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SEMINAR OF ECONOMIC AND SOCIAL STATISTICS
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On the Impact of Weather on German Hourly Power Prices

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[1]Graduate School of Risk Management, University of Cologne, Albertus Magnus Platz, 50923 Cologne, GERMANY

Abstract

The liberalization of electricity markets has triggered research in econometric modelling and forecasting of electricity spot prices. Moreover, both the demand and the supply of electricity are subject to weather conditions. Therefore, we examine the relation between hourly electricity spot prices from the European Energy Exchange and weather, represented by temperature and wind velocity. Furthermore, we assess whether the relation can be successfully exploited for forecasting. Thereby, we proceed in the framework of Markov regime-switching models which have become a workhorse in econometric modelling of electricity spot prices. As a result, we detect a strong relationship, on one hand. On the other hand, the significance of this relation for forecasting is confined to certain hours.

Keywords: Electricity spot prices, Weather, Markov regime-switching

JEL Classification: Q40, L94, C22.

1 Introduction

Until recently, the German electricity sector has been a vertically integrated industry and prices have been fixed by regulators. The progressing deregulation has triggered the launch of the European Energy Exchange in Leipzig (EEX). At the EEX, wholesale electricity spot prices for 24 hours of the following day are determined through an auction. These day-ahead prices are typically referred to as spot prices. A model, which accurately reflects, electricity spot price dynamics is of crucial interest for the delivery of power in day-ahead markets, because delivery is in need of reliable short-term spot price forecasts. Moreover, since spot prices are determined in auctions, good spot price forecasts enable the market participants to optimize their bidding strategies and to adjust their electricity production schedule in order to maximize their profits. Spot prices exhibit stylized facts which differ from those of other traded commodities and financial securities. Electricity cannot be stored and as a result enormous price fluctuations and extreme spikes, reflected by high volatility in electricity markets, are observed. Spikes are usually explained either by unexpected outages of large power plants or unpredicted changes of weather conditions. In most cases, spikes are very short-lived but they can also last for several days in a row. Furthermore, several seasonality cycles and mean reversion are typical of power prices.

The literature on power prices is still limited but fast growing. Important initial articles are those of Knittel and Roberts (2001),[23], and of Lucia and Schwartz (2002),[24]. Knittel and Roberts (2001),[23], evaluate the forecast performance of several univariate models using Californian power prices. Moreover, they successfully include temperature as covariate. Lucia and Schwartz (2002),[24], present analytic formulas for the pricing of power derivatives. In addition, they take seasonality and mean reversion into account. Escribano, Peña and Villaplana (2002),[10], suggest a very general jump model approach. They incorporate mean reversion, spikes and generalized autoregressive conditional heteroscedasticity (GARCH) in their approach for the modelling of power prices. GARCH models proposed by [4] generalizing the ARCH models proposed by [9] are the most popular models to capture volatility clustering in financial markets. Moreover, Cuaresma, Hlouskova, Kossmeier and Obersteiner (2004),[8], carry out a forecast study with several linear univariate time series models. They use data from the European Energy Exchange (EEX)in Germany. Moreover, Angeles Carnero, Koopman and Ooms (2003),[1], provide empirical evidence of periodic extensions of regression models with autoregressive fractionally integrated moving average disturbances for the analysis of daily spot prices. They apply their models to four different markets. Furthermore, Burger, Klar, Müller and Schindlmayer (2004),[7], derive a spot market model for hourly power prices at the EEX. They base their model on economic fundamentals of power prices in combination with a seasonal autoregressive integrated moving average approach. Rambharat, Brockwell and Seppi (2005),[5], propose a threshold autoregressive model for daily data from Pennsylvania. They incorporate a flexible mean reversion rate depending on temperature and spikes. Basic idea behind non-linear Markov regime-switching approaches in the spirit of Hamilton (1989),[16], is to model spikes as a separate regime. Modelling approaches based on regime-switching have been suggested and successfully applied for instance by Ethier and Mount (1998),[11], Huisman and Mahieu (2003),[19], De Jong and Huisman (2003),[20], Kosater and Mosler (2005),[22]. The latter focussed on the forecasting ability of Markov regime-switching models whereas the remaining authors stressed applicability in derivative pricing. More recently, Misiorek, Trück, Weron (2005),[26], found that Markov regime-switching failed to outperform linear approaches, except for some extremely spiky weeks, in forecasting spot prices from the California Power Exchange. However, Mount,Ning and Cai (2005) [27] show that daily price spikes in the Pennsylvania-New Jersey-Maryland (PJM) Power Pool can be very accurately predicted one day-ahead if load and the reserve margin are included in the model specification and transition probabilities are modelled as functions of load and the reserve margin.

Besides [5],[23] and [27], research has yet been focussed on modelling pure stochastic processes for spot prices and the logarithm of spot prices. Surprisingly, weather as one important driving force of electricity demand and hence electricity prices has been neglected, so far. Our aim in this article is to shed some light on the relation between temperature, wind and hourly spot prices from the EEX in Leipzig. Lower temperatures cause a higher need for heating and therefore, increase the electricity demand. Moreover, high temperatures can affect demand for electricity due to the need for cooling. Furthermore, also the supply of electricity can be subject to high temperatures. During the extraordinary hot summer in 2003, for example, the operation of thermal power plants in Germany was affected due to poor cooling conditions. German energy policy seeks to promote wind energy by subsidizing the creation and the operation of windmills. The goal is to replace parts of thermal electricity production and to establish wind energy instead. Furthermore, the trading of emission allowances with the aim to reduce CO_2 emissions is supposed to endorse renewable energy resources and therefore in particular wind energy. Wind energy, however, is exposed to large uncertainty. Hence, we expect that uncertainty and risk due to weather in spot markets for electricity will rise and become therefore of crucial importance. Furthermore, for short run forecasting for operational planning will have to explicitly take into account weather in order to provide reasonable results. Therefore, we attempt to specify the general impact of temperature and wind on hourly spot prices, on one hand. On the other hand, we try to quantify the relation between weather and the probability of the occurrence of spikes. Our procedure is similar to [27], whereas we include weather data in the model specification and make the transition probabilities a function of weather. In a further step, load and the reserve margin could be included in the specification, too. However for Germany, there are not yet any continuous time series available for load as well as the reserve margin, which we could use. We proceed in the framework of nonlinear Markov regime-switching models proposed in[19],[20] and [22]. These models are very well suited to achieve our second goal which is linking weather and the probability of the occurrence of a spike. According to [3], Markov regime-switching models tend to pick up too many spikes. This is especially true for the closely related models in [20] and [22], whereas the model proposed in [19] rather ignores spikes. In order to gain an utmost realistic insight, we do not merely rely on one model but choose to work with three approaches. For the sake of numerical tractability, we confine the autoregressive part of each model to consist of an autoregressive process of order one. Furthermore, results in [22] suggest that the data generating process for spot prices, proposed in [20], is very close to the true underlying spot price process. This presumption entails an estimation procedure which is confined to the simple autoregressive process of order one. Moreover, we do not incorporate GARCH, since the heteroscedasticity is caused by spikes on one hand and periodic autocorrelations, see [1], on the other hand. Moreover, our approach allows to disentangle normal prices from spikes much better than the standard procedure in [16] does. Thereinafter, we work in a multi-model framework consisting of 24 distinct hourly price series. Besides weekly seasonality, hourly prices exhibit a strong intra-day pattern. We neglect this intra-day pattern in this study because there is empirical evidence that a multi-model specification should be preferred instead, [6],[8],[25],[26]. We fit each of the three models to each of the 24 hourly price series. For each of the three models, three versions denoted by A,B and C are considered. Version A is the pure stochastic model. Whereas version B includes temperature and wind to model the general impact of weather on spot prices, analogously to [23]. In version C, we take into account the general impact on one hand and attempt to quantify the relation between weather and spike-occurrence probability, on the other hand. However, decisions of market participants are not based on actual measured weather data but its forecasts. Unfortunately, weather data provider merely archive the actual measured values, whereas forecasts are discarded. Therefore, we are forced to take the actually measured weather data as an approximation of the forecasts instead. In order to not overcharge this article, we confine ourselves to report some important results maximally for three selected hours. The complete results for the remaining hours of the day can be provided by the author on request. The results for these three hours fairly well reflect the essential insights of the study. With regard to the results, including temperature and wind velocity into the model specification improves the fit throughout all hourly prices. The benefit of including weather-dependent time-varying transition probabilities, however, is confined to some hours across the day depending on the model, we specify.

Furthermore, we carry out a forecasting study in the framework of the model proposed in [22]. In this study, we estimate the model of interest for a sub-sample of the given historical data. We carry out one-step ahead forecasts for observations held back at the estimation stage. In the following step, we augment the sub-sample which we use for estimation by one observation and carry out fore-

casting again. As a result, we obtain for the model of interest that versions B and C merely clearly outperform version A for hours from 19 to 6.
The remainder of the paper is organized as follows. In Section 2, we present the data. Moreover, we define a deterministic model component of the logarithm of the spot price. In Section 3, we introduce the considered stochastic models. Furthermore in Section 4, we present and discuss results of the empirical study. Section 5 concludes the paper and gives hints for further research.

2 Data

The EEX is the largest national power exchange in Europe. Besides the 24 hourly prices, blocks of hourly prices are traded at the EEX. In this paper, we use data including hourly price series in Euro from the EEX and hourly temperature time series measured in $0.1 \cdot C^{\circ}$ as well as hourly wind velocity time series measured in $0.1 \cdot meter/second$ from four measuring stations Hamburg, Holzdorf, Mendig and Ulm in Germany. We have chosen these four measuring stations in order to represent all parts of Germany. Hamburg is located in the North, Holzdorf represents the East, whereas Mendig can be found in the West of Germany. Finally, Ulm is located in the South of Germany. All data time series range from June 16$^{\text{th}}$ 2000 to December 31$^{\text{th}}$ 2004. Additionally, we use temperature forecasts and actually measured values from 7 a.m. 05/01/2005 to 6 a.m. 06/01/2005 from Ulm and wind velocity forecasts as well as actually measured data from Holzdorf for the same period. The data has been provided by the Deutscher Wetterdienst (DWD). In order to shed some light on the relation between prices and weather, we have carried out some preliminary least square regressions. As a result, we obtained that the fit and the explanatory power of data depends on the measuring station it is taken from. In our data, we see that among the measuring stations temperature data from Ulm provides the best fit. The reason for the good performance of temperature from Ulm is its geographical location. In the south, industrial electricity demand is higher than in other parts of Germany. Therefore, electricity demand from this area is more important than from other areas. However in the case of wind, Hamburg and Holzdorf perform best. As opposed to [23], we do not include the average of the measuring stations, but only include temperature data from Ulm. In the case of wind velocity, we mainly use data from Holzdorf and for some few hours data from Hamburg. Hamburg performs well because this town is very near to the North Sea where many off-shore windmills are settled. Holzdorf also offers good conditions for the operation of windmills. Understanding which conditions are appropriate for the operation of windmills requires to take into account some technical facts. A windmill does not start working unless a wind velocity of round about 4 meter/second is reached (see [13]). Once this velocity is reached and exceeded, produced electricity is proportional to the cube of the present wind velocity. This relation holds unless wind velocity reaches a value of round about

12 meter/second. At this point, we reach the maximal energy output. If wind velocity exceeds the value of 25 meter/second, windmills are switched off, for the sake of safety. Subfigures 1a and 1b show a scatter plot for temperature from Ulm and the logarithm of power prices as well as a scatter plot for wind velocity and the logarithm of power prices. In table 1 and 2, we present some descriptive statistics for the measured values at hour 12 from the four measuring stations. Table 2 shows why Ulm is a bad location for windmills, whereas Hamburg and Holzdorf offer good conditions. As aforementioned, we have bought forecasts as well as the measured values in order to assess the quality of the approximation of forecasts by measured data. Subfigures 1c and 1d show the actually measured weather data and its one day-ahead forecasts for the given period from Ulm and Holzdorf, respectively. Furthermore, we have examined the relationship between hourly prices and actually measured weather data as well as hourly prices and the one day-ahead forecasts. Subfigures 1e and 1f show the results. We see that the one day-ahead forecasts are similarly correlated with the hourly price as the actually measured data, except for some evening hours in the case of wind velocity.

Table 1: Descriptive Statistics for temperature in C° (hour 12).

	Ulm	Mendig	Hamburg	holzdorf
Mean	11.6061	13.7936	12.2398	13.3696
Median	11.8	13.6	12.35	13.6
Maximum	35.0	37.8	33.2	35.7
Minimum	-10.8	-7.3	-7.0	-10.8
Std. Dev.	8.8890	8.3105	7.8397	9.0626
Skewness	0.0094	0.0876	0.0364	0.0074
Kurtosis	2.1956	2.3446	2.3124	2.1892

Spot prices tend to fluctuate around a long term equilibrium. This fluctuation is due to shifts in demand caused by weather, for example. This tendency of spot prices to revert to a long term equilibrium is described as mean reversion. Let P_t with time-index $t \in \{1, \ldots, T\}$ be the spot price. A standard mean reverting process has the following specification.

$$P_t = P_{t-1} + \alpha \cdot (\mu_M - P_{t-1}) + u_t\,, \quad u_t \sim \mathcal{N}(0, \sigma^2)\,. \tag{2.1}$$

In equation (2.1) the parameter μ_M is the long term equilibrium for the spot price while α measures the speed of reversion from the current to the long term equilibrium. Electricity spot prices usually rather seem to follow a lognormal than a normal distribution. Therefore, most authors e.g. [7],[10], [20] prefer working with the logarithm of power prices instead of the original price series.

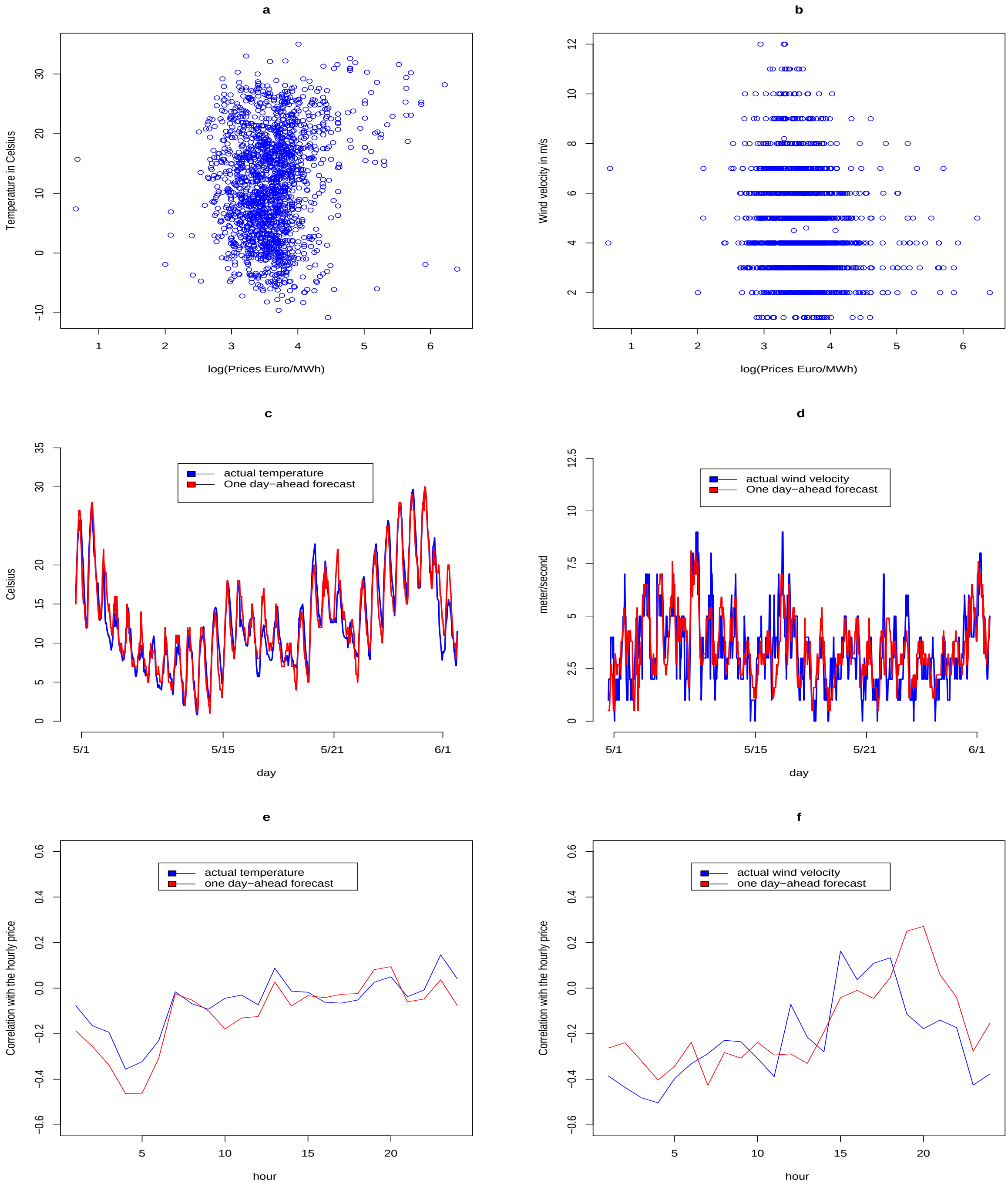

Figure 1: Scatter plots for temperature from Ulm and wind velocity from Holzdorf (hour 12) with the logarithm of the hourly price series (hour 12) (a,b), one day-ahead forecasts for temperature from Ulm and for wind velocity from Holzdorf (c,d), for 7 a.m. 1/05/2005 until 6 a.m. 1/06/2005, Correlation of measured temperature from Ulm and wind velocity from Holzdorf as well as its one day-ahead forecasts with the hourly price for 7 a.m. 1/05/2005 until 6 a.m. 1/06/2005 (e,f).

Table 2: Descriptive Statistics for wind velocity in meter/second (hour 12).

	Ulm	Mendig	Hamburg	Holzdorf
Mean	2.9167	4.1636	4.8483	4.5698
Median	2.7	4.0	4.7	4.0
Maximum	9.9	14.0	14.4	12.0
Minimum	0.0	0.0	0.0	1.0
Std. Dev.	1.3812	2.1739	2.0934	1.9613
Skewness	0.9459	0.6585	0.5424	0.7768
Kurtosis	4.1096	3.4869	3.3148	3.5896

In this paper, we follow their approach. According to [10] and [20] logarithm of power prices log(P_t) will be assumed to consist of two parts, a deterministic part denoted by f_t and a stochastic part X_t,

$$\log(P_t) = f_t + X_t. \tag{2.2}$$

According to [19],[20] and [22], we model the deterministic part of the logarithm of power prices as simply as possible but, on the other hand, still realisticly. In order to take into account the strong weekly seasonality, weekend dummy variables for Saturdays and Sundays as well as a sinusoidal term are included. Furthermore, we add a dummy variable for public holidays. Moreover, since the range of the data covers more than four years, we include a deterministic trend and a sinusoidal term to consider yearly seasonality. We have to mention that we do not drop the 29 th February in the leap year 2004 to maintain the weekly seasonality. The resulting error, however, is negligible. The deterministic part of the logarithm of power price f_t is specified as,

$$\begin{aligned} f_t \quad = \quad & \beta_1 \cdot dummy_{sat} + \beta_2 \cdot dummy_{sun} + \beta_3 \cdot dummy_{hol} + \beta_4 \cdot t \\ & + \gamma_1 \cdot \sin\left((\gamma_2 + t) \cdot \frac{2\pi}{365}\right) + \gamma_3 \cdot \sin\left((\gamma_4 + t) \cdot \frac{2\pi}{7}\right) . \end{aligned} \tag{2.3}$$

3 Stochastic Models

In this section, we outline the stochastic models which help us to examine the impact of temperature and wind on the logarithm of hourly spot prices at the EEX. For all models, we distinguish between a stable regime which reflects normal trading days, whereas the remaining regimes serve to model extraordinary periods with extreme prices. Moreover, we denote S as the regime parameter which takes M when power prices are in the stable regime and S else. For all three models, we assume for the stable regime,

$$X_{M,t} = \alpha \cdot \mu_M + (1-\alpha) \cdot X_{M,t-1} + u_{M,t}\,, \quad u_{M,t} \sim \mathcal{N}(0, \sigma^2) \qquad \text{stable regime}\,. \quad (3.1)$$

Thus, the three considered models only differ in the modelling of the spike regime. Therefore, we merely present how the spike regime is modelled. Moreover according to [20], we assume the regimes to be independent. The peculiarity of this assumption is that the autoregressive part is presumed to prevail in the stable regime only. This peculiarity is important for the maximum likelihood parameter estimation.

3.1 Model I: Mahieu and Huisman (2003)

Mahieu and Huisman (2003),[19], suggest a Markov regime-switching model for the logarithm of spot prices. Furthermore, they distinguish between a stable and two spike regimes. The idea of their model is that a deviation from the stable regime caused by an initial jump is immediately followed by a jump in the opposite direction. We denote the initial jump regime by $r_t = S$ and the subsequent reversal to the stable regime by $r_t = -S$. From the subsequent reversal regime, the price process moves back to the stable regime.

$$X_{r_t,t} = \mu_{r_t} + u_{r_t,t}\,, \quad u_{r_t,t} \sim \mathcal{N}(0, \sigma^2_{r_t}) \qquad \text{spike regimes} \qquad (3.2)$$

In equation (3.2), we follow [3] and model both spike regimes as random walk processes. Furthermore, we require according to the original model in [19] $X_{-S,t} \sim \mathcal{N}(-\mu_S, \sigma^2_S)$. Besides the same magnitude in average, initial jump and reversing jump distributions must also have equal standard deviations. Transition between the regimes is governed by the following transition matrix P,

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & q & 0 \\ 0 & 1-q & 0 \end{pmatrix} \quad . \qquad (3.3)$$

Thereby, q denotes the probability to stay in the stable regime.

3.2 Model II : De Jong and Huisman (2003)

In this model, see [20], besides the stable regime one spike regime is assumed. Contrary to Model I, this model allows for consecutive spikes. Thus, the price process can stay in the spike regime. This assumption is quite convenient with the fact that unforced outages can have a longer impact on spot prices than one day.

$$X_{S,t} = \mu_S + u_{S,t}\,, \quad u_{S,t} \sim \mathcal{N}(0, \sigma^2_S) \qquad \text{spike regime} \qquad (3.4)$$

Here in equation (3.4), we follow [20] and model the spike regime as a random walk. Transition between the states is now governed by a (2×2) transition matrix Π. Again, q denotes the probability to stay in the stable regime. Furthermore, p denotes the probability to stay in the spike regime.

$$\Pi = \begin{pmatrix} q & 1-p \\ 1-q & p \end{pmatrix} \tag{3.5}$$

3.3 Model III : Kosater and Mosler (2006)

Model III is a modification of Model II. Here, we assume that the probability of large upward spikes depends on the type of the day. Due to lower demand on weekends and public holidays, for example, large upward spikes are rather not to expect. However on weekends and public holidays, downward directed deviations from the stable regime are possible which are denoted as low spikes. In order to take into account different types of days, in this modified model high spikes and low spikes are distinguished. Practically, we decompose spikes by declaring an indicator function $\mathbf{1_H}$ which takes the value zero on holidays, weekend days, and two days before and after a holiday. All remaining days are candidates for high spikes only, so in these cases the indicator function takes value 1. This decomposition is valid for hours from 9 to 20. For the remaining hours, we mainly observe downward directed spikes only. However, even for the remaining hours, estimation does not reject the model. In[27], switching is made dependent on the reserve margin and load. However, the reserve margin cannot be used for forecasting downward directed spikes. Load may be helpful to some extent. Consequently, the decomposition of spikes depending on the type of the day, which is proposed in [22], or the introduction of a real third regime may improve the modelling of the spike regime.

$$\mathbf{1_H} = \begin{cases} 0 & \text{holiday, weekend, two days before and after a holiday}\,, \\ 1 & \text{else}\,, \end{cases} \tag{3.6}$$

$$X_{S,t} = \mathbf{1_H} \cdot (\mu_{S,H} + u_{S,H,t}) + (\mathbf{1} - \mathbf{1_H}) \cdot (\mu_{S,L} + u_{S,L,t}) \qquad \text{spike regime}\,. \tag{3.7}$$

Furthermore, we assume that the variances of the disturbances in the high spike regime denoted by $u_{S,H,t}$ and the low spike regime denoted by $u_{S,L,t}$ can differ. In particular, it holds $u_{S,H,t} \sim \mathcal{N}(0, \sigma^2_{S,H})$ and $u_{S,L,t} \sim \mathcal{N}(0, \sigma^2_{S,L})$. Transition between the regimes is the same as in the original Model II. Thus, the transition matrix is given by equation (3.5). A further possible modification of this model is to also allow for transition probabilities to depend on the type of the day. However, we found that this modification did not provide clearly better results. Therefore, we implemented the original model in [22]. We perform estimation and forecasting in `Eviews 5.1`[12]. Moreover, we use the implemented Berndt-Hall-Hall-Hausman [2] algorithm for numerical optimization. For all three models I to

III, estimation is carried out according to the methodology proposed in [20] and [22] which is to some extent different from the widely used methodology proposed in [16]. The approach proposed in [20] requires to look for the last logarithm of the spot price originating from the stable regime. Following [20] and for the sake of computational ease, we maximally go $K = 5$ steps back in time. However, for some hours more steps may be needed. For a more detailed discussion of the estimation methodology, see[22].

4 The Empirical Study

The empirical study divides into two parts. In the first part, we examine for each of the three stochastic models I to III whether the inclusion of weather data into the stochastic models provides a significant improvement in terms of fit compared to the pure models without weather. Furthermore in the second part, we carry out a forecasting study in the framework of model III to assess the quality of forecasts when weather data is included.
To model the relation between temperature and spot prices, we analogously proceed as in [23]. Besides temperature, we also add the square and the cube of temperature as explanatory variables in the regression equations below. Wind velocity is included as an additive linear term. We classify three versions A,B and C. Version A is the pure approach without weather and with constant transition probabilities according to equation (2.1). Moreover in version B, temperature and wind as explanatory variables are added to the deterministic components. So, our specification for version B is

$$\log(P_t) = X_t + f_t + w_t \,, \tag{4.1}$$

with

$$w_t = \delta_1 \cdot temp_t + \delta_2 \cdot temp_t^2 + \delta_3 \cdot wind_t + \delta_4 \cdot temp_t^3 \quad . \tag{4.2}$$

For version C, the specification is as in equations (4.1) and (4.2). Furthermore, we specify time-varying transition probabilities in terms of a Logit model. For further valid linking functions, see [14] and [15]. For the three models I to III, we replace q with $\dfrac{\exp(\mathbf{Z}\phi_{\mathbf{M}})}{1+\exp(\mathbf{Z}\phi_{\mathbf{M}})}$ and p with $\dfrac{\exp(\mathbf{Z}\phi_{\mathbf{S}})}{1+\exp(\mathbf{Z}\phi_{\mathbf{S}})}$ in equation (3.5). Moreover, we assume for the inner product $\mathbf{Z}\phi_{\mathbf{j}}$ of the vector of explanatory variables $\mathbf{Z}$ and the vectors of parameters $\phi_{\mathbf{j}}$ with $j \in \{M, S\}$,

$$\mathbf{Z}\phi_{\mathbf{j}} = \phi_{j,1} + \phi_{j,2} \cdot temp_t + \phi_{j,3} \cdot wind_t + \phi_{j,4} \cdot temp_t^2 + \phi_{j,5} \cdot temp_t^3 \,. \tag{4.3}$$

In equations (4.2) and(4.3), we choose the explanatory variables by testing their significance at the 5 %-level based on standard errors provided by the BHHH-algorithm. Only the significant explanatory variables are included in the specification. In some cases however, none of the estimates meets the criterion. In these cases, we include the most significant among them. Moreover, it is not clear to which extent standard errors provided by the numerical algorithm are reliable. Therefore while choosing the variables, we have also taken into account the loss in terms of fit after dropping a variable in the spirit of a likelihood ratio test.

4.1 Results on Model Fit

It rarely happens, however, hourly prices can be equal to zero. If we encounter those spot prices, we replace them by the average of the price at the same day one week before and the price at the same day one week ahead.
In general, estimation of the considered models is not straightforward. However for all models, estimation of versions denoted by C turns out to be most cumbersome. Furthermore, fitting of Model I to the hourly data is burdensome for all versions A to C. Moreover for Model I, we did not obtain significant estimates for μ_s for any hour and any version. Additionally, the standard deviation σ_s is extremely large compared with the other models. The model cannot cope with the occurrence of upward and downward directed spikes in night hours and on holidays. Downward and upward spikes sum to round about zero. This is the explanation for the not significant estimates of μ_s. Moreover, spikes, which are identified by the model, are rare and large sized which explains the extremely large standard deviation σ_s. The other estimates, however, are acceptable or even reasonable. Furthermore in Figure 2, we plot the resulting so-called smoothed probabilities for hour 12 for Models I and II (Model II and III provide similar smoothed probabilities.), respectively. Smoothed probabilities are calculated according to [21]. In tables 3 and 4, we provide selected results for hours 4, 12 and 22. In order to summarize the results for the general impact of weather on prices, we see that the linear temperature specification and wind provide significant negative estimates throughout all 24 hours. Negative parameter estimates of temperature reflect that negative temperature causes the demand, and therefore also prices, to rise due to the need for heating in winter, whereas moderate positive temperatures typically are accompanied by lower demand and therefore lower prices. Moreover, we have to bear in mind that these estimates merely reflect an average effect over the year. Therefore, to gain a deeper insight, investigations should be more detailed distinguishing summer and winter time or even take into account the monthly differences. A more detailed modelling of the general impact of weather, however, would increase complexity with regard to our second goal to link weather and spikes. We leave this more detailed examination for further research. The square of temperature mainly provides small and significant positive estimates in the late afternoon from about 16 or 17 until the evening hours 23. Moreover, the cube provides small and significant positive estimates in night hours from 1 up to 5 or 6 and then partly in the early afternoon hours. Comparing versions A and B, version B clearly outperforms version A throughout all 24 hours and models I to III. The Akaike and the Schwartz information criteria indicate that Model B should

be preferred. To compare versions B and C, we carry out a standard likelihood ratio test. This test may help us to infer whether time-varying transition probabilities in version C provide a better description of reality than fixed transition probabilities. The likelihood ratio test in the spirit of [14],[18] is

$$LR = 2 \cdot (L_C - L_B)\,. \tag{4.4}$$

L_C corresponds to the likelihood of version C, whereas L_B is the likelihood of version B. The null hypothesis of this likelihood ratio test is that version C does not outperform version B. Under the null hypothesis, LR is asymptotically χ^2-distributed with J degrees of freedom. Furthermore, J reflects the number of restrictions (maximally $4+4=8$), we impose, compared to version B. Moreover for models II and III, we often renounce to include the constant in equation (4.3) for the spike regime ($j = S$) due to the lack of significance. We treat these cases as if the constant was included. The results of the test are given in table 5. Whenever a dash appears in the table instead of a p-value, version B turned out to be the best specification for version C, too. Summarizing the results for the relation between weather and spikes, we can see that there is merely evidence for a significant relation for hours 1 to 8 which is, however, supported by all three models I to III. For the remaining hours we find only little, and depending on the model, evidence for a significant relation. Whenever estimation provides significant results for the wind parameter, it turns out that rising wind velocity reduces the transition probability to stay in the stable regime and augments the transition probability to stay in the spike regime, except for hours 13 and 14 for Models I and II and 12 and 14 for Model III. During these hours rising wind velocity augments the transition probability to stay in the stable regime for Model I. For Model II and III the transition probability to stay in the spike regime is reduced with increasing wind velocity. We also find a relation between temperature and regime probabilities. The relation is mainly confined to temperature in its linear specification. The square rarely and the cube very rarely provide significant estimates. Modelling of the relation between wind velocity and spot prices is not straightforward. Wind velocities below 4 meter/second are not significant for wind energy production but for spot prices. The supply side of electricity is affected when wind energy is not produced. For the main study, we assume that the impact is different for the wind velocities below 4 meter/second. In an additional short forecasting study, we assume that all wind velocities below 4 meter/second have the same impact on spot prices. Moreover, we also examine the case that no wind is added into the specification. The relation between wind velocity and spot prices still remains an interesting problem to tackle for further research.

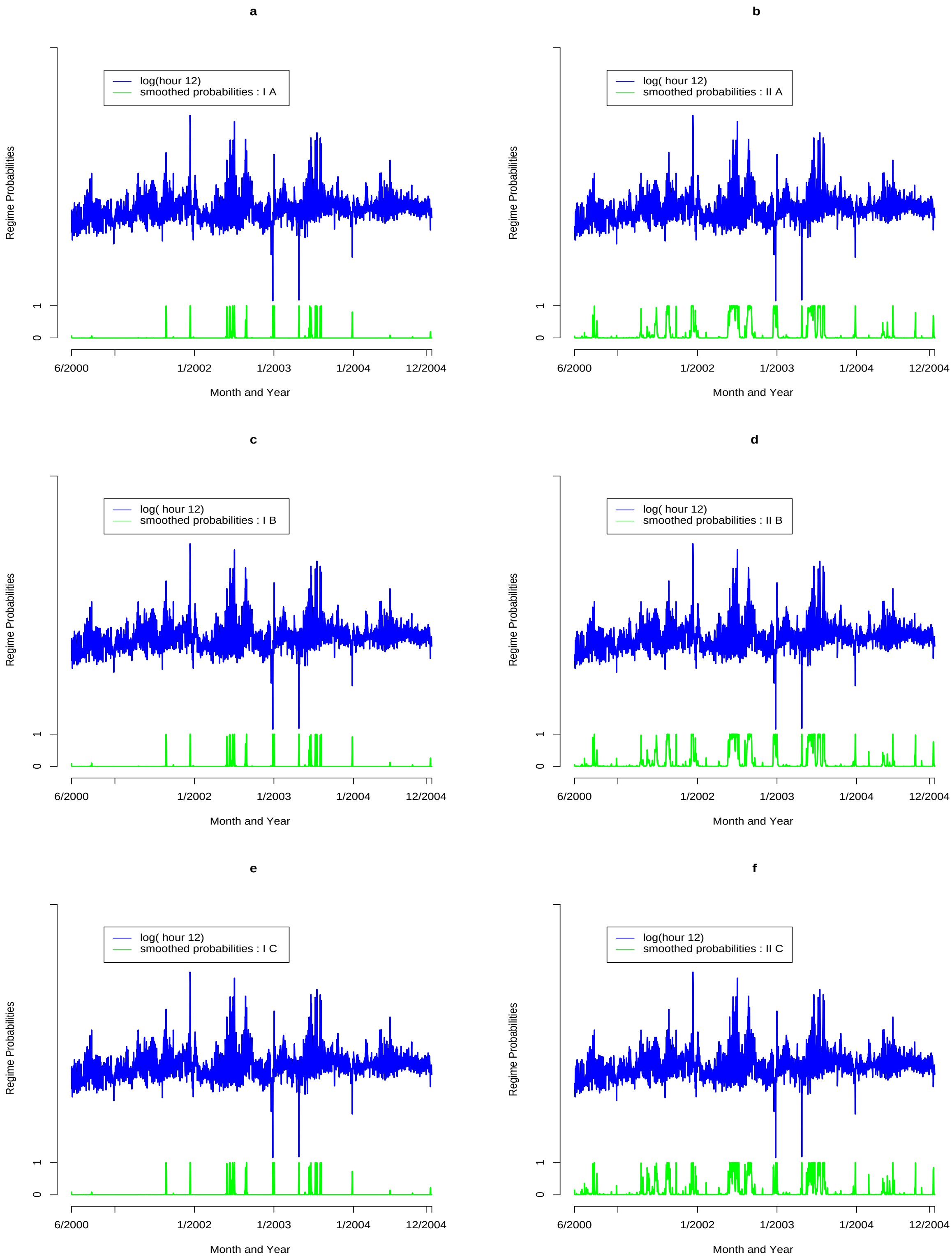

Figure 2: Plots of smoothed probabilities for the spike regime for Models I and II.

Table 3: Selected results of equation (4.2).(Temperature is measured in $0.1 \cdot C^{\circ}$, whereas wind velocity is measured in $0.1 \cdot meter/second$.)

Model	Hour	Version	δ_1	δ_2	δ_3	δ_4
I	4	B	-0.003 (0.0004)	-	-0.002 (0.0003)	$6{\cdot}10^{-8}$ ($1.2{\cdot}10^{-8}$)
		C	-0.003 (0.0004)	-	-0.002 (0.0004)	$5.9{\cdot}10^{-8}$ ($1.2{\cdot}10^{-8}$)
	12	B	-0.001 (0.0003)	-	-0.002 (0.0004)	$1.4{\cdot}10^{-8}$ ($2.6{\cdot}10^{-9}$)
		C	-0.001 (0.0003)	-	-0.002 (0.0004)	$1.3{\cdot}10^{-8}$ ($2.7{\cdot}10^{-8}$)
	22	B	-0.001 (0.0002)	$3.1{\cdot}10^{-6}$ ($9.1{\cdot}10^{-7}$)	-0.001 (0.0002)	-
		C	-0.001 (0.0002)	$3.1{\cdot}10^{-6}$ ($9.1{\cdot}10^{-7}$)	-0.001 (0.0002)	-
II	4	B	-0.002 (0.0003)	-	-0.002 (0.0003)	$4.6{\cdot}10^{-8}$ ($9{\cdot}10^{-9}$)
		C	-0.002 (0.0003)	-	-0.002 (0.0003)	$4.7{\cdot}10^{-8}$ ($8.9{\cdot}10^{-9}$)
	12	B	-0.001 (0.0002)	-	-0.002 (0.0003)	-
		C	-0.001 (0.0002)	-	-0.002 (0.0003)	-
	22	B	-0.001 (0.0002)	$3{\cdot}10^{-6}$ ($8{\cdot}10^{-7}$)	-0.001 (0.0002)	-
		C	-0.001 (0.0002)	$3{\cdot}10^{-6}$ ($8{\cdot}10^{-7}$)	-0.001 (0.0002)	-
III	4	B	-0.002 (0.0003)	-	-0.002 (0.0003)	$4.6{\cdot}10^{-8}$ ($8.9{\cdot}10^{-9}$)
		C	-0.002 (0.0003)	-	-0.002 (0.0003)	$4.6{\cdot}10^{-8}$ ($8.9{\cdot}10^{-9}$)
	12	B	-0.001 (0.0002)	-	-0.002 (0.0003)	$8.6{\cdot}10^{-9}$ ($2.6{\cdot}10^{-9}$)
		C	-0.001 (0.0002)	-	-0.002 (0.0003)	$8.9{\cdot}10^{-9}$ ($2.6{\cdot}10^{-9}$)
	22	B	-0.001 (0.0002)	$2.8{\cdot}10^{-6}$ ($7.8{\cdot}10^{-7}$)	-0.001 (0.0002)	-
		C	-0.001 (0.0002)	$2.9{\cdot}10^{-6}$ ($7.9{\cdot}10^{-7}$)	-0.001 (0.0002)	-

Table 4: Selected results of equation (4.3).(Temperature is measured in $0.1 \cdot C^\circ$, whereas wind velocity is measured in $0.1 \cdot meter/second$)

Model	Hour	$\phi_{M,1}$	$\phi_{M,2}$	$\phi_{M,3}$	$\phi_{M,4}$	$\phi_{M,5}$	$\phi_{S,1}$	$\phi_{S,2}$	$\phi_{S,3}$	$\phi_{S,4}$	$\phi_{S,5}$
I	4	5.628 (0.521)	−0.009 (0.004)	−0.034 (0.007)	-	-	-	-	-	-	-
	12	5.800 (0.444)	−0.010 (0.002)	-	-	-	-	-	-	-	-
	22	6.868 (0.810)	-	−0.033 (0.014)	-	-	-	-	-	-	-
II	4	3.710 (0.342)	-	−0.018 (0.008)	-	-	-	0.013 (0.003)	0.053 (0.012)	-	-
	12	3.788 (0.261)	-	-	$-2.9 \cdot 10^{-5}$ ($6.4 \cdot 10^{-6}$)	-	-	0.005 (0.001)	-	-	-
	22	4.509 (0.442)	-	−0.021 (0.008)	-	-	-	-	0.011 (0.010)	-	-
III	4	3.664 (0.346)	-	−0.018 (0.008)	-	-	-	0.012 (0.003)	0.053 (0.012)	-	-
	12	3.412 (0.194)	-	-	-	-	3.213 (0.652)	-	−0.035 (0.012)	-	-
	22	4.034 (0.431)	-	−0.019 (0.008)	-	-	-	-	0.009 (0.009)	-	-

Table 5: Results of the likelihood ratio test, equation (4.4). (Note, a dash means that version B turned out to be the best specification for version C, too.)

	Model I	Model II	Model III		Model I	Model II	Model III
Hour	p-value	p-value	p-value	Hour	p-value	p-value	p-value
1	0.033112	0.004384	0.005709	13	0.016965	$2.2 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
2	0.000364	$6.2 \cdot 10^{-5}$	$6.5 \cdot 10^{-6}$	14	0.177313	$9.1 \cdot 10^{-5}$	$6.1 \cdot 10^{-5}$
3	0.012490	$4.4 \cdot 10^{-5}$	$4.3 \cdot 10^{-5}$	15	-	0.007795	0.008060
4	0.000201	0.000466	0.000281	16	-	-	0.087488
5	0.000162	0.001592	0.000668	17	-	0.021068	0.108222
6	$7.6 \cdot 10^{-5}$	0.000232	0.000564	18	0.002670	$8.4 \cdot 10^{-5}$	0.069280
7	0.006246	$2.9 \cdot 10^{-5}$	0.000212	19	0.113846	0.008148	0.011562
8	0.002267	$5.4 \cdot 10^{-5}$	$8.3 \cdot 10^{-5}$	20	-	0.312519	0.080460
9	0.057089	0.004893	0.220671	21	0.050044	-	-
10	0.122456	0.080242	0.008537	22	0.115302	0.044157	0.003735
11	0.194924	$3.6 \cdot 10^{-5}$	$1.8 \cdot 10^{-5}$	23	0.016583	0.022848	0.044964
12	0.000137	$6.1 \cdot 10^{-7}$	0.025056	24	-	0.001783	-

4.2 A Forecast Comparison Study

In the preceding first part of the empirical study, we have found that there is a significant relation between weather and spot prices. In the second part of the

study, we want to examine whether this relation can be exploited for forecasting the one-day ahead spot price.
Among models I to III, Model III copes best with upward and downward spikes. Therefore, results in [22] and the estimation results of Model I strongly suggest that among the considered models Model III is best suited for forecasting. Therefore, we carry out the study with Model III.
First, we introduce our forecasting methodology for version A, because our procedure deviates from the standard approach in [16]. For versions B,C estimation and prediction are carried out according to equations (4.1) to (4.3). Let $\xi_{(T|\psi_T)}$ be the vector of smoothed probabilities at time T and ψ_T the information set at T. Moreover, let Π be the transition matrix according to equation (3.5). The one-step ahead forecasts for the smoothed probabilities are computed as follows,

$$\xi^f_{T+1} = \begin{pmatrix} \xi^f_{M,T+1} \\ \xi^f_{S,T+1} \end{pmatrix} = \Pi \cdot \xi_{(T|\psi_T)}. \tag{4.5}$$

V_{T+1} is defined as the vector containing the conditional expectations $E[X_{S_{T+1},T+1}|\psi_T]$ for each regime.

$$V_{T+1} = \begin{pmatrix} E[X_{M,T+1}|\psi_T] \\ E[X_{S,T+1}|\psi_T] \end{pmatrix} \tag{4.6}$$

with $E[X_{S,T+1}|\psi_T] = \mu_{S,H} \cdot \mathbf{1_H} + \mu_{S,L} \cdot (1 - \mathbf{1_H})$. Finally, the forecast results as

$$X^f_{T+1} = V^f_{T+1} \cdot \xi^f_{T+1}. \tag{4.7}$$

In this framework, the two regimes are assumed to be independent. Therefore, the forecast for the stable regime is

$$E[X_{M,T+1}|\psi_T] = \mu_M \cdot \alpha + (1 - \alpha) \cdot E[X_{M,T}|\psi_T]. \tag{4.8}$$

Following [22], we set $E[X_{M,T}|\psi_T] = X_T$ and use the actual value X_T as forecast origin. Additionally, we determine $E[X_{M,T}|\psi_T]$ as follows. First, we look for the last logarithm of spot price which belongs to the stable regime. Starting at X_T, we look for the last logarithm of spot price with a smoothed probability smaller than 0.5 to be in the spike regime. Let X_{T-i} be the stochastic part of such a logarithm of the spot price. Then, we replace the actual value X_T by its forecast based on X_{T-i} with $i \in \{0, 1, \cdots, T-1\}$ as the forecast origin. By this, we approximate the forecast $E[X_{M,T+1}|\psi_T]$ by $E[X_{M,T+1}|\psi_{T-i}]$.

$$\begin{aligned} E[X_{M,T+1}|\psi_{T-i}] &= \mu_M \cdot \alpha + (1-\alpha) \cdot \left((1-\alpha)^i \cdot X_{T-i} + \mu_M \cdot (1-(1-\alpha)^i)\right) \\ &= (1-\alpha)^{i+1} \cdot X_{T-i} + \mu_M \cdot (1-(1-\alpha)^{i+1}). \end{aligned} \tag{4.9}$$

The one-step ahead forecast in both cases is thus

$$P^f_{T+1} = \exp\left(X^f_{T+1} + f^f_{T+1}\right). \tag{4.10}$$

The advantage of our alternative approach is that we avoid forecasts for the stable regime based on spikes. However, one drawback is that the prediction error rises. Moreover, we renounce to exploit the forecast of the deterministic component at T. A possible procedure to avoid the loss in terms of seasonality is to first remove the deterministic effects from the actual time series. The stochastic model is then fitted to the data from which deterministic have been removed, as in [26]. Anticipating the results of the forecasting study, we found that the new approach presented in this paper outperforms the methodology in [22] for hours from 19 to 6. Spot prices for hours 21 to 6 do not differ much throughout the different types of days because demand is always low. Therefore, for these hours the deterministic component is not as important as for the hours from 9 till 20. We always report the best of all forecasts provided by the two methods. We can proceed this way because the best performing method performs best for all three versions A to C. Additionally, we have also assessed the performance of version C including the constant in equation (4.3) whenever the constant turned out to be not significant in the first part of the study. We have only obtained slightly better forecasts for hours 1 and 5. Here, we have neglected the intra-day correlations of hourly prices. However in [8], the authors show that it may be an asset to include cross-correlations between hours in to the model specification. In the presented models, other hourly price series can be included as explanatory variables similar to temperature or wind velocity to reflect relations between hours during a day. However, the inclusion of lagged regressors of the same price series in this framework is not possible. In such a case, the approach in [16] should be used instead.

4.3 Results of the Forecasting Study

In this forecast comparison study, we carry out and evaluate ex ante forecasts in terms of the root mean square error (RMSE) and the mean absolute error (MAE). All given information available at time T is exploited and, by this, we use all known electricity prices up to T to estimate the parameter values. This proceeding is reasonable, since electricity prices exhibit strong seasonality and autocorrelation that are estimated the better the more data is available. The forecasting procedure is close to that of [8] applied to hourly prices and is described below. The given dataset is divided into an in-sample period which includes observations from 6/16/2000 to 9/21/2004 at the beginning. Moreover, the out-of-sample period ranges from 9/22/2004 to 12/30/2004. The forecasting experiment is designed as follows. We use in-sample data to estimate the parameters of the model III version of interest. Then, we make out-of-sample one-step ahead forecasts and evaluate them. The in-sample period is then enlarged by one observation and again forecasts for the out-of-sample period are made and evaluated. We repeat this procedure 100 times. This forecasting study is carried out using the logarithm of all hourly 24 price series. As aforementioned, we use the actually measured values at the day the forecast is made for, instead of the forecast which we do not possess. P_{T+1} denotes the actual observed price at time $T+1$, while P^f_{T+1} refers to the predicted price at time $T+1$.The measures used for comparison are

$$RMSE = \sqrt{\frac{1}{100} \cdot \sum_{i=1}^{100} \left(P_{T+1,i} - P^f_{T+1,i} \right)^2}, \tag{4.11}$$

$$MAE = \frac{1}{100} \cdot \sum_{i=1}^{100} \left| P_{T+1,i} - P^f_{T+1,i} \right|. \tag{4.12}$$

The two-steps ahead hardly and the three- steps ahead forecasts not at all resemble the measured values for both temperature and wind velocity. Therefore, we merely carry out one-step ahead forecasts. For practical application, meteorologists provide forecasts up to six days ahead. Subfigures 3a and 3b show the results of the forecasting study.The results of the main study suggest to use weather data for forecasting prices from hours 19 to 6. For the remaining hours, the incorporation of weather data does not necessarily provide better forecasts. Subfigures 3c and 3d show a comparison of forecast errors for versions A and B as well as versions A and C for hour 12. We plot the absolute deviation of the forecastP^f_t from the actual price P_t, $|P_t - P^f_t|$.

Furthermore, to understand the results of the study, we have carried out the following regressions.

$$P_t = \eta_1 + \eta_2 \cdot temp_t + \eta_3 \cdot temp_t^2 + \eta_4 \cdot temp_t^3 + \eta_5 \cdot wind_t \tag{4.13}$$

$$\log(P_t) = \eta_1 + \eta_2 \cdot temp_t + \eta_3 \cdot temp_t^2 + \eta_4 \cdot temp_t^3 + \eta_5 \cdot wind_t \tag{4.14}$$

In subfigure 3e, we present the results of equations (4.13) and (4.14). Moreover in a second step, we have extracted some outliers from the electricity spot prices. Then, we regressed the remaining prices according to equations (4.13) and (4.14). The results of the confined regressions are plotted in subfigure 3f. Subfigures 3e and 3f show a very low coefficient of determination R^2 for those hours where versions B and C fail to outperform version A.

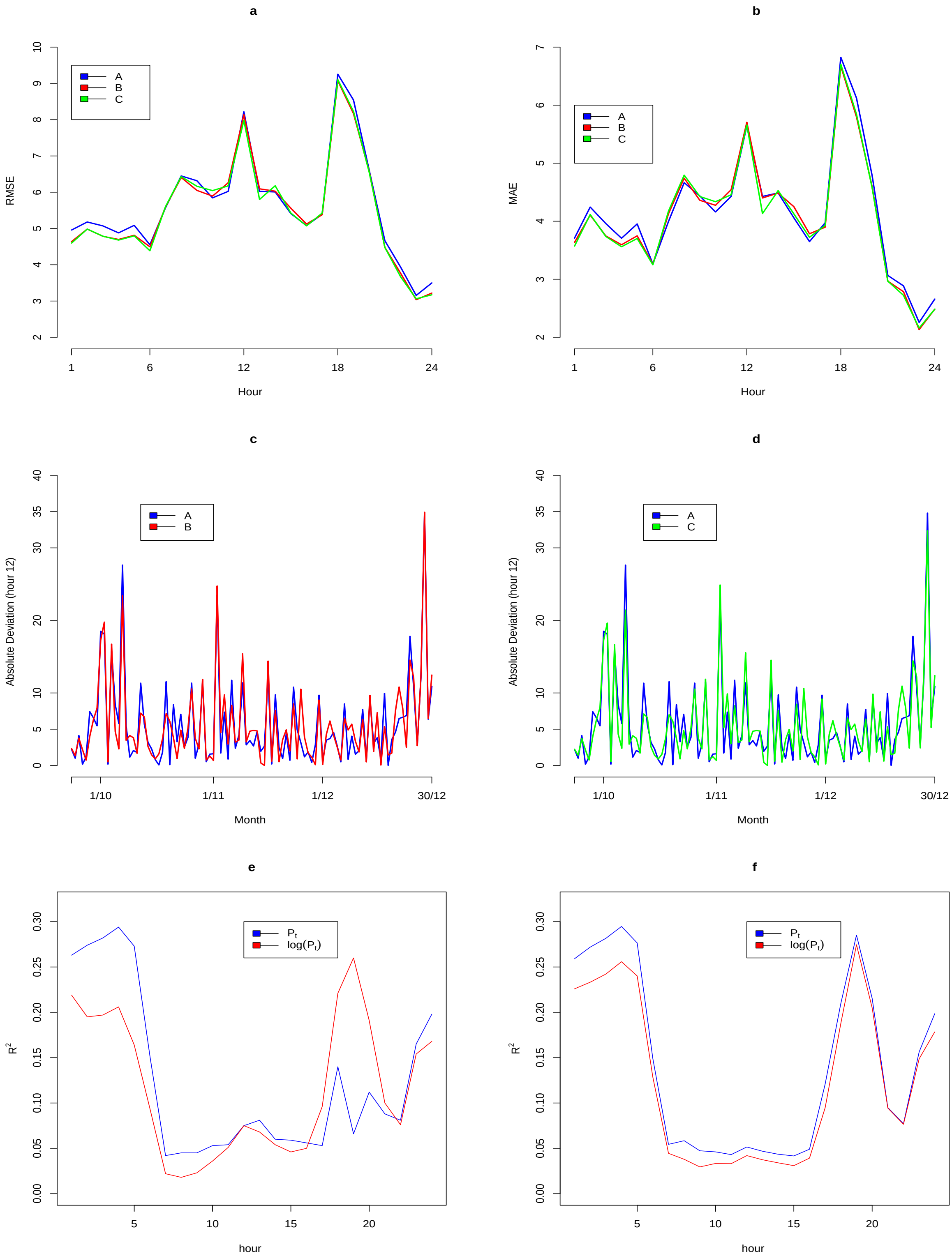

Figure 3: Results of the forecasting study (a,b), Comparison of forecast errors for hour 12 (c,d), Results of equations (4.13) and (4.14).

4.4 An Additional Small Forecasting Study

To analyze the relevance of wind velocity for forecasting, we also carry out a small forecasting study for hours 4,12 and 22. Thereby, we carry out the study without wind velocity. Secondly, we transform wind velocity in order to take into account that wind velocities below the margin of 4 meter/second are not significant for production. The transformation is implemented as follows,

$$wind_{trans} = \begin{cases} 0 & : \quad \text{wind velocity } < 4\,, \\ \text{wind velocity} - 4 & : \quad 4 \leq \text{ wind velocity } \leq 12\,, \\ 8 & : \quad \text{wind velocity } > 12\,. \end{cases} \tag{4.15}$$

Furthermore, we examine whether taking the deviation of temperature from a long run average instead of the absolute values provides better forecasts. Unfortunately, we only possess temperature data for about four and a half years from which we try to calculate a long run average. Of course, this may be too short for a valid examination.

Finally, we also assess the performance of the unrestricted models incorporating all possible regressors for f_t as well as for the time-varying transition probabilities according to equations (4.2) and (4.3). Wind velocity and temperature are left unchanged compared with the main study. The results of the additional study, summarized in tables 6 and 7, suggest that wind velocity is relevant for forecasting. Moreover, we see that instead of actual wind velocity, we should implement a transformation which takes into account the technical conditions of the operation of windmills. Among the selected hours, wind exhibits the strongest impact on spot prices at hour 4 according to tables 3 and 4. Exactly for this hour, we observe the largest difference between the results of the main study and the results obtained by including the transformation of wind velocity. This fact even strengthens our statement. The label modified temperature in tables 6 and 7 denotes the results of the study where absolute temperature is replaced with its deviation from its long run average. The results of the modified temperature can only be compared to the values of the study without wind, because we have run the study without including wind velocity. It is not clear whether forecasts are better if we take the deviation of temperature from a long run average instead of the absolute values. However, it turns out that we should use the absolute values for time-varying transition probabilities in equation (4.3) according to the results for version C for hour 4. Finally, incorporating all regressors may be an asset for forecasting, on one hand. On the other hand, we run risk heavy losses due to overfitting. The results for version C for hour 4 for all regressors are labelled with a star because we have taken the results based on X_T and equation (4.10), as opposed to the remaining results for hour 4. The method which has yield the remaining results, has yield an RMSE and an MAE almost three times higher than for the remaining hours in the case of hour 4.

Furthermore, we want to compare the results with results of other models. Therefore, we also carry out one-step ahead forecasts with three further linear models denoted by Model IV to VI and a non-linear model in the spirit of Hamilton, presented in [22]. Model IV is an autoregressive model of order one for the stochastic part of the logarithm of the spot price X_t. In Model V compared with Model IV, we additionally include an autoregressive term of order seven to take into account the strong weekly seasonality. Finally in Model VI, we specify the most sophisticated among the linear models, namely a seasonal autoregressive integrated moving average process (ARIMA(1,0,1)$\times$ SARIMA$(1,0,1)_7$) to capture mean reversion and weekly seasonality. Moreover, we also examine the impact of weather on hourly spot prices in the framework of these models IV to VII. Again, we denote three model versions A to C. For the linear models IV to VI, we specify version B according to Model III B, see table 3. However, for Model VII we include all possible regressors in version B and C, respectively.

Model IV:

$$X_t = \alpha \cdot \mu_M + (1-\alpha) \cdot X_{t-1} + u_t \ , \qquad u_t \sim \mathcal{N}(0, \sigma^2). \tag{4.16}$$

Model V:

$$X_t = (\alpha_1 - \alpha_2) \cdot \mu_M + (1-\alpha_1) \cdot X_{t-1} + \alpha_2 \cdot X_{t-7} + u_t \ , \qquad u_t \sim \mathcal{N}(0, \sigma^2). \tag{4.17}$$

Model VI:

$$\begin{aligned} X_t \ &= \ \alpha_1 \cdot (1-\alpha_2) \cdot \mu_M + (1-\alpha_1) \cdot X_{t-1} + u_t - \theta \cdot u_{t-1} - \omega \cdot (u_{t-7} - \theta \cdot u_{t-8}) \\ &\quad + \alpha_2 \cdot (X_{t-7} - (1-\alpha_1) \cdot X_{t-8}) \ , \qquad u_t \sim \mathcal{N}(0, \sigma^2). \end{aligned} \tag{4.18}$$

Model VII :

$$X_{\{S_t=j\},t} = \mu_{\{S_t=j\}} + (1-\alpha) \cdot \left(X_{\{S_{t-1}=i\},t-1} - \mu_{\{S_{t-1}=i\}} \right) + u_{\{S_t=j\},t}, \tag{4.19}$$

$$\mu_S = \mathbf{1_H} \cdot (\mu_{S,H}) + (\mathbf{1} - \mathbf{1_H}) \cdot (\mu_{S,L}) \, , \tag{4.20}$$

$$u_{S,t} = \mathbf{1_H} \cdot (u_{S,H,t}) + (\mathbf{1} - \mathbf{1_H}) \cdot (u_{S,L,t}) \, , \tag{4.21}$$

with $j, i \in \{M, S\}$, $u_{M,t} \sim \mathcal{N}(0, \sigma^2)$, $u_{S,H,t} \sim \mathcal{N}(0, \sigma^2_{S,H})$, $u_{S,L,t} \sim \mathcal{N}(0, \sigma^2_{S,L})$.

and the transition matrix according to equation (3.5). Analogously to Model III, we assume a stable regime denoted by M and a spike regime S in (4.19). Furthermore, we also distinguish low and high spikes in (4.20) and (4.21). The indicator function $\mathbf{1_H}$ is the same as in (3.6). Note that in Model VII, stable and spike regime are not assumed to be independent as in Models I to III, since spikes are presumed to affect the mean reverting regime. Forecasting is carried out according to equation (4.10) for the three selected hours 4,12 and 22. The logarithm of the spot price for hour 12 is characterized by alternating periods of very high volatility due to spikes and calm periods. Therefore, we carry out forecasting not only in the very calm period from 9/22/2004 to 12/30/2004 but additionally for the period from 5/30/2003 to 9/8/2003. Estimation of parameters for the second spiky period is based on the sub-sample from 6/16/2000 to 5/29/2003 at the beginning. Tables 8 and 9 present the results of the comparison of the out-of-sample prediction performance of Model III with Models IV to VII. Results for the first calm period are denoted by I and for the spiky period by II. Summarizing the outcome of the study, Model VI performs very well across the three hours 4,12 and 22. For the calm period I of hour 12, Model VI even clearly outperforms the non-linear Models III and VII. For the spiky period, however, the opposite is true. Moreover, the study proves that non-linear models are valuable competing methods to sophisticated linear models with still hidden potential. This hidden potential lies in linking non-linear Markov regime-switching models to economic explanatory variables such as load and reserve margin as pointed out in [27].

Table 6: RMSE for three selected hours.

Hour	Version	without wind	transformed wind	modified temperature	all regressors	main study
4	B	4.7592	4.5523	4.8081	4.6947	4.6976
	C	4.6647	4.4624	4.7908	4.8561*	4.6799
12	B	8.3188	8.0940	8.2638	8.1419	8.1354
	C	-	8.1386	-	7.4634	7.9809
22	B	3.8345	3.7837	3.8907	3.6846	3.7694
	C	-	3.6807	-	3.6430	3.6746

Table 7: MAE for three selected hours.

Hour	Version	without wind	transformed wind	modified temperature	all regressors	main study
4	B	3.5809	3.4649	3.6383	3.5927	3.5946
	C	3.5308	3.3740	3.6701	3.6417*	3.5600
12	B	5.6936	5.6405	5.6413	5.7210	5.7052
	C	-	5.6422	-	5.4245	5.6578
22	B	2.8119	2.7653	2.8347	2.7171	2.7864
	C	-	2.7105	-	2.6902	2.7234

Table 8: RMSE for three selected hours.(Best results are emphasized in bold.)

Hour	Version	Model IV	Model V	Model VI	Model VII	main study
4	A	5.0278	5.2642	4.7019	4.8675	4.8787
	B	5.1751	4.9722	**4.4573**	4.6435	4.6976
	C	-	-	-	4.6221	4.6799
12 (I)	A	9.1684	7.5489	**6.5104**	8.4697	8.2181
	B	9.2547	7.9515	6.5924	8.5021	8.1354
	C	-	-	-	8.4735	7.9809
12 (II)	A	58.5012	63.3161	59.7819	**55.7688**	58.7594
	B	59.5612	59.9693	59.0457	56.3788	58.4863
	C	-	-	-	65.3322	58.4381
22	A	4.2596	4.3630	4.1946	4.3415	3.9399
	B	4.2012	4.2049	3.8962	4.2047	3.7694
	C	-	-	-	4.2093	**3.6746**

Table 9: MAE for three selected hours. (Best results are emphasized in bold.)

Hour	Version	Model IV	Model V	Model VI	Model VII	main study
4	A	3.9010	4.1855	3.6309	3.6603	3.7063
	B	4.0989	4.0134	**3.5237**	3.5273	3.5946
	C	-	-	-	3.5676	3.5600
12 (I)	A	6.0674	4.8699	**4.3461**	5.8522	5.6423
	B	6.3950	5.4537	4.3494	5.9518	5.7052
	C	-	-	-	5.8264	5.6578
12 (II)	A	29.4071	30.7435	32.0217	**27.9622**	29.8095
	B	30.2657	30.9074	30.7669	28.0153	29.0674
	C	-	-	-	30.9345	28.7559
22	A	3.0330	3.2473	2.9634	3.1333	2.8862
	B	2.8914	3.0306	2.7675	3.0295	2.7864
	C	-	-	-	3.0383	**2.7234**

5 Conclusion

Weather is an important driving force of electricity demand and therefore also electricity prices. In this paper, we examine the relation between hourly prices from the EEX and weather represented by temperature and wind velocity. Our investigation consists of two parts. First, we explore whether a relation can be detected in the given data. Secondly, we examine if this relation can be exploited for forecasting of future spot prices. In the first part, we work in the framework of three established Markov regime-switching approaches. Thereby, we try to capture the general impact of weather on hourly spot prices, on one hand. Additionally, we model the time-varying transition probabilities as functions of temperature and wind velocity with the aim to link weather and spikes. As a result, we find that including weather data with the aim to model the general impact of weather on hourly spot prices yields better results in terms of fit than the pure stochastic models. Furthermore, the incorporation of time-varying transition probabilities provides an additionally significant improvement in terms of fit for some hours. The forecasting experiment, however, is carried out with the best suited of the three models, only. Moreover, the forecasting experiment reveals that weather data should be used for forecasting prices in the hours from 19 until 6. During the remaining hours, including weather into the model does not necessarily provide better forecasts. Due to emission allowances and the tendency towards renewable energy in electricity production however, inclusion of weather will presumably become an asset for forecasting hourly prices throughout the whole day in the future. Furthermore, we have shown that non-linear Markov regime-switching models are an asset in forecasting with still hidden potential compared with linear models. For further research, load and the reserve margin should be incorporated in a good model specification. The general impact of weather on prices should be specified more precisely taking into account the four seasons or even the different months of the year. This study has revealed the importance of wind velocity for modelling and forecasting of spot prices. Therefore, research should also focus on modelling the relation between spot prices and wind velocity.

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