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Multivariate Dispersion, Central Regions and Depth: The Lift Zonoid Approach

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Abstract:

The book introduces a new representation of probability measures – the lift zonoid representation – and demonstrates its usefulness in multivariate analysis. A measure on the Euclidean d-space is represented by a convex set in $(d+1)$-space, its lift zonoid. This yields an embedding of the d-variate measures into the space of symmetric convex compacts in $\mathbb{R}^{d+1}$. The embedding map is positive homogeneous, additive, and continuous. It has many applications in data analysis as well as in inference and in the comparison of random vectors.

First, lift zonoids are useful in multivariate data analysis in order to describe an empirical distribution by central (so-called trimmed) regions. The lift zonoid trimmed regions range from the convex hull of the data points to their mean and characterize the distribution uniquely. They give rise to a concept of data depth related to the mean. Both, the trimmed regions and the depth, have nice analytic and computational properties. They lend themselves to multivariate statistical inference, including nonparametric tests for location and scale.

Secondly, for comparing random vectors, the Hausdorff distance between lift zonoids defines a measure metric and the set inclusion of lift zonoids defines a stochastic order. The lift zonoid order reflects the dispersion of random vectors and is slightly weaker than the multivariate convex order. The lift zonoid order and related orderings have a broad range of applications in stochastic comparison problems in economics and other fields.