}\zeta_0\frac{l_t^{\zeta_1}}{n_t^{\zeta_2}} \\
&\quad +\frac{n_t c_0}{(1-\tau_w)(1-\alpha)k_t^{\alpha}} && (12)\\
l_t &= \frac{\beta}{\gamma}\left(\kappa+\frac{c_0}{(1-\tau_w)(1-\alpha)k_t^{\alpha}}\right)n_t && (13)\\
k_{t+1} &= \frac{(1-\tau_w)(1-\alpha)k_t^{\alpha}(1-l_t-n_t\kappa)}{n_t(1-l_{t+1}-n_{t+1}\kappa)}-\frac{c_0}{1-l_{t+1}-n_{t+1}\kappa}, && (14)
\end{aligned}$$

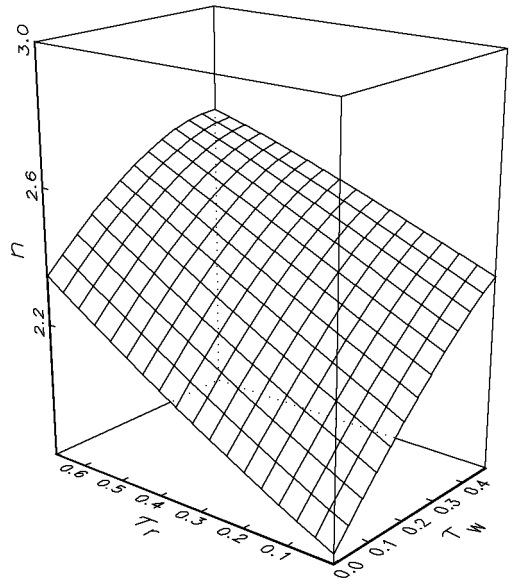
where

$$\begin{aligned}
\zeta_0 &= \left(\frac{1}{\phi\beta\omega^{1-\psi_z}}\right)^{\frac{1}{\psi_c}},\\
\zeta_1 &= \frac{1-\beta(1-\psi_z)}{\psi_c},\\
\zeta_2 &= \frac{\gamma(1-\psi_z)}{\psi_c}.
\end{aligned}$$

**Steady State**

In the steady state, the endogenous variables are constant: $k_{t+1} = k_t = k$, $l_{t+1} = l_t = l$, and $n_{t+1} = n_t = n$. Thus equations (12) to (14) reduce to an ordinary

Figure 1: Steady state of $n$

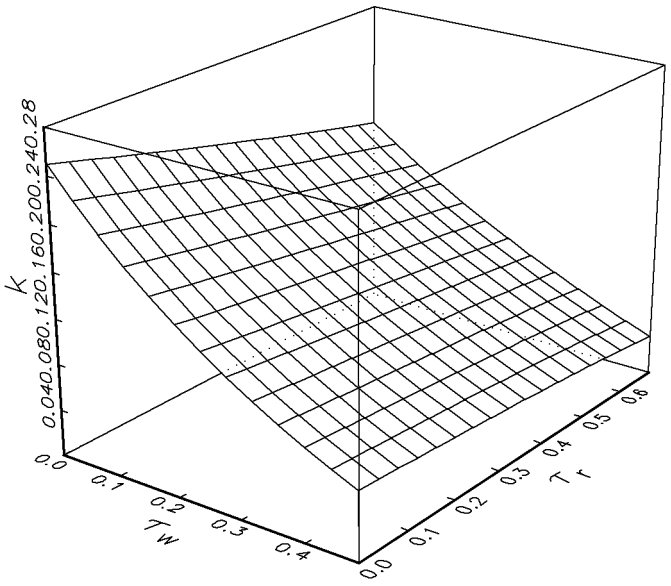



nonlinear equation system which can be solved for the steady state values $k$, $l$, and $n$ numerically.

Figures 1 to 3 depict the steady state values of the three endogenous variables as functions of the tax rates $\tau_w$ and $\tau_r$. The parameters are $\alpha = 0.3$, $\beta = 0.5$, $\gamma = 0.4$, $\phi = 1$, $\omega = 1$, $\kappa = 0.15$, $c_0 = 0.001$, and $\psi_c = \psi_z = 0.3$. As can be seen the fertility $n$ and the time devoted to home production $l$ depend positively on the tax rates, whereas the capital intensity decreases for higher tax rates.

The numerical results are tested for various parameters and are found to be only sensitive with regard to the goods costs of children, $c_0$, in relation to the opportunity costs, $\kappa w$. Changing the fix costs of raising children does not only alter the level of the steady state surface but also its shape. For a sufficiently high ratio of goods costs to wages fertility, $n$, is negatively related to the labor income tax for high tax rates, $\tau_w$. However, this will merely be the case for unreasonably high values of $\alpha$, $\phi$, $\omega$, and $c_0$, and low values of $\kappa$. In summary, income taxation, at least for reasonable values of the tax rates $\tau_w$ and $\tau_r$, reduces capital accumulation in the economy and intensifies population growth. In order to develop the market sector, these effects of taxation should be balanced by

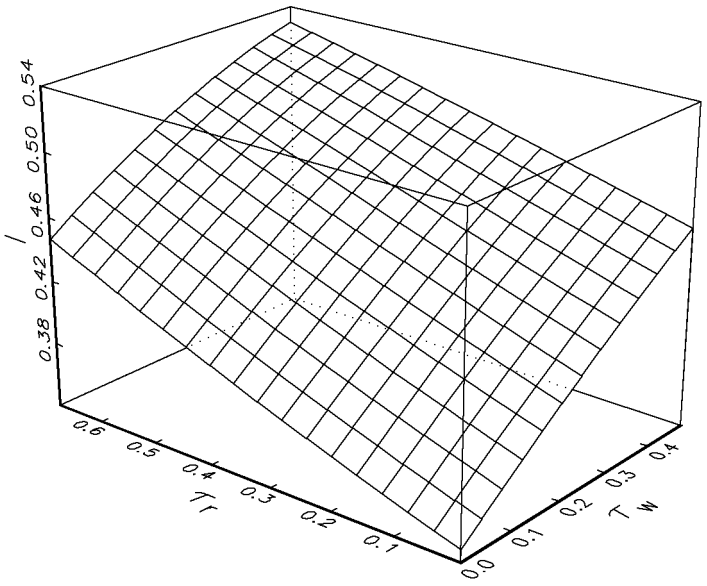
Figure 2: Steady state of $k$




population policies, e.g. raising of the goods costs of children, $c_0$.

The government collects tax revenues from both labor and capital income. The revenues from a wage tax as a function of the tax rate, $\tau_w$, are depicted by figure 4. The revenue function looks similar to the Laffer curve. As the tax rate $\tau_w$ increases, both time spent on market activities and capital intensity decline as resources are shifted into the non-market sector. For sufficiently high tax rates, the pre-tax wage income starts to fall and, beyond a certain threshold value of $\tau_w$, the wage tax revenues decrease as well. Further, for given government expenditures per capita, pure capital income taxation is found to result in higher fertility rates, $n$, and non-market activities, $l$, than pure labor income taxation for a wide range of parameters. Surprisingly, capital intensity is also higher for pure capital taxation than for pure labor taxation. A tax on capital income reduces the after-tax marginal product of capital. Moreover, a wage tax causes non-market activities to increase. Consequently, the marginal product of capital declines. In addition, savings have to increase in the new capital market equilibrium to keep the capital stock per capita constant due to the increased population growth. In conclusion, income taxation of both labor income and capital income are found

Figure 3: Steady state of $l$

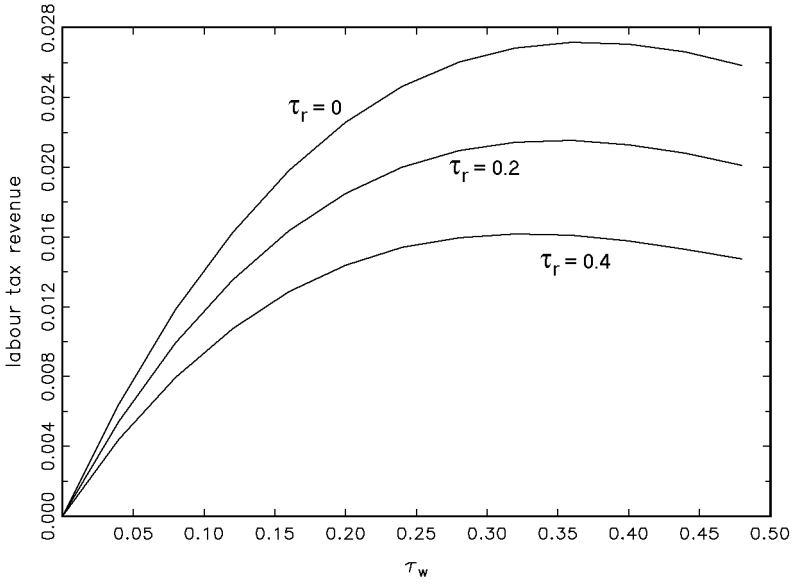



to be detrimental to the development of the market sector.

Our model also allows for analyzing the economic effects of technological progress and productivity changes. Natural resources are subject to overexploitation in many developing countries, e.g., overgrazing might cause soil erosion. Consequently, productivity of the agricultural sector and the subsistence production declines. In our model, this corresponds to a reduction in $\omega$, which reduces the incentives to work in home production, and, consequently, lowers fertility. The same effect, of course, arises if technological progress raises the productivity of labor in the market sector.

Concerning stability it turns out that the steady state is a saddle point equilibrium. For most of the plausible parameter combinations two of the eigenvalues of the equation system (12) to (14) are outside the unit circle and only one is inside. Since we assume perfect foresight, and given the initial capital endowment $k(0)$, this amounts to a single equilibrium paths leading towards the steady state equilibrium. However, if both tax rates are high it may happen for some parameter combinations that two eigenvalues are inside the unit circle. In this case there is no unique equilibrium path. In the sequel we will exclude these incidents as they

Figure 4: Steady state labor tax revenue per capita $\tau_w(1-l-n\kappa)w_t$



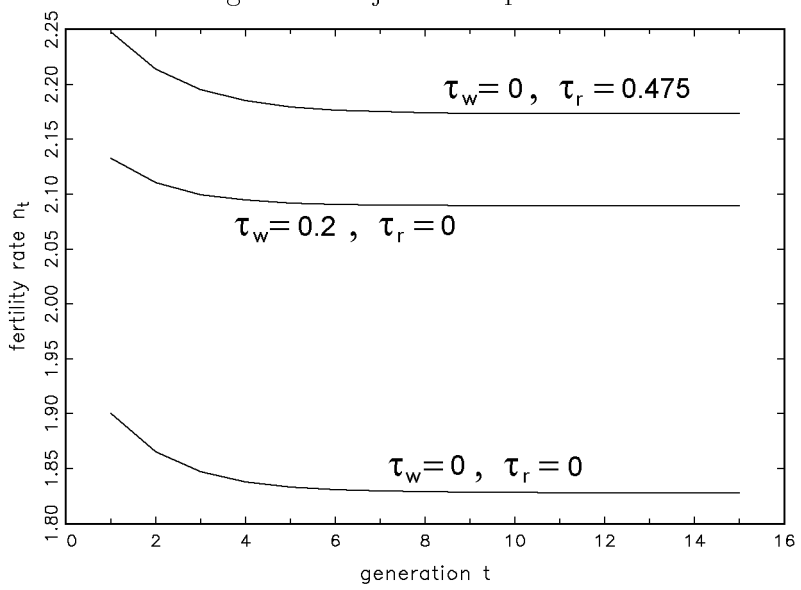

are unlikely to occur in reality.

# 3 Adjustment Paths

The economy moves along the equilibrium saddle path at all times due to perfect foresight. Whenever the parameters of the system are changed, for instance an increase in the tax rates, households adjust their fertility decision and the time they spend in home production such that the economy jumps onto the new (unique) saddle path. As the dynamic system is fairly complex the adjustment paths cannot be derived analytically but only numerically. In order to compute the adjustment path for a certain parameter combination we first calculated the steady state values of the endogenous variables $k_t, l_t$ and $n_t$ and slightly perturbed the capital intensity (by an order of $10^{-5}$). We then solved the nonlinear dynamic system (12) – (14) backwards to arrive at $k_{t-1}, l_{t-1}, n_{t-1}$. Iterating this procedure some twenty times yields the adjustment path of the endogenous variables.

Figures (5) – (7) display the adjustment paths for the endogenous variables. In the

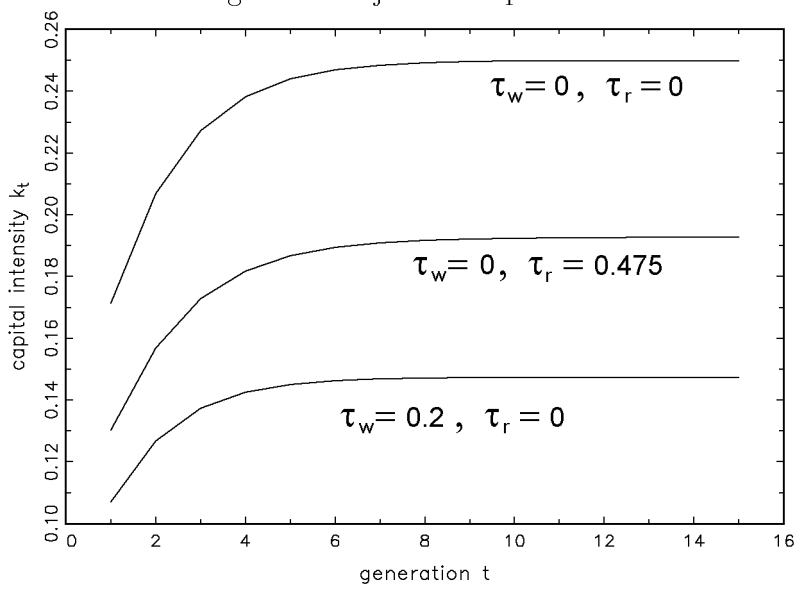
Figure 5: Adjustment paths of $n$




early stages of economic development, capital intensity is low. The bottom line shows the development towards the no-tax equilibrium; the other two lines depict adjustment towards equilibrium with labor income tax only ($\tau_w = 0.2, \tau_r = 0$) and capital income tax only ($\tau_w = 0, \tau_r = 0.475$), respectively. By choice of these parameters, the steady state government revenues per capita, $\tau_w(1 - l_t - n_t\kappa)w_t$ and $\tau_r r_t k_t(1 - l_t - n_t\kappa)$, respectively, are equal.

If the initial capital intensity is too low compared to the steady state equilibrium population growth and the time devoted to home production will initially be higher than their equilibrium values. During the process of economic development fertility will decline as depicted by figure 5. This demographic transition is well in accordance with empirical findings. According to the studies of Barro and Sala-I-Martin (1995), fertility declines with rising income for countries characterized by a per capita GDP in excess of \$800 (in 1985 US dollar). Likewise, time spent on non-market activities, $l_t$, will decline whereas the capital intensity increases during economic development.

Figure 6: Adjustment paths of $k$

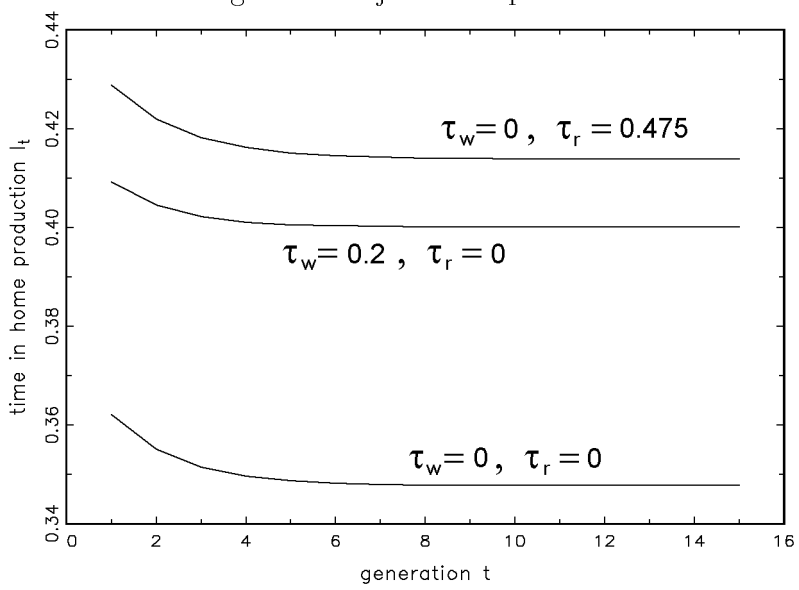



# 4 Conclusion

In a country characterized by a significant share of non-market activities, fiscal policy affects both capital accumulation and population growth. Income taxation of capital and labor provide incentives to shift resources between the market and the non-market sector of the economy. Consequently, as the size of the home production sector increases, fertility increases and less capital is accumulated. In order to intensify market activities the policy implications are straightforward: a rise in the opportunity costs of children and a reduction in the income tax rate, preferably in the labor income tax rate.

Our model could further be interpreted as a two-sector model. In this case, the time spent on non-market activities denotes the fraction of the population working in the rural area and the time spent on market activities denotes the urban population working in manufacturing. In such a dual economy, agricultural markets are assumed to be less developed than manufacturing markets and rural income is subject to tax evasion. A similar argument as in the case for home production can be made for fiscal and population policies.

Figure 7: Adjustment paths of $l$




Our emphasis has been on the normative analysis of taxation during economic development. In developing countries, however, government often faces significant administrative costs and difficulties to enforce taxes. Therefore, the choice of taxes are limited. This is also evident from the fact that value added tax accounts for a higher fraction of government revenues in developing countries than in developed countries, e.g., Ahmad and Stern (1988). However, in our simple model of a closed economy, this does not pose a problem as income and sales taxation are equivalent; of course, we abstracted from the possibility of capital flight which might be a serious argument in favor of wage taxation rather than capital taxation.

Finally, we must mention two caveats of our analysis. First, we do not discuss welfare implications of the different fiscal policies but only consider the policy implications on market activities and population growth. For this purpose, we would have to specify how the government uses the tax proceeds. Second, we take the productivity of the two sectors as given and only look at the economic effects of exogenous shocks. However, in the long run, technological progress and know-how might change productivity in both the agricultural sector and the manufacturing sector, and, most likely, to a different degree. Since the market sector might take

better advantage of the division of labor in comparison to the non-market sector it seems plausible to assume that productivity grows faster in the market sector. As a consequence, wages increase and individuals will shift resources to the market activity as observed in the process of economic development of many countries. Simultaneously, the need for children as an input into home production declines and thus the quality rather than the quantity of children becomes important to the parents.

# References


**Ahmad, E.; Stern, N. (1988):** "Taxation for Developing Countries," in: H. Chenery and T.N. Srinivasan (eds.), Handbook of Development Economics, Vol. II, 1004-92.

**Barro, R.J. (1990):** "Government Spending in a Simple Model of Endogenous Growth," Journal of Political Economy, vol. 98, S103-S125.

**Barro, R.J.; Becker, G.S. (1989):** "Fertility Choice in a Model of Economic Growth," Econometrica, vol. 57, 481-501.

**Barro, R.J.; Sala–I–Martin, X. (1992):** "Public Finance in Models of Economic Growth," Review of Economic Studies, vol. 59, 645-661.

**Barro, R.J.; Sala–I–Martin, X. (1995)** , Economic Growth, McGraw-Hill: New York.

**Becker, G.S. (1965),:** "A Theory of the Allocation of Time," The Economic Journal, vol. 75, 493-517.

**Benhabib, J.; Rogerson, R.; Wright, R. (1991):** "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," Journal of Political Economy, vol. 99, 1166-1187.

**Caballé, J.; Santos, M.S. (1995):** "On Endogenous Growth with Physical and Human Capital," Journal of Political Economy, vol. 101, 1042-67.

**Dasgupta, P. (1992):** "Poverty, Resources and Fertility: The Household as a Reproductive Partnership," Development Economics Research Programme Discussion Paper Series, No. 37.

**Dasgupta, P. (1994):** "Poverty and the Environment: Is There a Trade-off?," in: L. Campigli, L. Pineschi, D. Siniscalco, T. Treves, (eds.), The Environment after Rio, Graham & Trotman: London.

**Dasgupta, P.; Mäler, K.G. (1993):** "Poverty, Institutions, and the Environmental Resource Base," Development Economics Research Programme Discussion Paper Series, No. 48.

**Diamond, P.A. (1965):** "National Debt in a Neoclassical Growth Model," American Economic Review, vol. 55, 1125-50.

**Drazen, A.; Eckstein, Z. (1988):** "On the Organization of Rural Markets and the Process of Economic Development," American Economic Review, vol. 78, 431-443.

**Greenwood, J.; Hercowitz, Z. (1991):** "The Allocation of Capital and Time over the Business Cycle," Journal of Political Economy, vol. 99, 1188-1214.

**Grüner, H.-P.; Heer, B. (1995):** "Taxation of Income and Wealth in a Model of Endogenous Growth," Public Finance, forthcoming.

**Harris, J.R.; Todaro, M.P. (1970):** "Migration, Unemployment and Development: A Two-Sector Analysis," American Economic Review, vol. 60, 126-42.

**Lucas, R.E. (1988):** "On the Mechanics of Economic Development," Journal of Monetary Economics, vol. 22, 3-42.

**Rebelo, S. (1991):** "Long-Run Policy Analysis and Long-Run Growth," Journal of Political Economy, vol. 99, 500-521.

**Romer, P.M. (1986):** "Increasing Returns and Long Run Growth," Journal of Political Economy, vol. 98, 1002-37.