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Local versus Global Assessment of Mobility*

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Abstract: The common approach to measuring income mobility is to compute a mobility index, which reduces the information about income changes contained in the joint distribution of incomes into a scalar. Information about "local" income changes is aggregated into a "global" mobility index. We derive an approximation to the aggregation rule for the important class of so-called stability indices. By comparing global mobility estimates and local distributional change between the USA and Germany, we explain the empirical puzzle observed by Burkhauser et al. (1997) who find that Germany has more income mobile than the USA. We show that the relative global mobility ranking of the two countries is driven by the first-period poors.

Keywords: Mobility indices; aggregation rules; influence function; quantile-lines

JEL classification: D31, D63, I32

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1 Introduction

The common approach to measuring income mobility is to compute a mobility index, which reduces the information about income changes contained in the joint distribution of incomes into a scalar. Information about "local" income changes is aggregated into a "global" mobility index. However, the aggregation rule is not transparent for one important and popular class of mobility indices – the class of so-called stability indices introduced by Shorrocks (1978a). This paper provides an easily interpretable approximation to this rule.

The usefulness of measuring income or earnings mobility is expressed in the recent interest amongst policy makers[1] and researchers (cf. Atkinson, Bourguignon, and Morrisson (1987) for a summary). In the context of observed large increases in cross-sectional inequality in the USA, Britain, and most other OECD countries (Atkinson, Rainwater, and Smeeding (1994)), it has also been argued that high mobility should reduce concerns about inequality at a point in time, as incomes averaged over time should be more equally distributed than per-period incomes.

The practical usefulness of mobility measurement, however, has been called into question by the striking results of some recent studies which compare income mobility in the USA and Germany (Burkhauser, Holtz-Eakin, and Rhody (1997) and Burkhauser and Poupore (1997)). Germany is a useful choice for comparison with the US since, in contrast to the US, the German labor market is characterized by a relatively centralized wage bargaining. Further, the German social welfare system is much more generous. Hence, one would expect both lower income inequality and lower income mobility in Germany. In their empirical studies the authors find that, as expected, per-period inequality is high in the USA and low in Germany. But when it comes to mobility, Germany is often ranked to be as mobile as and sometimes *more mobile* than the USA, contrary to received wisdom which attributes a high extent of mobility to the US and takes Germany to be a country in stasis. This observation raises the question of whether this ranking is an artefact of the specific statistical measurement technique or whether the received wisdom is just wrong.

The mobility index maps the joint distribution of income into a scalar, and this paper therefore examines both the image of the map and the map itself. The examination of the former can be dressed in an empirical coat in form of the question of whether the patterns of local income changes can be substantially different between two different countries (or regions or periods), despite the mobility index producing similar global mobility assessments. We propose a graphical method which separates out distinct aspects of mobility, which global mobility indices typically mix up. As regards the map itself, we derive an approximation to the aggregation rule for the class of stability indices, which simply states that measured mobility is obtained by

[1] Policy makers have started to appreciate the importance of a dynamic perspective: "In the past, analysis ... has focused on static, snapshot pictures of where people are at a particular point in time. Snapshot data can lead people to focus on the symptoms of the problem rather than addressing the underlying processes which lead people to have or be denied opportunities" (H.M. Treasury (1999), p.5)

integrating weighted distributional changes:

$$\int w_M(x;F)d(F(x)-G(x))$$

where G is the distribution of time-averaged income, and F is a reference distribution which we take to be a mixture of marginal distributions. $w_M(x;F)$ is a weighting function, capturing the mathematical structure of the mobility measure M, which also depends on the reference distribution. The weighting function does not depend on information about changes in personal income, which is exclusively contained in the term $d(F(x)-G(x))$. This difference constitutes the notion of local distributional change captured by the stability index.

These tools are applied to an examination of the extent of income mobility in the USA and Germany. We can attribute the striking observation made by Burkhauser et al. to two re-inforcing factors: (i) The local profiles of income changes in Germany and USA profiles differ markedly. In particular, the lowest income group is substantially more mobile in Germany than in the USA. Poverty is rapidly overcome in Germany, whereas it is persistent in the USA. All other income groups are more mobile in the USA. (ii) The aggregation rule places large weights on local income changes at the bottom of the distribution. The relative global mobility ranking of the USA and Germany is therefore driven mainly by this income group. The depiction of the weighted distributional change, $w_M(x;F)d(F(x)-G(x))$, for both countries leads directly to this conclusion.

This paper is organized as follows: Section 2 is devoted to the global analysis, in which we define the class of mobility indices to be examined, and we report global mobility measures for both Germany and the USA. Our results confirm the empirical puzzle which attributes often a higher extent of mobility to Germany than to the USA. Section 3 contains the local analysis. We first examine the image of the mobility map and then proceed to investigate the map itself, deriving an easily interpretable approximation to the aggregation rule. Section 4 concludes.

2 Global mobility indices

A mobility index seeks to quantify the extent of income changes by mapping the joint distribution of incomes into a single scalar. It therefore offers a "global" assessment of mobility by aggregating "local" income changes according to some rule. This section presents the class of global mobility measures to be examined, and reproduces the empirical puzzle observed by Burkhauser et al. in their comparison of the USA and Germany which calls into question the practical usefulness of measuring mobility.

One important class of mobility measures, introduced by Shorrocks (1978a), often employed in the applied literature comprises stability indices. These indices measure the reduction of income inequality occurring when the accounting period is extended beyond a single period. The stability index is based on the comparison between inequality of (time-)averaged income and a weighted sum of single period income inequalities. The more inequality is reduced by looking at long-term rather than short-term incomes, the higher is income mobility.

More precisely, let Y_t denote the random variable personal income received in period t which is drawn from distribution F_{Y_t} and has realization y_t. We restrict our attention to the two-period case in order to simplify the exposition. All methods can be extended to larger accounting periods. The joint distribution of incomes in both periods is denoted by F_{Y_2,Y_1}, the conditional distribution by $F_{Y_2|Y_1}$. Let $G = G_{0.5(Y_1+Y_2)}$ denote the distribution of average income received by a person. Define the functional $\mu_\alpha(F) = \int y^\alpha dF(y)$. For an accounting window of two periods, the stability index M_I based on an inequality measure I is defined by

$$M_I = 1 - \frac{I(G)}{\lambda_1 I(F_{Y_1}) + \lambda_2 I(F_{Y_2})} \tag{1}$$

where λ_t is the weight attached to period t. These weights sum to one. The typical choice is $\lambda_t = \mu_1(F_{Y_t})/[\mu_1(F_{Y_1})+\mu_1(F_{Y_2})]$, the share of total aggregate income accruing in period t. Sometimes the numerator in (1) is defined to be the inequality of total income. Since I is scale-invariant, this equals inequality of time-averaged income.

Our choice for the inequality index is the Generalized Entropy measure (or Theil's index) defined by

$$GE_\alpha(F_{Y_t}) = \begin{cases} \frac{1}{\alpha^2-\alpha}\left[\mu_\alpha(F_{Y_t})\mu_1(F_{Y_t})^{-\alpha} - 1\right] & \alpha \notin \{0,1\} \\ \mu_1(F_{Y_t})^{-1}\mu_1(F_{Y_t \ln Y_t}) - \ln \mu_1(F_{Y_t}) & \alpha = 1 \\ -\mu_1(F_{\ln Y_t}) + \ln(\mu_1(F_{Y_t})) & \alpha = 0 \end{cases} \tag{2}$$

since its sensitivity properties can be controlled by the parameter α. The smaller is α, the larger is the sensitivity of the inequality index to the lower tail of the income distribution. This index, however, is not monotonic in α.

The usual estimator of the inequality index is based on the empirical distribution function, $GE_\alpha(\widehat{F_{Y_t}})$, which in turn yields the usual estimator of the stability index $\widehat{M_{GE_\alpha}}$. However, income is not identically distributed since sample inclusion probabilities vary between households. This is easily remedied in the estimation by using the weighted empirical distribution function, the weights being the inverse of the sample inclusion probabilities

$$\widehat{F_{Y_t}}(x) = \frac{1}{\sum_{j=1}^n w_t^{(j)}} \sum_{j=1}^n w_t^{(j)} 1(Y_t^{(j)} \leq x)$$

where $Y_t^{(j)}$ denotes income of person j received in period t and $w_t^{(j)}$ the person's sample weight.

For the empirical analysis, the income concept used in this paper is real equivalized post-tax post-benefit household income, assumed to be shared equally amongst household members. We examine personal income mobility, since household composition may change over time. In order to account for different household sizes and economies of scale, incomes are needs-adjusted using the OECD equivalence scale by dividing incomes by the square root of household size. Incomes are evaluated at 1996 prices. This income definition is the most comprehensive single measure of economic welfare available. The data are taken from the US Panel Study of Income

Table 1: Global mobility indices

Years	$\widehat{M_{GE_{-1}}}$	$\widehat{M_{GE_0}}$	$\widehat{M_{GE_1}}$	$\widehat{M_{GE_2}}$
	USA (PSID)			
1984–1985	0.1741	0.0998	0.0771	0.0757
1985–1986	0.1662	0.0992	0.0786	0.0780
1986–1986	0.1488	0.0924	0.0762	0.0775
1987–1988	0.1504	0.0926	0.0765	0.0794
1988–1989	0.1525	0.0933	0.0759	0.0780
1989–1990	0.1528	0.0902	0.0728	0.0749
1990–1991	0.1815	0.0992	0.0760	0.0773
1991–1992	0.2639	0.1209	0.0917	0.0961
	Germany (GSOEP)			
1984–1985	0.1805	0.1268	0.1076	0.1036
1985–1986	0.1391	0.1090	0.0959	0.0937
1986–1987	0.1374	0.1056	0.0925	0.0904
1987–1988	0.1308	0.0999	0.0865	0.0837
1988–1989	0.1344	0.1051	0.0943	0.0947
1989–1990	0.1412	0.1116	0.1004	0.1005
1990–1991	0.1424	0.1078	0.0924	0.0882
1991–1992	0.1559	0.1096	0.0900	0.0843

Dynamics (PSID) and the German Socio-Economic Panel (GSOEP) in their incarnations as the "Equivalent Data Files" distributed by Syracuse University[2]. The data provider generates comparable income variables from the raw data. We examine the (calendar) time period 1983 to 1992, a period during which the USA and Germany moved through a largely synchronized business cycle. The accounting period for our mobility analysis comprises always two subsequent years in order to simplify exposition. However, the methods can be easily extended to larger time periods. Our sample selection criteria[3] yield samples in excess of 10 000 persons. For instance, the German sample for 1983/84 contains 12 526 persons in 4 578 households, and the US sample 17 705 persons in 6 322 households.

Table 1 reports two-year mobility estimates for the USA and Germany as the sensitivity parameter α increases from -1 to 2. This rise tends to lower the mobility estimate for both countries. The table suggests that, surprisingly, Germany and the USA exhibit a similar extent of mobility. The precise mobility ranking depends both on the sensitivity parameter used and the time-period considered. For $\alpha \in \{0, 1, 2\}$

[2]See http://www.diw-berlin.de/ soep/ soepe.htm and http:// www-cpr.maxwell.syr.edu/ gsoep/ equivfil.htm.

[3]The data is obviously affected by unobservable measurement error, with consequences for estimation. Cowell and Schluter (1998) have shown that most mobility measures are not robust against outliers, so that a sufficient large single outlier can render the mobility measure uninformative. To prevent this, we have trimmed the data symmetrically by deleting households whose income falls below the 1% quantile or exceeds the 99% quantile. The number of deletions is small but the gain from robustification is likely to outweigh the loss of information.

Germany is deemed more mobile than the USA but for the last period 1991/92 when the USA value jumps up. For $\alpha = -1$, the USA is deemed more mobile except for the first period. This result (mirroring those of Burkhauser, Holtz-Eakin, and Rhody (1997) and Burkhauser and Poupore (1997)) is rather surprising and contrary to the received wisdom. It raises the question of whether the similarity of the global mobility estimates is based on similar profiles of local distributional change.

3 Local mobility assessment

To understand why a global mobility index produces a certain result we need to look at local income changes. Two questions have to be addressed. First, who moves where? And second, how are the local income changes aggregated into a global index? The former question is tackled in section 3.1; we present a graphical method which depicts local distributional changes using quantile-lines. The aggregation problem is dealt with in section 3.2.

3.1 Quantile-lines and local distributional change

The distribution determining local income changes around some first period income level y_1 is characterized by the conditional distribution $F_{Y_2|Y_1}(.|y_1)$. A concise representation depicts selected quantiles of the distribution, rather than the entire distribution itself. Letting y_1 move through the income domain we obtain "quantile-lines", i.e. the loci of the selected quantiles. The depiction of the quantile-lines of $F_{Y_2|Y_1}$, as in Figures 1 and 2, is insightful, because it separates out two distinct features of mobility[4], typically mixed up by mobility indices, which correspond to their slope and spread[5]:

1. Predictability and randomness: the position of the quantile-lines relative to the 45 degree line capture what is perhaps the intuitive meaning of the word (im)mobility. At one extreme is perfect predictability. The income distribution exhibits perfect immobility if all quantile lines coincide with the 45 degree line; all income receivers have maintained their relative income positions and incomes are unchanged. Similarly, by reflecting all incomes at the median, so that all n ordered income receivers swap income positions deterministically by interchanging the ith and the $n-i$th incomes, all quantile lines would coincide with the minor diagonal. In the latter case, income would change substantially, but this change is entirely deterministic. The other extreme is perfect randomness: horizontal quantile lines reveal that the distribution of second period incomes is independent of first period incomes. This feature of mobility is captured by the slope of the quantile-lines, which may differ between different income groups. In order to separate out this aspect of mobility, Figures 1 and 2 also depict the locus of the conditional mean $E(Y_2|Y_1)$.

[4]See also Shorrocks (1978b): "Interest in mobility is not only concerned with movement but also with predictability....movement and predictability issues are not always in accord.."(p.1016).

[5]We assume that $F_{Y_2|Y_1}$ is continuous, so that these features can only vary smoothly over y_1.

2. Variability: this aspect is captured by the vertical spread of the quantile lines, which is just the spread of the conditional distribution $F_{Y_2|Y_1}(.|y_1)$. This spread constitutes the local (ex-ante) variability or riskiness of second period incomes seen from the viewpoint of the first period. Different income groups may have to bear different income risk. We have isolated this aspect by depicting the locus of the conditional variance $Var(Y_2|Y_1)$.

In the empirical implementation, the conditional distributions are estimated non-parametrically using standard kernel density estimation techniques which employ a Gaussian kernel and a subjective choice of bandwidth (cf. also Trede (1998)). In order to enable a comparison between the patterns of income change across countries, incomes are measured relative to the country-specific mean income in the first period. Figures 1 and 2 depict the loci of the 0.1, 0.3, 0.5 and 0.7 quantiles. Since the quantile-lines exhibit a remarkable empirical regularity, we have chosen to depict only two periods, 1983/84 and 1990/91. The quantile-lines for both countries are J-shaped, cluster and spread around the 45 degree line. Those for the USA cross the German quantile-lines from below, as do the loci of the conditional mean and variance. Thus both aspects of mobility are present.

The Figures make clear that the local pattern of income changes is very different[6] between the USA and Germany – despite the fact that global mobility is measured to be roughly the same. They suggest that there are three relevant income groups defined in terms of first period income: low incomes (with relative incomes below 0.5, which is a poverty line often used in the empirical poverty literature), middle and high incomes. The quantile-lines for middle incomes in the USA are slightly above those for Germany, for higher incomes they move perceptibly further apart. These American income groups have higher conditional means than their German counterparts, but the distributional spread as measured by the conditional variance is roughly the same.

The most pronounced and, from a welfare point, the most important difference concerns the low income group. All German quantile-lines for this group lie substantially above those for the USA, as do the conditional means and variances. Those Germans who are poor in the first period experience large proportional income gains which move about half of this group close but slightly below the poverty line, the other half beyond it. Upward mobility of the German poor is thus considerable. By contrast, year-to-year poverty in the USA is highly persistent. Income improvements are only slight, and most people in poverty in the first period do not exit this state, as the 0.7 quantile-line only crosses the 0.5 threshold if first period incomes are close to 0.5.

Thus, the local profiles of income changes are quite different between the USA and Germany, the most pronounced difference being the substantially greater local mobility of those poor in the first period. The other income groups appear more mobile in the USA. Since the extent of measured mobility is similar, this observation raises the question of how the local income changes are aggregated in the global

[6] The method of statistical inference outlined in the appendix confirms that these differences are statistically significant.

mobility measure. In particular, is it one single income group which drives this mobility assessment ? This issue is addressed in the next section.

3.2 The aggregation rule

Although aggregate or "global" income mobility, measured in terms of stability indices, is similar in Germany and the USA, the conditional densities $F_{Y_2|Y_1}$ happen to be quite different. The most marked local difference concerns the low income group. However, such a picture cannot reveal the actual quantitative contribution to the aggregate mobility index made by this group. Thus it would be desirable to have an economically insightful aggregation rule, which makes transparent how local income changes are aggregated into the "global" mobility index. Such a rule then makes possible a comparison between the relative quantitative contributions of various income groups in one country or between similar groups across countries.

The aggregation rule, to be derived in this section as a first order approximation, simply states that measured mobility is obtained by integrating weighted distributional changes:

$$M_I = \int w(x; I, F) d(F(x) - G(x)) \tag{3}$$

where $G = G_{0.5(Y_1+Y_2)}$ is the distribution of time-averaged income, and F is a reference distribution which we choose to be the mixture of single period distributions $F = 0.5F_{Y_1} + 0.5F_{Y_2}$. $w(x; I, F)$ is a weighting function, capturing the mathematical structure of the mobility measure, which depends on the inequality index I chosen for the stability index and the reference distribution. The weighting function does not depend on information about local changes in personal income, which is exclusively contained in the term $d(F(x) - G(x))$. This difference constitutes the notion of distributional change captured by the stability index. In particular, substantial income improvements amongst those poor in the first period put a large distance between F and G, so $w(x; I, F)d(F(x) - G(x)) > 0$ within this low income interval (provided $w(x; I, F) > 0$). If income jumps are short range and poverty is persistent, then $w(x; I, F)d(F(x) - G(x))$ can become negative in this interval.

This aggregation rule differs markedly from decomposition rules proposed in the literature, which depend on the additive decomposability of a suitably chosen inequality index into "within-group" and "between-group" components after partitioning the sample into discrete income groups[7]. In contrast to such decompositions, our proposed rule has the advantage of (i) being linear, (ii) not requiring arbitrary discretizations, (iii) clearly showing the impact of distributional change, and (iv) being easily depicted graphically.

We proceed with deriving the aggregation rule for the stability index (1) with the usual weights $\lambda_t = \mu_1(F_{Y_t})/[\mu_1(F_{Y_1}) + \mu_1(F_{Y_2})]$, per period mean income divided by average total income. The key insight is to find a convenient approximation to $I(G)$. Consider two points H and F in the space $\mathfrak{F}$ of all income distributions. The Gateaux

[7]See, for instance, Shorrocks (1980) or Cowell (1980), and Buchinsky and Hunt (1999).

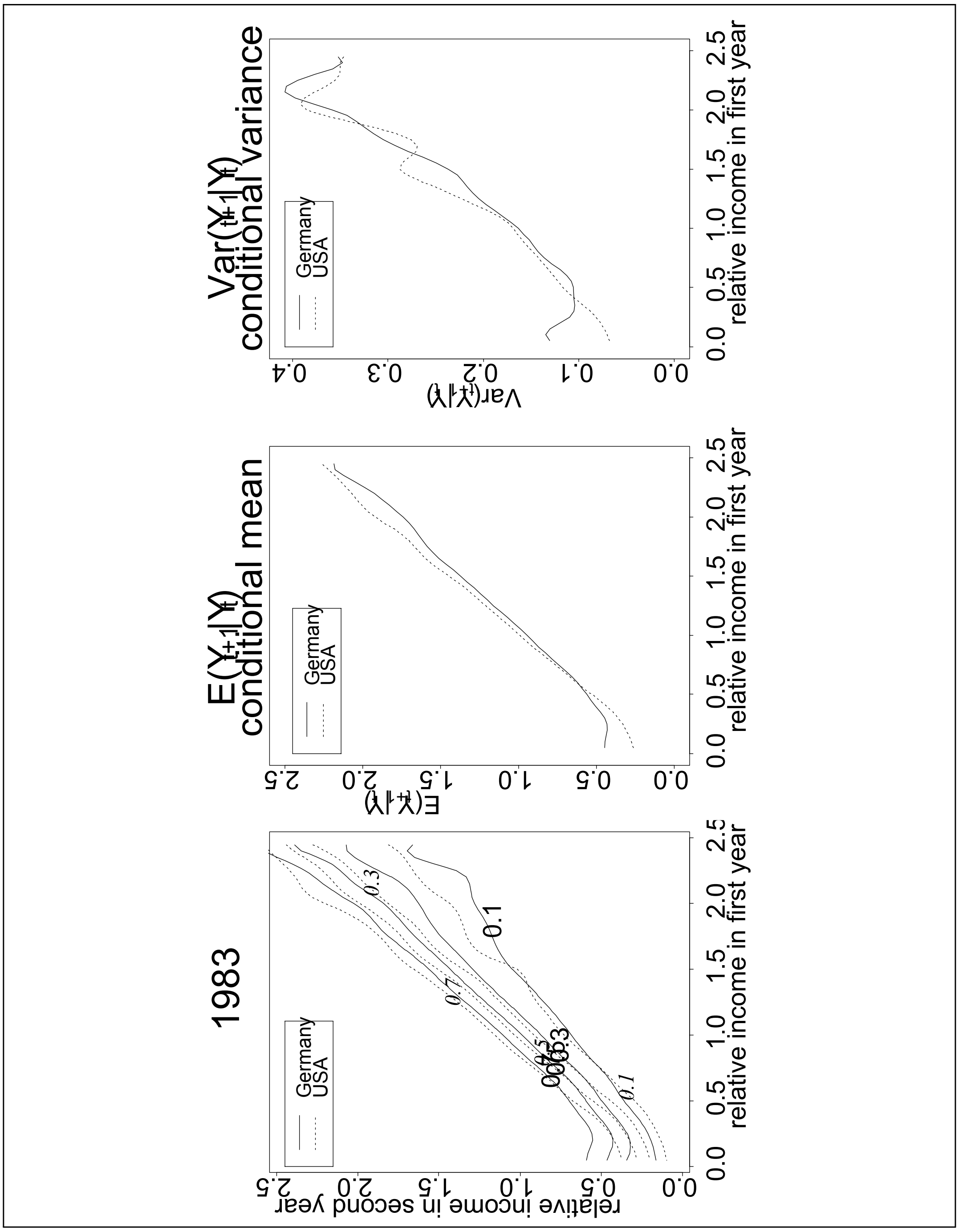


Figure 1: Quantile-lines, and the loci of the conditional means and variances for the period 1983/84. Italicised numbers in the first panel refer to quantiles of the US conditional distribution.

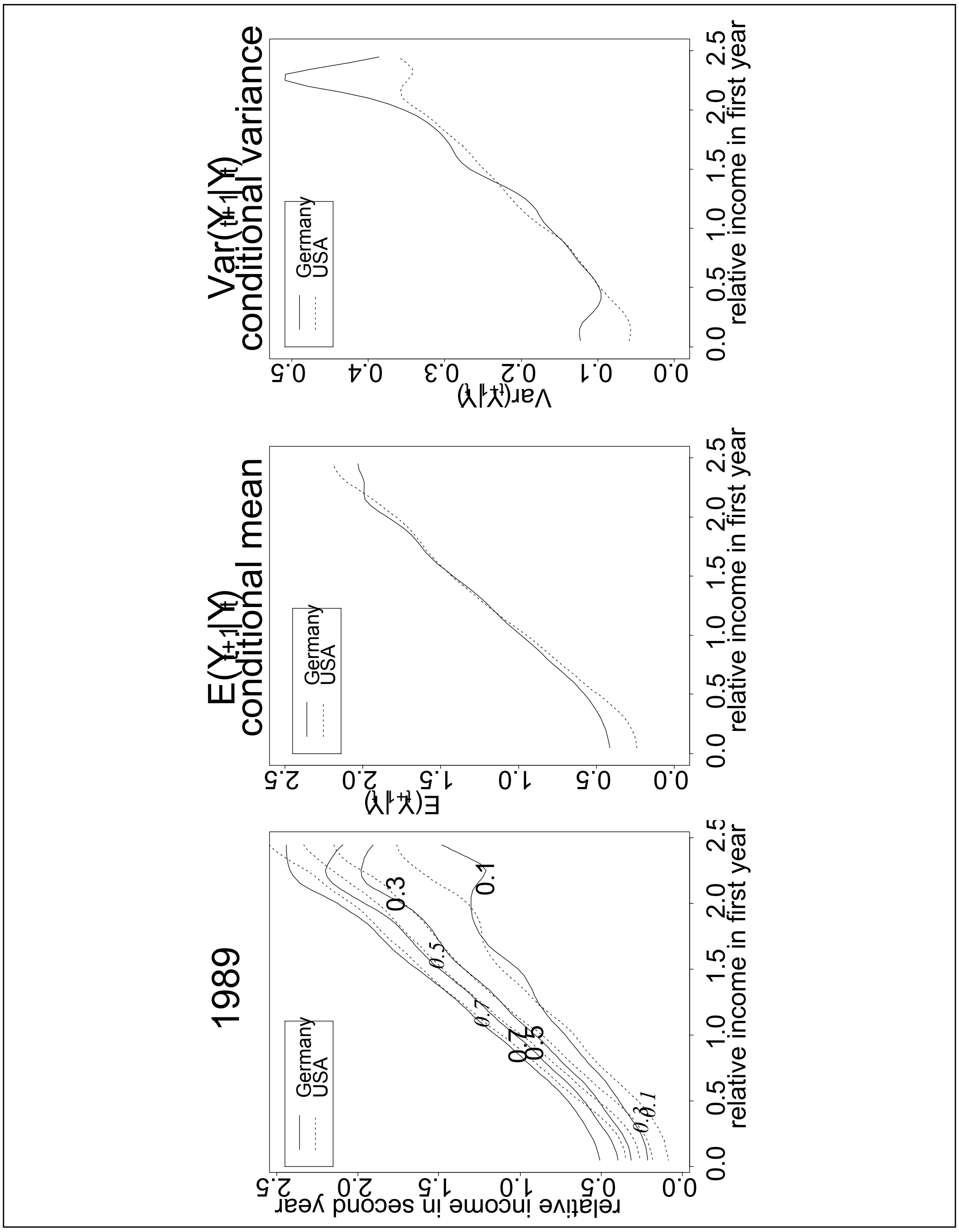


Figure 2: Quantile-lines, and the loci of the conditional means and variances for the period 1989/90. Italisised numbers in the first panel refer to quantiles of the US conditional distribution.

differential of I at F in the direction of H is defined by (see also Serfling (1980))

$$\begin{aligned} d_1 I(F; H-F) &= \lim_{\varepsilon \downarrow 0} \frac{I(F+\varepsilon(H-F)) - I(F)}{\varepsilon} \\ &= \frac{d}{d\varepsilon} I(F+\varepsilon(H-F))|_{\varepsilon=0} \end{aligned} \tag{4}$$

The special case of $H(y) = 1_x(y)$, which puts a mass of one on point x, yields the influence function of the functional I

$$IF(x; I, F) = \frac{d}{d\varepsilon} I(F+\varepsilon(1_x - F))|_{\varepsilon=0} \tag{5}$$

The influence function is a measure of the sensitivity of the inequality index to incomes at point x. In the particular case of the generalized entropy index $I = GE_\alpha$ a simple differentiation yields

$$IF(x; GE_\alpha, F) = A(F) + B(F)x^\alpha + C(F)x, \qquad \alpha \notin \{0, 1\} \tag{6}$$

where $A(F) = GE_\alpha(F) + 1/\alpha$, $B(F) = \mu_1(F)^{-\alpha}[\alpha^2 - \alpha]^{-1}$,
and $C(F) = -\mu_1(F)^{-1}[\alpha GE_\alpha(F) + (\alpha - 1)]$.

A first order Taylor expansion, valid under the usual regularity conditions, of the inequality functional $I(G)$ about F yields

$$\begin{aligned} I(G) &= I(F) + d_1 I(F; G-F) \\ &= I(F) + \int IF(x; IF) d(G(x) - F(x)) \end{aligned} \tag{7}$$

It remains to choose a convenient reference distribution F. Consider $\lambda_1 I(F_{Y_1}) + \lambda_2 I(F_{Y_2})$. Since the weights sum to one and per-period inequality does not change much from year to year, this sum is well approximated by $I(F)$, where $F = \lambda_1 F_{Y_1} + (1-\lambda_1)F_{Y_2}$ is the mixture of the single period income distributions.[8] Empirically, mean (real) personal income does not change much from year to year. Using this fact and (7) in (1) yields the first order approximation to the aggregation rule

$$\begin{aligned} M_I &= -\frac{1}{I(F)} \int IF(x; I, F) d(G(x) - F(x)) \\ &= \int w(x; I, F) d(F(x) - G(x)) \end{aligned} \tag{8}$$

with $w(.) = IF(.)/I(.)$. Mobility is thus the aggregated weighted differences between the reference distribution F and the distribution of time-averaged income G. The weighting function is proportional to the influence function of the inequality index, and thus reflects the dependence of the mobility index on the inequality index. It

[8] The approximations are rather accurate. For instance, using the GSOEP income data for 1983/84 the Genralized Entropy index with parameter $\alpha = 2$ is $GE_2(G) = 0.07194$ for average income and $GE_2(F) = 0.08064$ for the mixture. The approximation of $GE_2(G)$ by means of (7) is $GE_2(G) = 0.07177$.

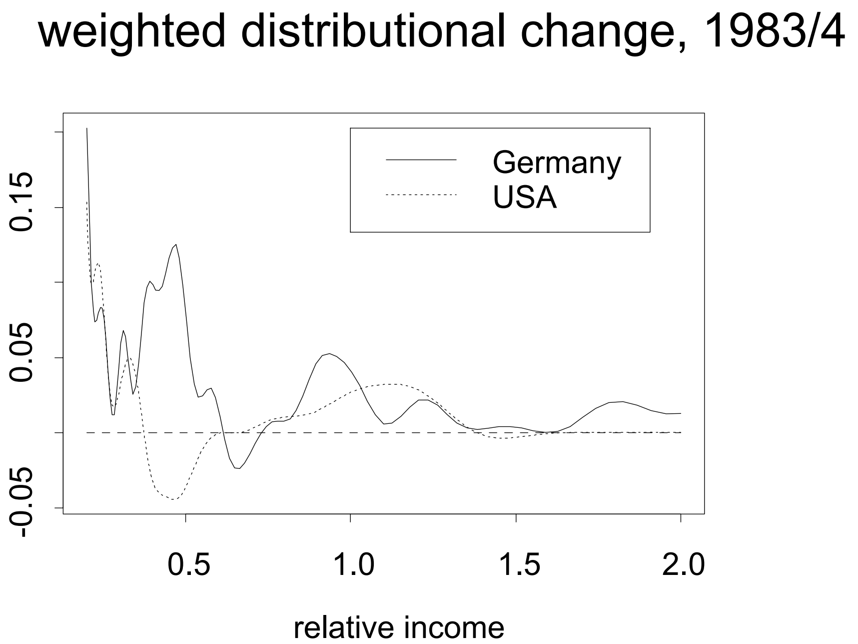
measures the sensitivity of the inequality index to point x, and thus weights the contribution of distributional change at income level x to overall mobility. Since the inequality index GE_α is non monotonic in α , it follows that the weighting function and thus the mobility index M_{GE_α} is not monotonic in α.

It should be emphasized that the aggregation rule is a first order approximation to the mobility index. This is adequate for most empirical analyses, since distributions of (real) personal income change only slightly over time, and the distribution of time-averaged income is "close" to the per-period reference distribution. This is certainly the case for the empirical analysis presented below. However, higher order Gateaux differentials can be used to improve the approximation should the distance between the distributions be excessively large.

The empirical implementation requires estimation of the distributions G and F. Non-parametric estimators are obtained using standard kernel density estimation techniques. In order to account for the skewness of the income distributions , the estimation was carried out on log income. A cross-validation method determined the choice of the bandwidth. Since we are not only concerned with the local contributions of various groups to the global mobility index, but also with their comparison between the USA and Germany, we depict $w(x; GE_\alpha, F)d(F(x) - G(x))$ in the common metric which measures incomes relative to country-specific average income $0.5(\mu_1(F_{Y_1}) + \mu_2(F_{Y_2}))$. We have chosen $I = GE_2$ as the inequality measure. Figure 3 depict the results for selected years. The shape of the local contributions to the global mobility index change only little over time. The pattern of income changes thus exhibits a striking empirical regularity.

Consider first Germany. The prominent feature of the figure is that people on low incomes (relative incomes less than 0.5) make the largest contribution to measured mobility. Other income groups make less pronounced contributions. These observations confirm what the conditional distributions, depicted in Figures 1 and 2 , suggested – large proportional income improvements on average for the low income group – but we are now in a position to quantify precisely their contribution to the mobility index. As regards the local contributions to mobility in the USA, the largest contribution comes from middle incomes whereas the low income group makes a negative contribution, suggesting persistent poverty. The US poor experience only little income improvements. Comparing F and G, the predominantly short range income movements amongst the poor renders dG larger than dF, so $w(x; GE_\alpha, F)d(F(x) - G(x)) < 0$ within this range. In terms of moving from per period income to time-averaged income, entries into this interval exceed exits from it. In other words, the income improvement is typically not large enough for them to exceed the typical poverty line at 0.5.

The contrast between the American and German local mobility profiles is thus striking. Although both countries are measured to exhibit a similar extent of mobility, it is a different income group which drives the mobility index. Welfare assessments of distributional change will differ greatly. People on low income contribute substantially to mobility in Germany, whilst poverty in the USA is persistent.



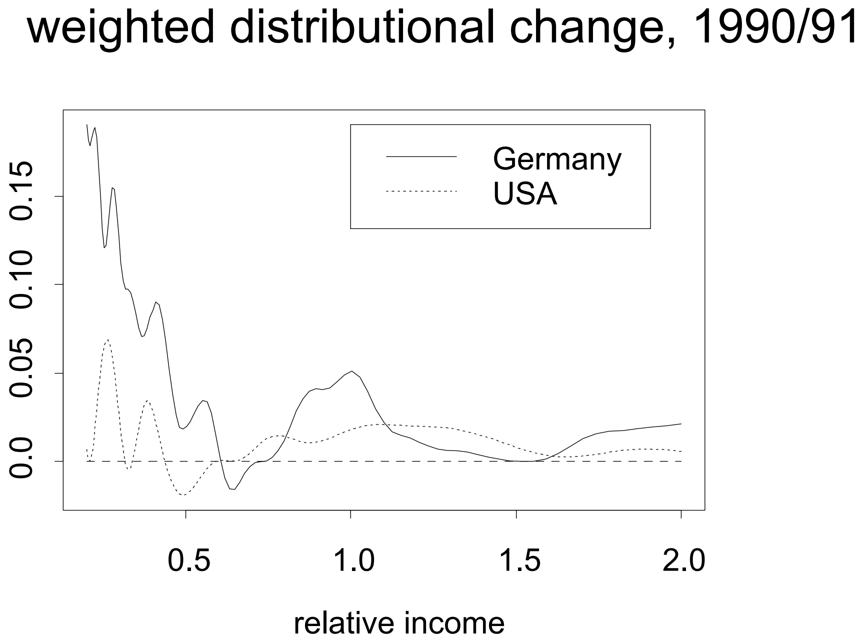


Figure 3: The local contributions of distributional change to the stability index M_{GE_2}. Depicted is $w(x; GE_2, F)d(F(x) - G(x))$. G is the distribution of time-averaged income, and the reference distribution F is the mixture $0.5(F_{Y_1} + F_{Y_2})$.

4 Conclusion

The common approach to measuring income mobility is to compute a mobility index, which reduces the information about income changes contained in the joint distribution of incomes into a scalar. Information about "local" income changes is aggregated into a "global" mobility index. This paper has shown the importance of a "local" analysis, in which we have examined separately the image of the mobility map using a graphical method. As regards the map itself, we have derived an easily interpretable approximation to the aggregation rule for the important class of so-called stability indices. By comparing global mobility estimates and local distributional change between the USA and Germany, we can explain the empirical puzzle observed by Burkhauser et al. (1997) who find that Germany has more income mobile than the USA. We have shown that the relative global mobility ranking of the two countries is driven by the first-period poors.

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A Statistical inference for the quantile lines

This appendix outlines a non-parametric method for making inferences for the quantile-lines, which are depicted in Figures 1 and 2 above. We compute confidence bands around the estimated studentized quantile line using the bootstrap rather than asymptotic confidence bands, because of the superior performance of the former in finite samples. Thus we bootstrap an asymptotically pivotal statistic, see e.g. Hall (1992). Consider the conditional distribution $F_{Y|X}$ and fix $X = x$ to simplify notation. Let the estimated quantile be denoted by $\widehat{y_p}$ which is defined by $p = \widehat{F_{Y|X}}(\widehat{y_p}|x)$ where $\widehat{F_{Y|X}}$ is our non-parametric estimate of $F_{Y|X}$. Assume that we have an estimator of the variance of $\widehat{y_p}$, denoted by $\widehat{Var}\left(\widehat{y_p}\right)$. We then bootstrap $\widehat{y_p}\left(\widehat{Var}\left(\widehat{y_p}\right)\right)^{-0.5}$: repeating this computation for a large number of independent drawings from the empirical distribution yields a bootstrap distribution, whose α and $(1-\alpha)$ order statistic determine the lower and upper bound of the α-confidence band. Finally, we rescale these bounds by multiplication with an estimate of the square-root of the variance of the original quantile line. As an example, Figure 4 depicts a 95% confidence band for the 0.1-quantile line for Germany for the year 1984/85 using 300 bootstrap replications.

We proceed to derive $\widehat{Var}\left(\widehat{y_p}\right)$ in three steps.

1. Since $p = \widehat{F_{Y|X}}(\widehat{y_p}|x)$, a first order Taylor expansion around y_p yields

$$\left(\widehat{y_p} - y_p\right) = \left[\frac{d}{dy_p}\widehat{F_{Y|X}}\right]^{-1}\left(\widehat{F_{Y|X}}(y_p|x) - F_{Y|X}(y_p|x)\right)$$

with $\frac{d}{dy_p}\widehat{F_{Y|X}} = \widehat{f_{X,Y}}(x, y_p)/\widehat{f_X}(x)$. It follows that

$$Var\left(\widehat{y_p}\right) = \left[\widehat{f_{X,Y}}(x, y_p)dz/\widehat{f_X}(x)\right]^{-2} Var\left\{\widehat{F_{Y|X}}(y|x)\right\}$$

2. Our estimator $\widehat{F_{Y|X}}$ is an integrated density estimator

$$\widehat{F_{Y|X}}(y|x) = \int_0^y \widehat{f_{Y|X}}(z|x)dz = \int_0^y \widehat{f_{Y,X}}(z, x)dz/\widehat{f_X}(x)$$

Write $\widehat{F_{Y|X}} = g\left(\widehat{\theta}\right)$ with $\widehat{\theta} = \left(\int_0^y \widehat{f_{Y,X}}(z, x)dz, \widehat{f_X}(x)\right)$. Collect the covariances of $\widehat{\theta}$ in the matrix σ. A first order Taylor expansion around θ then yields $Var\left\{\widehat{F_{Y|X}}(y|x)\right\} = \left(\partial g(\theta)/\partial\theta\right)|_{\theta=\widehat{\theta}}\sigma\left(\partial g(\theta)/\partial\theta\right)|_{\theta=\widehat{\theta}}^{\top}$.

3. It remains to derive σ. Consider first $\int_0^y \widehat{f_{X,Y}}(x, z)dz$ which, using a Gaussian kernel with density ϕ and cdf Φ and bandwidth h, is given by

$$\int_0^y \widehat{f_{X,Y}}(x, z)dz = n^{-1}h^{-1}\sum_i \phi\left(\frac{x - X_i}{h}\right)\Phi\left(\frac{y - Y_i}{h}\right)$$

We have

$$Var\left\{\int_0^y \widehat{f_{X,Y}}(x,z)dz\right\} = n^{-1}h^{-2}Var\left\{\phi\left(\frac{x-u}{h}\right)\Phi\left(\frac{y-v}{h}\right)\right\}$$

Next consider

$$\begin{aligned} E\left\{\phi\left(\frac{x-u}{h}\right)^2\Phi\left(\frac{y-v}{h}\right)^2\right\} &= \int\int\phi\left(\frac{x-u}{h}\right)^2\Phi\left(\frac{y-v}{h}\right)^2 f_{X,Y}(u,v)dudv \\ &= h^2\int\phi\left(a\right)^2\left\{\int\Phi\left(b\right)^2 f_{X,Y}(x-ah,y-bh)db\right\}da \end{aligned}$$

Integrate the bracketed term by parts and expand the densities to obtain

$$\begin{aligned} & E\left\{\phi\left(\frac{x-u}{h}\right)^2\Phi\left(\frac{y-v}{h}\right)^2\right\} \\ = \;& h\int\phi\left(a\right)^2\left\{\int 2\Phi(b)\phi(b)\left\{\int_0^y f_{X,Y}(x-ah,z)dz - \int_{y-bh}^y f_{X,Y}(x-ah,z)dz\right\}\right\} \\ = \;& h\int_0^y f_{X,Y}(x,z)dz\int\phi\left(a\right)^2 da\int 2\Phi(b)\phi(b)db + O(h^2) \end{aligned}$$

$E\left\{\phi\left(\frac{x-u}{h}\right)\Phi\left(\frac{y-v}{h}\right)\right\}$ can be derived in a similar fashion. It follows that

$$\begin{aligned} & Var\left\{\int_0^y \widehat{f_{X,Y}}(x,z)dz\right\} \\ = \;& n^{-1}h^{-2}\left[h\int_0^y f_{X,Y}(x,z)dz\int\phi\left(a\right)^2 da\int 2\Phi(b)\phi(b)db - \left(h\int_0^y f_{X,Y}(x,z)dz\right)^2\right] \\ = \;& n^{-1}h^{-1}\int_0^y f_{X,Y}(x,z)dz\int\phi\left(a\right)^2 da\int 2\Phi(b)\phi(b)db + O(n^{-1}) \end{aligned}$$

Similarly we obtain

$$Var\left\{\widehat{f}(x)\right\} = n^{-1}h^{-1}f(x)\int\phi(z)^2dz + O(n^{-1})$$

and

$$cov\left(\int_0^y \widehat{f_{X,Y}}(x,z)dz, \widehat{f_X}(x)\right) = n^{-1}h^{-1}\int_0^y f_{X,Y}(x,z)dz\int\phi\left(a\right)^2 da + O(n^{-1})$$

The last three equations define the covariance matrix σ.

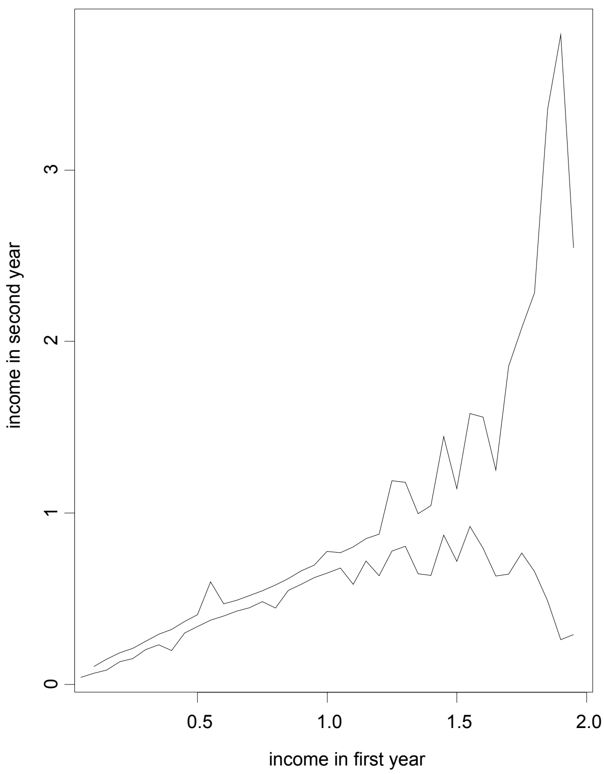

Figure 4: 95% confidence band of the 0.1-quantile line for Germany in 1983/1984

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