DISCUSSION PAPERS IN STATISTICS AND ECONOMETRICS

SEMINAR OF ECONOMIC AND SOCIAL STATISTICS
UNIVERSITY OF COLOGNE

Nr. 2/06

Cross-City Hedging with Weather Derivatives using Bivariate DCC GARCH Models

by

Peter Kosater

December 2006

DISKUSSIONSBEITRÄGE ZUR STATISTIK UND ÖKONOMETRIE

SEMINAR FÜR WIRTSCHAFTS- UND SOZIALSTATISTIK
UNIVERSITÄT ZU KÖLN

Albertus-Magnus-Platz, D-50923 Köln, Deutschland

DISCUSSION PAPERS IN STATISTICS AND ECONOMETRICS

SEMINAR OF ECONOMIC AND SOCIAL STATISTICS
UNIVERSITY OF COLOGNE

Nr. 2/06

Cross-City Hedging with Weather Derivatives using Bivariate DCC GARCH Models

by

Peter Kosater [1]

December 2006

[1]Graduate School of Risk Management, University of Cologne, Albertus Magnus Platz, 50923 Cologne, GERMANY

Abstract

As monopolies gave their way to competitive wholesale electricity markets, volumetric risk came into play. Electricity supplier can buy weather derivatives to protect from volumetric risk due to unexpected weather conditions. However, contracts can only be negotiated for weather variables measured at few selected locations. To hedge their specific risk, electricity supplier have to correlate their risk with the risk at tradeable locations. In this paper, we concentrate on temperature derivatives. More precisely, we examine if and how bivariate GARCH models with dynamic conditional correlations can help in modelling correlation between two distinct temperature time series. The knowledge of correlation dynamics between the temperature time series enables an electricity supplier to correlate his risk with the risk of a traded city and to construct a sensible hedge. It turns out that the application of bivariate DCC GARCH models to three German temperature time series provides encouraging results.

1 Introduction

Many sectors of the economy such as energy, agriculture, retail and tourism are exposed to weather risk. The earnings of producers of ice cream and energy companies, for example, are very much depending on the weather conditions, they are faced with. To cope with the volumetric risk due to uncertain weather conditions, weather derivatives have become a common instrument. These instruments allow electricity suppliers to protect their earnings from warm winters or ice cream producers from cold summers. Especially in the USA, the market for weather derivatives, as well over- the- counter as exchange-based, is fast growing.

As of September 1999, the Chicago Mercantile Exchange, also referred to as CME, began listing futures and options on temperature indices of ten cities across the USA. Today, indices for eighteen U.S. cities are available. Besides, contracts on indices for nine European and two Japanese cities can be struck. These cities have been chosen based on population, the variability in their seasonal temperatures and the activities in over- the- counter markets. The total number of contracts traded was 4165 in 2002 and 14234 in 2003. Through September 2005, there were 630 000 weather contracts traded with a notional value of 22 billion dollars.

Weather derivative instruments include weather swaps, options, option collars and short straddles, to mention a few among them. The payoffs of these instruments may be linked to various underlying meteorological variables such as average temperature, minimum temperature, maximum temperature, heating degree days and cooling degree days, as well as wind speed, rainfall and sunshine.

Here, we concentrate on temperature derivatives, since about 90 % of the traded derivatives are based on temperature. To be more specific, we focus on contracts written on heating degree day (HDD) and cooling degree day indices (CDD), respectively. HDD indices can be used to protect from a bland winter, whereas CDD indices are designed to hedge against a cold summer.

A degree day measures how much a day's average deviates from 65° F (or 18.33° C) a level of outdoor temperature considered to be utmost comfortable by the utility industry. The idea behind this choice is that, for each degree below 65° F, more energy is needed for heating. By contrast, for each degree above 65° F, more energy is needed to power air conditioners. Most contracts are written on the accumulation of HDDs or CDDs over a calendar month or a season so that one contract can hedge against revenue fluctuations over the concerned period. Moreover, so-called energy degree day indices (EDD) are additionally offered by the CME. These contracts allow for more flexibility. For example, a different level than 65° F can be specified.

More precisely, we denote the daily HDD = $\max[0, 65^\circ \text{ F} - \text{daily average temperature}]$, whereas for the daily CDD, we denote, CDD $= \max[0, \text{daily average temperature} - 65^\circ \text{ F}]$. Note that daily average tempera-

ture is computed as the average of the maximum and minimum temperatures on a certain day. Further basic elements of contracts with HDDs or CDDs as underlyings are the accumulation period and the station which records temperatures used to construct the underlying variable. Finally, the so-called tick size has to be determined. The tick size indicates the amount of money attached to each HDD or CDD, respectively.
How does trading develop outside the USA ? At the Eurex in Germany, the Deutsche Börse Group had offered heating degree days (HDD) and cooling degree days (CDD) indices since December 2000 for thirty European cities. Among these cities were the German cities Berlin, Essen, Frankfurt and Hamburg. In the meantime, the Eurex has withdrawn from this market due to the lack of standardized contracts and liquidity. Moreover, reliable and comprehensive weather data is not as easily available as in the USA. Additionally, the relevance of air conditioning in the summertime is not as pronounced as in the USA. Consequently, the demand for CDDs is much lower than in the USA. Attempts to establish an exchange-based trading of weather derivatives have failed in other European countries either. Therefore in Europe, trading of weather derivatives mainly takes place over the counter.
When we talk about valuation of temperature derivatives, we have to bear in mind that temperature as underlying has some salient characteristics. Since it is a meteorological variable rather than a traded asset, the conventional risk-neutral, arbitrage-free valuation methodology does not apply. By contrast, theoretically adequate valuation can only be based on an equilibrium model which takes into account the stochastic dynamics of the underlying as well as the risk aversion of the investors.
Another open question is whether the HDDs, CDDs should be directly modelled for each contract. Cao and Wei (2000) argue that direct modelling of the HDDs, CDDs has certain shortcomings. Instead, modelling the daily temperature enables us to handle temperature contracts of any maturity and for any season. Moreover, estimation of model parameters has to be carried out only once. By contrast, direct modelling of HDDs and CDDs requires a separate estimation procedure for each contract taking into account the season and the maturity of the contract due to the nature of temperature behavior. As a result, modelling temperature rather than the HDDs and CDDs seems more adequate.
Literature on weather derivatives is rather scanty. In the following paragraphs, we report some important contributions, at least in our opinion, on temperature derivatives.
To start with, Dischel (1998) and Brody et al. (2002), propose to simulate future behavior of temperature as a continuous time or discrete time stochastic process which takes into account the salient features of temperature such as mean reversion and seasonality. These processes can be then fitted to data and used to value any contingent claim by taking expectation of the discounted future payoff. Davis (2001) puts forward to value temperature derivatives based on HDDs

and CDDs in an equilibrium framework. Besides the stochastic dynamics of temperature, the author takes into account optimal consumption and investment rules when he derives explicit pricing formulas for the valuation of swap rates and option values. Torro et al. (2003) model air temperature behavior in Spain combining techniques for the modelling of short-term interest rates with a generalized autoregressive conditional heteroscedastic (GARCH) time series approach. They suggest to create a population-weighted index of daily temperatures from four different measuring stations to compute HDDs or CDDs. Furthermore, Cao and Wei (2004) propose an equilibrium framework linking the temperature uncertainty and the economy's aggregate output therein. They suggest a serially correlated bivariate-process for the temperature and the aggregate output. Finally, their framework allows to address the market price of weather risk. They apply their framework to temperature from five CME-traded cities in the USA. Campbell and Diebold (2005) take a simple but systematic time series approach to modelling and forecasting daily average temperature in 10 U.S. cities. They find strong evidence that point and density forecasts from their approach prove useful for participants in the weather derivatives market.

In addition, Taylor and Buizza (2004) and Taylor and Buizza (2005) compare temperature density forecasts from time series models with atmospheric models in terms of short-run predictions one up to ten days ahead. They find evidence that so-called weather ensemble density forecasts of daily midday temperature data recorded at five locations in the UK outperform forecasts provided by time series models. Weather ensemble forecasts consist of multiple future scenarios for a weather variable generated from atmospheric models. In a second step, Taylor and Buizza (2005) assess forecasts of the conditional mean and quantiles of the density of the payoff of a 10 day-ahead put option provided by univariate time series models, on one hand, and from atmospheric models, on the other hand. Again, the obtained results suggest to use weather ensemble forecasts.

In this article, we intend to particularly address aspects of multivariate analysis and cross-city hedging as put forward by Campbell and Diebold (2005). Trading of temperature derivatives requires to fix the station which records the temperature data that is used to compute the payoff of the derivative. At the CME, contracts are struck on data from few selected measuring stations to ensure liquidity. Campbell and Diebold (2005) argue that hedging of weather risk in remote locations is only possible if the risk of the remote location is highly correlated with the risk of a location for which a liquid market exists. Since HDDs and CDDs are computed at a daily basis, a multivariate model which captures daily correlation dynamics between locations may provide a very rich picture of reality and be therefore a very useful tool for risk management. In previous work of Torro et al (2003), Cao and Wei (2004) and Campbell and Diebold (2005) on univariate modelling, the authors have revealed that temperature displays rich dynamics such as yearly seasonality as well in the conditional mean as in the conditional variance. Consequently, it is probably naive to assume that the

conditional correlation between two locations is the same in winter as in summer. We may rather expect the opposite to be true.
At the CME, temperature derivatives based on data from nine European cities, among them Berlin and Essen, can be traded. Henceforth, if EnBW an important electricity supplier in the south-west of Germany plans to hedge his volume risk in the area of Stuttgart at the CME, for example, he must be aware of the correlation dynamics of daily average temperature at Stuttgart and the traded cities. Consequently, the correlation dynamics between the series from Stuttgart-Echterdingen and Berlin may be of special interest for EnBW. However, the modelling of correlation dynamics between temperature time series has not been paid much attention to, so far. Maybe this is one possible reason why many investors prefer to negotiate customized contracts over the counter on data from the region of their interest rather than to engage in standardized contracts on data from traded cities.
Following and extending the previous work of Torro et al (2003) and Campbell and Diebold (2005) on univariate GARCH models, we choose a bivariate GARCH framework. In more detail, our focus in this paper is twofold. On one hand, we want to assess the ability of bivariate GARCH models with dynamic conditional correlations in modelling time-varying correlation dynamics between temperature time series. On the other hand, we aim to apply the elaborated methodology to help an investor to correlate his own exposure with tradeable cities. As we mentioned before, knowledge of these correlation dynamics is the key to constructing a sensible hedge.
Previous contributions of Campbell and Diebold (2005), Franses et al.(2001), Torro et al. (2003), Taylor and Buizza (2004) and Tol (1996) show that generalized autoregressive conditional heteroscedastic (GARCH) models are useful in modelling and forecasting of univariate temperature time series.
First, we fit a univariate GARCH model in spirit to Franses et al.(2001) and Campbell and Diebold (2005) to our temperature time series. Moreover in a second step, we move on to a bivariate GARCH framework to examine the correlation dynamics between different locations. Recently, several multivariate GARCH models have been designed to allow for parsimony or to guarantee a positive definite covariance matrix, or often both. We should keep in mind, that these models have originally been conceived to model dynamics of financial time series. Temperature time series display yearly seasonality in the conditional variance, which is a salient feature compared with financial time series. Consequently, the incorporation of seasonality dynamics in the existing multivariate GARCH framework is the main task that we are faced with the modelling of temperature series.
A thorough analysis of different multivariate GARCH approaches has revealed that dynamic conditional correlation models, abbreviated DCC, are well suited for modelling correlations between temperature time series.
DCC models offer a high degree of flexibility in modelling the conditional variance

and conditional correlation dynamics. A further advantage of DCC models is the numerical stability, also for higher dimensions, due to a two-step estimation procedure that can be applied.
The remainder of the article is organized as follows. In Section 2, we present the temperature data and some descriptive statistics. Furthermore in Section 3, we discuss the considered univariate time series model and fit it to our given temperature data. In Section 4, we present and discuss selected bivariate GARCH models with dynamic conditional correlations. In addition, we fit them to the temperature data and discuss the results. How the elaborated methodology can be applied is described in Section 5. Finally, Section 6 concludes the article and gives hints for further research.

2 Data

The data comprises actually measured daily average temperature (measured in C°) from three measuring stations ranging from January 1$^{\text{st}}$ 1991 to April 29$^{\text{th}}$ 2005.
One of the measuring stations is located at Stuttgart, namely Echterdingen. Moreover, we have decided to take data from Berlin (-Tempelhof) and Hamburg (-Fuhlsbüttel). The data from Berlin is used to compute European CDDs and HDDs at the CME. Therefore, this choice is quite natural. Hamburg and Echterdingen, however, are located in the north and south of Germany. We expect temperature at other locations to exhibit correlation dynamics in between these two. Daily average temperature have been computed as the arithmetic mean of daily maximum and daily minimum temperature series. Finally, according to Campbell and Diebold (2005) and Taylor and Buizza (2004), we have discarded the 29^{th} February in leap years.
In the original series of Echterdingen, we have found three extremal observations larger than 43 C°. In our opinion, these values must be wrong. Therefore, we have replaced these aberrant observations by the average of temperatures observed one year before and one year after.
In table 2.1, we present some descriptive statistics for the three daily average temperature time series, whereas table 2.2 shows the correlations between the different temperature series. The estimated Kurtosis ranges between 2.27 and 2.44 and is far below 3 the value for the normal distribution. This is due to the different levels of temperature in winter and summer. By this, the distribution of these temperature series rather resembles a two-component mixture of normals than a normal distribution. In order to motivate our modelling approach in Section 4.3, we analyze the data from Echterdingen. The remaining temperature series exhibit similar time series characteristics. Consequently, we treat them analogously. Hence, subfigure 2.1a shows the series of the station at Echterdingen, while subfigure 2.1b presents the histogram of this series together with the superimposed

estimated normal density. As aforementioned, the empirical distribution seems slightly bimodal. The correlations between the temperature time series are positive and exceed 0.9.
Subfigure 2.1c shows the autocorrelation function which resembles a cosine function. This shape of the autocorrelation function indicates a strong yearly seasonality in the data. Finally, subfigure 2.1d suggests that the conditional mean of the considered time series should be modelled by a low- ordered autoregressive moving average process (ARMA).

Table 1: Descriptive Statistics on Temperature Series in C° from The Three Selected Stations.

	S-Echterdingen	Berlin	Hamburg
Mean	9.58	9.85	9.32
Median	9.70	9.85	9.20
Maximum	27.55	29.55	29.60
Minimum	-13.90	-14.5	-15.05
Std. Dev.	7.52	7.87	7.02
Skewness	-0.14	-0.09	-0.07
Kurtosis	2.33	2.27	2.44
Jarque-Bera	113.59	122.55	72.88

Table 2: Correlations of Temperature Series in C° from The Three Selected Stations.

	S-Echterdingen	Berlin	Hamburg
S-Echterdingen	1.000		
Berlin	0.942	1.000	
Hamburg	0.927	0.975	1.000

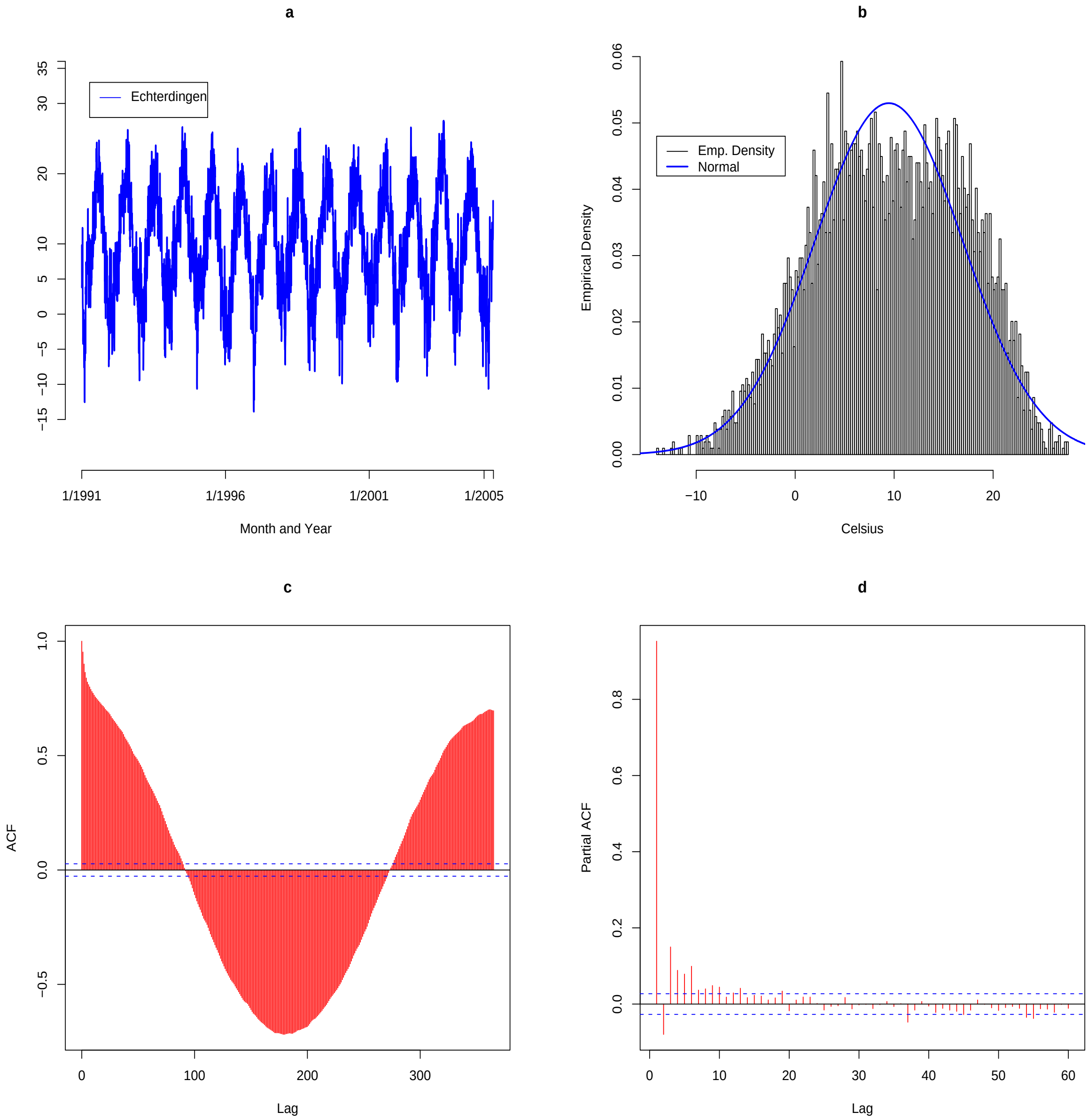

Figure 1: Daily average temperature from Echterdingen(01/01/1991 until 04/29/2005 (a), Histogram for Echterdingen (b), Autocorrelation function (ACF) for Echterdingen (c), Partial autocorrelation function (PACF)for Echterdingen (d).

3 Univariate Modelling

Our modelling approach of the average daily temperature time series is substantially inspired by the work of Franses et al.(2001) and Campbell and Diebold (2005). As opposed to Campbell and Diebold (2005), we prefer an autoregressive moving average process ARMA(1,1) for the two temperature series except for Hamburg. In the case of Hamburg, we opt for an ARMA(2,1). In addition, we specify the yearly seasonality with a Fourier series. Our approach for the two locations, except for Hamburg, is summarized in equation (3.1). The specification for Hamburg is presented in equation (3.2).

$$T_t = \lambda_{c,1} \cos(2\pi \frac{d_t}{365}) + \lambda_{s,1} \sin(2\pi \frac{d_t}{365}) + \rho_1 \mu_m + (1 - \rho_1) T_{t-1} + \epsilon_t + \theta \epsilon_{t-1}, \quad (3.1)$$

$$T_t = \lambda_{c,1} \cos(2\pi \frac{d_t}{365}) + \lambda_{s,1} \sin(2\pi \frac{d_t}{365}) + (\rho_1 - \rho_2)\mu_m + (1 - \rho_1) T_{t-1} + \rho_2 T_{t-2} + \epsilon_t + \theta \epsilon_{t-1}, \quad (3.2)$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$. Figure 3.1 presents some results on equation (3.1) for data from Echterdingen. Subfigures 3.1a and 3.1b show that the residuals do not exhibit any notable pattern of autocorrelation. However, the subfigures 3.1c and 3.1d suggest that the squared residuals are sligthly autocorrelated. Furthermore, subfigure 3.1e shows that the residuals are leptokurtic. Hence, the results put forward to model the conditional variance. Since the introduction of the (generalized) autoregressive conditional heteroscedasticity model (G)ARCH by Engle (1982) and Bollerslev (1986), a plethora of GARCH models has been proposed to take into account volatility clustering and the asymmetric effect of news on volatility.

For our purposes, we have chosen the approach of Franses et al. (2001) and link the potential asymmetry to a daily repeating step function d_t. The advantage of this model is that potential asymmetry is directly linked to its seasonal source.

Following Campbell and Diebold (2005), we divide the conditional variance h_t into a seasonal and a GARCH part denoted σ_t^2.

$$h_t = Seasonal_t + \sigma_t^2 \quad (3.3)$$

$$Seasonal_t = \sum_{q=1}^{Q} \left(\lambda_{c,q+1} \cos(2\pi q \frac{d_t}{365}) + \lambda_{s,q+1} \sin(2\pi q \frac{d_t}{365}) \right) \quad (3.4)$$

In equation (3.4), we set $Q = 2$ for all three stations. while the seasonal part of the conditional variance follows from equation (3.5) below.

$$\sigma_t^2 = \omega + \alpha \left(\epsilon_{t-1} - \gamma_1 - \gamma_2 d_t - \gamma_3 d_t^2 \right)^2 + \beta \sigma_{t-1}^2 \quad (3.5)$$

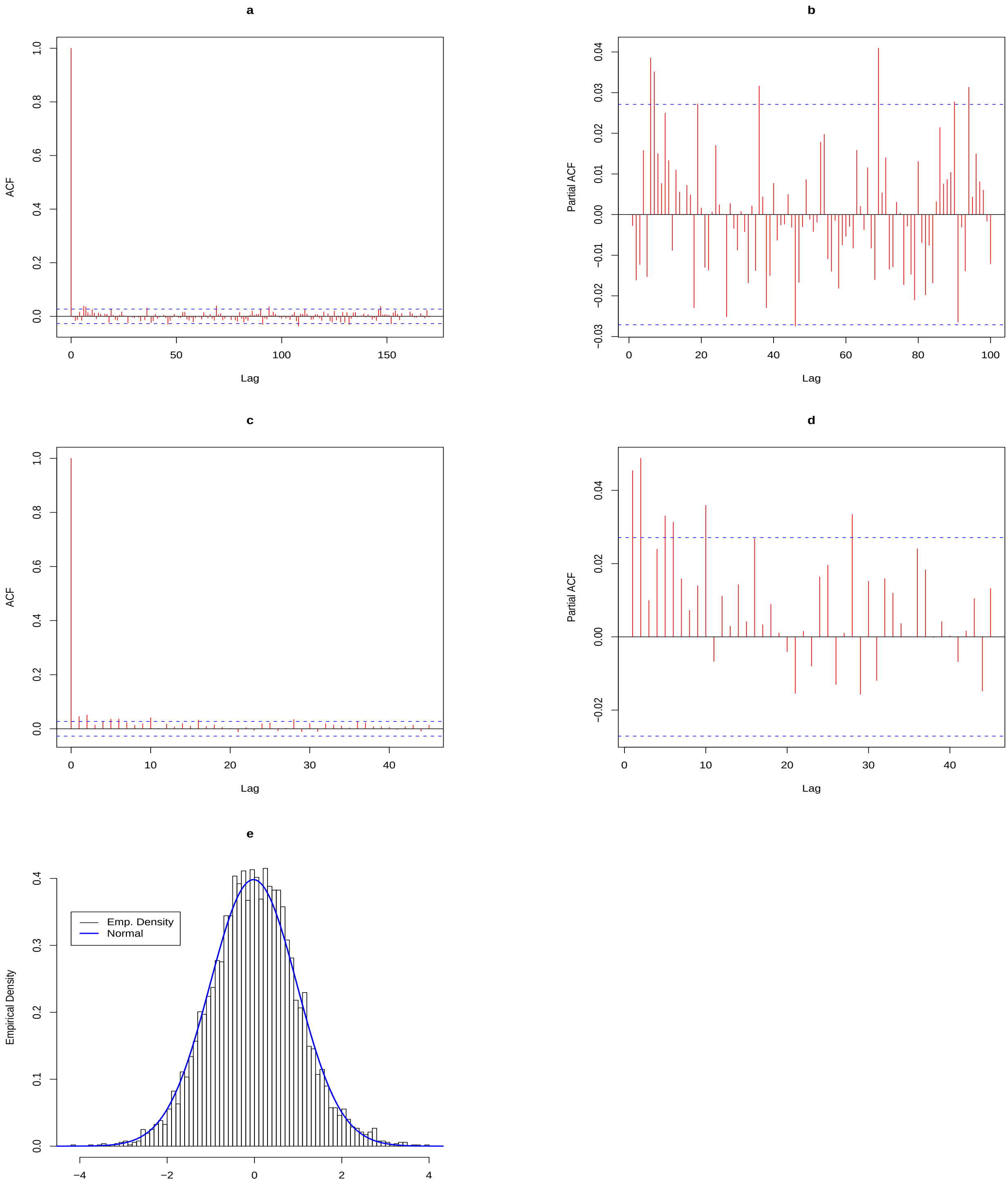

Figure 2: ACF of residuals (a), PACF of residuals (b), ACF of squared residuals (c), PACF of squared residuals (d), plot of standardised squared residuals (e),histogram of residuals (f), all for equation (3.1).

3.1 Results on Model Fit

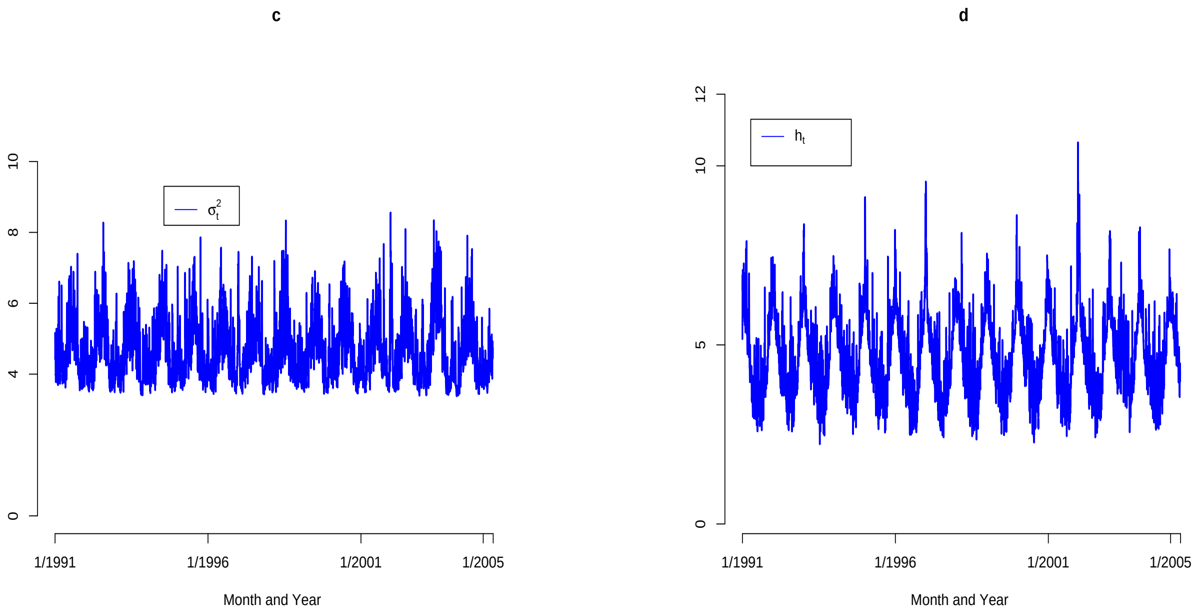

Figure 3: σ_t^2: (a), h_t (b), both series for Echterdingen.

The parameter estimates of equations (3.1-3.5) fitted to each of the three time series are collected in table 3.1. Besides the parameter estimates, we also report Kurtosis, Skewness and the value of the Ljung-Box statistics for the standardized residuals u_t and the squared standardized residuals u_t^2. The standardized residuals pass the test only at a level between 1% and 2% for Echterdingen and Berlin. We renounced to add an additional AR(2) term due to the lack of significance. The squared standardized residuals do not exhibit any notable autocorrelation. The coefficients of determination R^2 indicate a very good in-sample fit. However, the standardized residuals are still skewed and to a smaller extent leptokurtic. The deviation from normality indicates that a more sophisticated distribution which allows to take into account the skewness of standardized residuals should be investigated in further research. Finally, figure 3.2 shows the estimated conditional variance series generated from the estimated model for the temperature time series of Echterdingen.

Table 3: Summary In-sample Fit: Equations (3.1-3.5).

	Results		
	Echterdingen	Berlin	Hamburg
$\lambda_{s,1}$	-2.6569 (0.1882)	-2.7211 (0.1972)	-2.7071 (0.1878)
$\lambda_{c,1}$	-8.8657 (0.2033)	-9.4561 (0.2124)	-8.1187 (0.2007)
μ	9.7903 (0.1424)	10.0809 (0.1478)	9.5875 (0.1403)
ρ_1	0.2649 (0.0115)	0.2502 (0.0113)	0.4817 (0.1146)
ρ_2	-	-	0.1888 (0.0932)
θ	0.2233 (0.0168)	0.2133 (0.0164)	0.3659 (0.1111)
ω	1.1120 (0.3375)	0.8829 (0.2691)	1.2091 (0.2228)
α	0.0370 (0.0110)	0.0342 (0.0106)	0.0682 (0.0133)
γ_1	2.9282 (1.1958)	2.0431* (1.2375)	3.6364 (0.8753)
γ_2	-0.0753 (0.0201)	-0.0762 (0.0234)	-0.0751 (0.0139)
γ_3	$2 \cdot 10^{-4}$ ($5.4 \cdot 10^{-5}$)	$2 \cdot 10^{-4}$ ($6.4 \cdot 10^{-5}$)	$2 \cdot 10^{-4}$ ($3.8 \cdot 10^{-5}$)
β	0.6665 (0.0796)	0.7083 (0.0649)	0.5994 (0.0580)
$\lambda_{s,2}$	0.3804 (0.1665)	0.4893 (0.1931)	0.5166 (0.1158)
$\lambda_{c,2}$	1.9234 (0.3679)	1.6577 (0.5304)	0.9674 (0.2817)
$\lambda_{s,3}$	-0.2037^* (0.1337)	-0.3073 (0.1766)	-0.2640^* (0.1403)
$\lambda_{c,3}$	0.1863^* (0.1661)	0.1319^* (0.1766)	-0.1751^* (0.1803)
Skewness	0.0546	0.1755	0.2538
Kurtosis	3.2585	3.1384	3.1134
Jarque-Bera	17.1549	31.0090	58.9014
LL	-11372.32	-11384.98	-11325.05
R^2	0.9685	0.9706	0.9660
adj. R^2	0.9684	0.9705	0.9658
LB Q(7) for u_t	16.184	13.941	12.710
LB Q(7) for u_t^2	2.1367	1.3347	2.8629

*Note that * means not significant at the 5 % level.*

4 Bivariate Modelling

Research on multivariate GARCH models is very active due to their relevance for many financial applications such as asset pricing, portfolio selection, hedging and risk management. Quite alike the development in univariate GARCH modelling, we can observe several different more or less sophisticated approaches to multivariate GARCH models. For a comprehensive survey on multivariate GARCH models, we refer to Bauwens et al. (2006).
We continue with our approach from univariate modelling and assume the conditional covariance matrix H_t to consist of a seasonal part and a GARCH(1,1) part denoted Σ_t.

$$H_t = Seasonal_t + \Sigma_t \qquad \text{with } \Sigma_t = \begin{pmatrix} \sigma_{11,t}^2 & \sigma_{12,t} \\ \sigma_{21,t} & \sigma_{22,t}^2 \end{pmatrix} . \tag{4.1}$$

The seasonal term of $h_{ii,t}$, with $i \in \{1,2\}$, is specified as in the univariate case with $Q = 2$,

$$Seasonal_{ii,t} = \sum_{q=1}^{Q} \left(\lambda_{ii,c,q+1} \cos(2\pi q \frac{d_t}{365}) + \lambda_{ii,s,q+1} \sin(2\pi q \frac{d_t}{365}) \right) . \tag{4.2}$$

Here, we work in the framework of so-called dynamic conditional correlation models. Dynamic conditional correlation models have certain advantages. They allow to include seasonality in the conditional variance specification without running risk of numerical problems which is neither guaranteed by any version of VEC models proposed by Bollerslev et al. (1988) nor by any version of BEKK models advocated by Baba et al.(1991), respectively. Secondly, the specification of the conditional variance is not confined to be a standard GARCH(1,1) according to Bollerslev (1986). Therefore, we can take into account the asymmetry displayed by the temperature series due to the different impact of temperature higher than expected and lower than expected on the conditional variance. Orthogonal GARCH models, see van der Weide (2002), are also flexible with the specification of the conditional variance. However, orthogonal GARCH models are a special case of BEKK models. Consequently, these models do not possess distinct parameters, which exclusively govern the correlation dynamics. We opt for utmost flexibility in modelling. This is especially true with respect to correlation dynamics, the main subject of our study. Dynamic conditional correlation models warrant an utmost flexible modelling compared with the remaining approaches such as VEC and BEKK models.

4.1 The DCC Model Class

Dynamic conditional correlation models allow to separately specify the individual conditional variances, on one hand, and the conditional correlation matrix or

another measure of dependence between the individual series, like a copula of the conditional joint density, on the other hand.
First attempts to design dynamic conditional correlation models have been undertaken by Engle (2002), Tse and Tsui (2002), Christodoulakis and Satchell (2002). The dynamic conditional correlation model class nests the popular constant correlation coefficient model introduced by Bollerslev (1990). The main advantage of DCC models over VEC or BEKK models is parsimony in parametrization which alleviates estimation and allows to overcome the curse of dimensionality for higher than the bivariate case. A disadvantage is that theoretical results on stationarity, ergodicity and moments cannot be easily derived as for VEC and henceforth also BEKK models.
The center piece of this model class is the fact that H_t can be decomposed as follows,

$$H_t = D_t R_t D_t \,, \tag{4.3}$$

where D_t is the diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{ii,t}}$ on the i^{th} diagonal and R_t is the (possibly) time-varying correlation matrix. This class of models was originally designed to allow for two-step estimation of the typically high dimensional matrix H_t in the context of portfolio optimization, where very many assets are involved.
In the first step univariate volatility models are fitted for each of the assets or temperature series and estimates of $h_{ii,t}$ are obtained. In the second step, parameters of the conditional correlation are estimated given the estimated parameters and conditional variances from the first step.
Unfortunately, model parameters are not simultaneously estimated by means of quasi maximum likelihood and therefore inefficient. However, Engle and Sheppard (2001) show that consistency and asymptotic normality of the parameter estimates of the two-step DCC estimator closely follow the results for GMM.
For the bivariate DCC model, H_t can be expressed as,

$$H_t = \begin{pmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{pmatrix} \begin{pmatrix} 1 & r_{12,t} \\ r_{21,t} & 1 \end{pmatrix} \begin{pmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{pmatrix} . \tag{4.4}$$

Since correlations lie between -1 and 1, these models must include a rescaling procedure. The models of Engle (2002), Tse and Tsui (2002) are very similar. Therefore, we work only with the model of Engle (2002) for some reasons. This model is easier to handle in terms of specification and forecasting. In addition, Capiello et al. (2003) present an asymmetric extension to the model put forward by Engle (2002). Asymmetry in correlation dynamics between temperature time series may potentially be a very important feature.
More precisely, the evolution of the correlation in the model of Engle (2002) is

given by,

$$Q_t = \begin{pmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{pmatrix} \quad (4.5)$$

$$Q_t^* = \begin{pmatrix} \sqrt{q_{11,t}} & 0 \\ 0 & \sqrt{q_{22,t}} \end{pmatrix} \quad (4.6)$$

$$R_t = (Q_t^*)^{-1} Q_t (Q_t^*)^{-1} \quad (4.7)$$

$$Q_t = (1-\phi-\psi)\bar{Q} + \phi \mathbf{u_{t-1}} \mathbf{u_{t-1}}' + \psi Q_{t-1}\,, \quad (4.8)$$

where ϕ and ψ are scalars, whereas $\bar{Q} = E[\mathbf{u_t u_t'}]$ is the unconditional correlation matrix of the $u_{i,t} = \frac{\epsilon_{i,t}}{\sqrt{h_{ii,t}}}$. Obviously, the matrix $(Q_t^*)^{-1}$ is used for rescaling.
As aforementioned, Capiello et al. (2003) propose an asymmetric extension to the model of Engle (2002). The evolution of Q_t is now supposed to be,

$$Q_t = \left(\bar{Q} - \Phi'\bar{Q}\Phi - \Psi'\bar{Q}\Psi - \Upsilon' E[\eta_\mathbf{t}\eta_\mathbf{t}']\Upsilon\right) + \Phi'\mathbf{u_{t-1}u_{t-1}'}\Phi + \Psi' Q_{t-1}\Psi + \Upsilon' \eta_\mathbf{t-1}\eta_\mathbf{t-1}'\Upsilon\,. \quad (4.9)$$

Here, we denote $\eta_\mathbf{i,t} = \min(\mathbf{u_{i,t}}, \mathbf{0})$. Additionally, we substitute the expectations $\bar{Q} = E[\mathbf{u_t u_t'}]$ and $E[\eta_\mathbf{t}\eta_\mathbf{t}']$ with their sample analogues $\frac{1}{T}\sum_{t=1}^{T} \mathbf{u_t u_t'}$ and $\frac{1}{T}\sum_{t=1}^{T} \eta_\mathbf{t}\eta_\mathbf{t}'$, respectively.
Here, we only consider the bivariate scalar asymmetric model version, where ϕ, ψ and υ are scalars. In our opinion, this is no shortcoming, since our data does not support a more elaborate parametrization. The interesting specification is expressed in equation (4.10),

$$Q_t = \left(\bar{Q} - \phi\bar{Q} - \psi\bar{Q} - \upsilon E[\eta_\mathbf{t}\eta_\mathbf{t}']\right) + \phi\mathbf{u_{t-1}u_{t-1}'} + \psi Q_{t-1} + \upsilon\eta_\mathbf{t-1}\eta_\mathbf{t-1}'\,. \quad (4.10)$$

Dynamic conditional correlation models are still the subject of very active ongoing research. Here, we only present and exploit very fundamental models and methodology.
To name some of the recent contributions in this field, Hafner and Franses (2006) suggest semi-parametric modelling of conditional correlations. Pelletier (2006) puts forward to extend the dynamic conditional correlation framework to regime switching dynamic conditional correlation models. Teräsvirta (2005) considers a smooth transition conditional correlation model that allows the conditional correlations to vary between two extremes. Other authors pursue the modelling of the dependence between financial time series rather than the correlation, see Patton (2000), and Jondeau and Rockinger (2001). The investigation of the more sophisticated approaches and model extensions is left as a challenging task for further research.

4.2 Flexible Dynamic Correlations models

Due to the rescaling procedure which ensures that conditional correlations lie between -1 and 1, it is not possible to include a Fourier series to capture potential yearly seasonality in the conditional correlation series in the framework of Engle (2002). This is also true for the framework of Tse and Tsui (2002).
Christodoulakis and Satchell (2002) put forward a potential remedy which warrants full flexibility in modelling conditional correlation dynamics. The authors use the Fisher transformation of the correlation coefficient to ensure that $-1 \leq r_{12,t} \leq 1$. More precisely, Christodoulakis and Satchell (2002) suggest

$$r_{12,t} = \frac{\exp(2r^*_{12,t}) - 1}{\exp(2r^*_{12,t}) + 1}\,. \tag{4.11}$$

Moreover, Baur (2006) proposes a transformation which is described in equation (4.12),

$$r_{12,t} = \frac{r^*_{12,t-1}}{\sqrt{1 + (r^*_{12,t-1})^2}},\,. \tag{4.12}$$

Finally, we compute the covariance $h_{12,t}$ according to equations (4.13) and (4.14). We determine the correlation $r_{12,t}$ using one of the transformations presented in equations (4.11) and (4.12).

$$\begin{aligned} r^*_{12,t} &= \omega_{12} + \phi u_{1,t-1}u_{2,t-1} + \psi r^*_{12,t-1} + \upsilon \eta_{1,t-1}\eta_{2,t-1} \\ &\quad + \lambda^*_{c,1}\cos(2\pi\frac{d_t}{365}) + \lambda^*_{s,1}\sin(2\pi\frac{d_t}{365}) \qquad (4.13) \\ h_{12,t} &= r_{12,t}\sqrt{h_{11,t}h_{22,t}} \qquad (4.14) \end{aligned}$$

4.3 Two- Step Estimation Procedure

The DCC models as well as the FDC models are estimated by means of the two step estimation procedure suggested in Engle (2002) and quasi maximum likelihood, respectively. Here, we outline the two- step procedure referring to the original contribution in Engle (2002). The results of the univariate study put forward that assuming $\epsilon_t|\mathcal{F}_{t-1} \sim \mathcal{N}(0, H_t)$, where $\mathcal{F}_{t-1}$ is the information set at time $t-1$, is a possible choice. The logarithmic likelihood $L(\varphi, \theta)$ for this estimator can be expressed as,

$$
\begin{aligned}
L(\varphi,\theta) &= -\frac{1}{2}\sum_{t=1}^{T}\Big(n\log(2\pi)+\log|H_t|+\epsilon_{\mathbf{t}}{}'H_t^{-1}\epsilon_{\mathbf{t}}\Big)\\
L(\varphi,\theta) &= -\frac{1}{2}\sum_{t=1}^{T}\Big(n\log(2\pi)+\log|D_tR_tD_t|+\epsilon_{\mathbf{t}}{}'D_t^{-1}R_t^{-1}D_t^{-1}\epsilon_{\mathbf{t}}\Big) \quad (4.15)\\
L(\varphi,\theta) &= -\frac{1}{2}\sum_{t=1}^{T}\Big(n\log(2\pi)+2\log|D_t|+\log|R_t|+\mathbf{u_t}{}'R_t^{-1}\mathbf{u_t}\Big)
\end{aligned}
$$

The parameters in D_t are denoted φ, whereas the additional parameters in R_t are denoted θ. Furthermore n is the number of assets or in our case temperature series. Henceforth, in our study n equals 2 ($n = 2$). To implement the two- step estimation strategy, Engle (2002) suggests to replace R_t by the identity matrix to obtain a consistent estimator in the first step of the estimation procedure. In such a case, the univariate quasi logarithmic likelihood function $QL_1(\varphi)$ becomes

$$QL_1(\varphi) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi)+\sum_{i=1}^{n}\left(\log(h_{ii,t})+\frac{\epsilon_{i,t}^2}{h_{ii,t}}\right)\right). \quad (4.16)$$

The first step provides estimates $\hat{\varphi}$. In the second step of the estimation procedure, we estimate the remaining parameters θ conditioned on the estimates from the first step. Since parameters φ are determined, the relevant part for estimation in the second step is the quasi logarithmic likelihood denoted $QL_2(\theta|\hat{\varphi})$.

$$QL_2(\theta|\hat{\varphi}) = -\frac{1}{2}\sum_{t=1}^{T}\Big(\log|R_t|+\mathbf{u_t}{}'R_t^{-1}\mathbf{u_t}\Big) \quad (4.17)$$

In the bivariate case, $L_C(\theta)$ can be quite simply written as,

$$QL_2(\theta|\hat{\varphi}) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log(1-r_{12,t}^2)+\frac{(u_{1,t}^2+u_{2,t}^2-2r_{12,t}u_{1,t}u_{2,t})}{(1-r_{12,t}^2)}\right). \quad (4.18)$$

To compare this two- step likelihood with the logarithmic likelihood of other models, we can compute its value as follows,

$$L(\varphi,\theta) = QL_1(\varphi)+QL_2(\theta|\hat{\varphi})+\frac{1}{2}\sum_{t=1}^{T}\mathbf{u_t}{}'\mathbf{u_t}\ . \quad (4.19)$$

4.4 Results on Model fit

We estimate the logarithmic likelihood given in equation (4.20). For simplicity, standardized residuals are assumed to be normally distributed,

$$\log L(\theta,\varphi) = -\frac{T}{2}\log(2\pi)-\frac{1}{2}\log(|H_t|)-\frac{1}{2}\epsilon_{\mathbf{t}}{}'H_t\epsilon_{\mathbf{t}}\ . \quad (4.20)$$

Temperature derivatives based on temperature from Berlin are traded at the CME. Therefore, we have designed two pairs, Echterdingen-Berlin and Hamburg-Berlin, for the empirical study. We estimate all four models with the two- step method put forward by Engle (2002) and described in subsection 3.3. Additionally, we take the estimates of the two- step procedure as starting values and carry out a simultaneous quasi maximum likelihood estimation of the likelihood in equation (4.20). Estimates of parameters which enter the correlation equations are collected in tables 4.1 to 4.4. Here, the remaining parameters are of minor interest, therefore we do not explicitly address them, but they can be obtained upon request. However for the two- step method, the remaining parameters are given in table 3.1. In addition, figure 4.1 shows the estimated conditional correlations for the pairs Echterdingen-Berlin and Hamburg-Berlin provided by the five considered versions of dynamic conditional correlation models.
The estimates of the quasi maximum likelihood estimation provide higher values of the logarithmic likelihood throughout all models compared with the two- step method. The in-sample results of the four DCC models do not indicate, that any model version clearly performs best.
For the pair Echterdingen-Berlin, the symmetric DCC model provides the highest in-sample fit. Conditional correlations between the temperature series from Echterdingen and Berlin only display weak yearly seasonality. Consequently, the inclusion of seasonality in the flexible dynamic conditional correlation models is of minor importance. By contrast, conditional correlations between Hamburg and Berlin seem to display a very pronounced yearly seasonality. As a result, the flexible dynamic correlation models outperform the dynamic conditional correlation models for the pair Hamburg-Berlin in terms of fit. In our study, we see no evidence that an asymmetric component as suggested by Capiello et al. (2003) is necessary to model conditional correlation dynamics.
Furthermore, the flexible dynamic correlation models provide very similar results. The parameter estimates of ψ are very small throughout all four dynamic conditional correlation models. For the two flexible dynamic correlation models, the parameter estimates of ϕ are large and significant for the pair Echterdingen-Berlin. By contrast, they are nonsignificant for the pair Hamburg-Berlin. For the pair Hamburg-Berlin, it seems, that it is only the yearly seasonality that really counts throughout the flexible dynamic correlations models.
In practice, an energy supplier may often wish to isolate his volumetric risk to more than one location. In such a case, the analysis can become trivariate and even higher dimensional which disqualifies the flexible dynamic correlation models because they only work in the bivariate case, whereas DCC models in spirit to Engle (2002) are designed for high dimensional multivariate GARCH, too.
In the bivariate case, conditional correlations can strongly differ across the different model versions. This seems to be particularly true if conditional correlations display strong seasonality patterns as in the case of Hamburg and Berlin.
In such a case, one may wish to base model selection not only on in-sample fit.

In addition, the density of bivariate temperature time series generated by the different DCC model versions could be estimated by means of a Monte Carlo simulation. These estimated densities could be then compared to the empirical density of the actually measured temperature data.
Diebold, Hahn and Tay (1999)advocate a more sophisticated approach which entails to evaluate multivariate density forecasts using an integral transform dating back to Rosenblatt (1952).

Table 4: Summary In-Sample Fit : DCC Models by Engle (2002) and Capiello et al. (2003), (Two- step estimation).

	Symmetric DCC		Asymmetric DCC	
	Echterdingen-Berlin	Hamburg- Berlin	Echterdingen-Berlin	Hamburg- Berlin
ϕ	0.084 (0.013)	0.091 (0.010)	0.083 (0.014)	0.085 (0.011)
ψ	0.421 (0.110)	0.398 (0.092)	0.408 (0.110)	0.321 (0.094)
υ	-	-	0.011^* (0.024)	0.041^* (0.021)
LL	-22189.25	-21078.05	-22189.15	-21076.53
SC	8.5410	8.1190	8.5426	8.1201

Note that * *means not significant at the 5 % level.*

Table 5: Summary In-Sample Fit: DCC Models by Engle (2002) and Capiello et al. (2003), (QML).

	Symmetric DCC		Asymmetric DCC	
	Echterdingen-Berlin	Hamburg- Berlin	Echterdingen-Berlin	Hamburg- Berlin
ϕ	0.068 (0.013)	0.059 (0.018)	0.068 (0.014)	0.063 (0.011)
ψ	0.473 (0.120)	0.652 (0.078)	0.473 (0.120)	0.674 (0.071)
υ	-	-	-0.002^* (0.026)	-0.021^* (0.014)
LL	-22122.79	-20836.03	-22122.85	-20835.81
SC	8.5189	8.0280	8.5205	8.0296

Note that * *means not significant at the 5 % level.*

Table 6: Summary In-Sample Fit : Satchell and Christodoulakis (2002), equation(4.4.11).

	Two-step estimation		QML	
	Echterdingen-Berlin	Hamburg- Berlin	Echterdingen-Berlin	Hamburg- Berlin
ω_{12}	0.140 (0.048)	0.950 (0.191)	0.152 (0.061)	1.334 (0.349)
ϕ	0.042 (0.009)	0.019 (0.007)	0.033 (0.009)	0.010^* (0.008)
ψ	0.662 (0.108)	-0.196^* (0.228)	0.675 (0.122)	-0.411^* (0.358)
υ	-0.007^* (0.019)	0.070 (0.022)	0.004^* (0.019)	0.040^* (0.023)
$\lambda^*_{s,1}$	0.013^* (0.007)	0.024^* (0.018)	0.013^* (0.008)	0.054^* (0.030)
$\lambda^*_{c,1}$	0.009^* (0.006)	0.175 (0.040)	0.009^* (0.007)	0.214 (0.062)
LL	-22202.82	-21074.28	-22131.05	-20827.10
SC	8.5528	8.1241	8.5286	8.0311

Note that * *means not significant at the 5 % level.*

Table 7: Summary In-Sample Fit : Baur (2006), equation(4.4.12).

	Two-step estimation		QML	
	Echterdingen-Berlin	Hamburg-Berlin	Echterdingen-Berlin	Hamburg- Berlin
ω_{12}	0.155 (0.052)	1.040 (0.211)	0.158 (0.063)	1.479 (0.420)
ϕ	0.049 (0.011)	0.029 (0.010)	0.039 (0.011)	0.017^* (0.012)
ψ	0.636 (0.111)	-0.177^* (0.223)	0.674 (0.120)	-0.349^* (0.368)
υ	-0.004^* (0.022)	0.095 (0.030)	0.003^* (0.021)	0.062 (0.036)
$\lambda^*_{s,1}$	0.010^* (0.007)	0.034^* (0.025)	0.010^* (0.008)	0.077^* (0.044)
$\lambda^*_{c,1}$	-0.004^* (0.022)	0.230 (0.053)	0.003^* (0.021)	0.299 (0.093)
LL	-22202.50	-21074.72	-22130.92	-20827.83
SC	8.5526	8.1243	8.5285	8.0315

Note that * *means not significant at the 5 % level.*

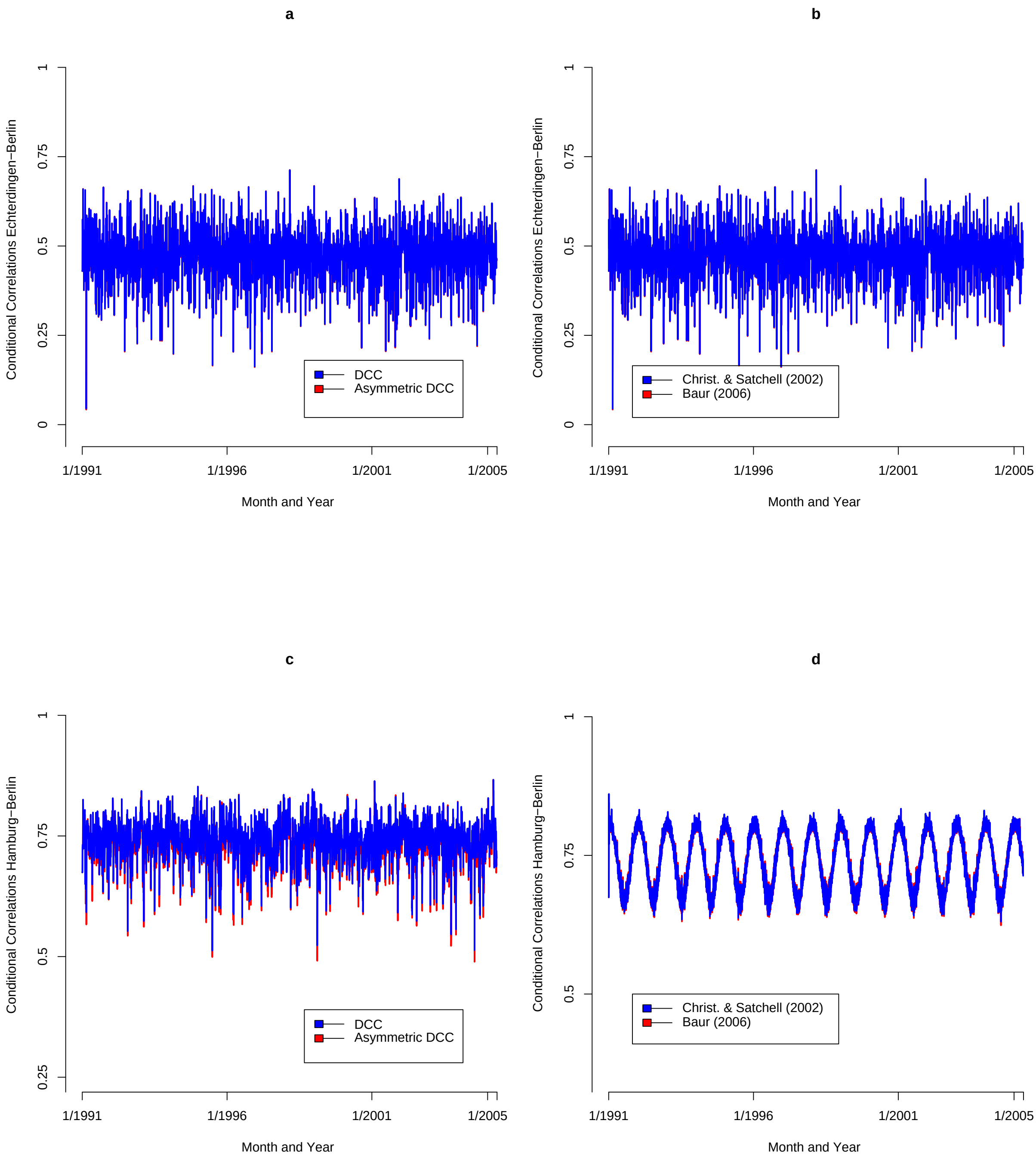

Figure 4: Conditional correlations estimated by QML: DCC and Asymmetric DCC Echterdingen-Berlin (a),FDC Echterdingen-Berlin (b), DCC and Asymmetric DCC Hamburg-Berlin (c), FDC Hamburg-Berlin (d).

5 Cross-City Hedging

In this section, we want to discuss how the presented methodology can be used. We do this from the angle of an electricity supplier who wants to hedge his volume risk at non-traded locations such as Echterdingen or Hamburg constructing a hedge based on HDDs or CDDs computed and accumulated on the temperature measured in Berlin. Recall, HDDs and CDDs are computed as follows,

$$HDD(t_1, t_2) = \sum_{t=t_1}^{t_2} \max(18.33^\circ - Y_{1,t}, 0) \;, \tag{5.1}$$

$$CDD(t_1, t_2) = \sum_{t=t_1}^{t_2} \max(Y_{1,t} - 18.33^\circ, 0) \;, \tag{5.2}$$

where t_1 denotes the beginning while t_2 marks the end of the accumulation period and $Y_{1,t}$ is the daily average temperature measured at the traded station in Berlin. Let $Y_{2,t}$ be the non-traded location which, in our case, is Echterdingen or Hamburg. For the sake of simplicity and according to our preceding assumptions, we assume that $Y_{1,t}$ and $Y_{2,t}$ are conditional bivariate normal distributed according to,

$$\begin{pmatrix} Y_{1,t} \\ Y_{2,t} \end{pmatrix} = \begin{pmatrix} E[Y_{1,t}|\mathcal{F}_{t-1}] \\ E[Y_{2,t}|\mathcal{F}_{t-1}] \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix} \sim \mathbf{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} \right) . \tag{5.3}$$

The assumption of bivariate normality is to some extent heroic but not completely unrealistic given our univariate studies. Furthermore, it offers the advantage that the distribution of $\{Y_{1,t}|Y_{2,t} = y_{2,t}\}$ is a univariate normal distribution,

$$\{Y_{1,t}|Y_{2,t} = y_{2,t}\} \sim N\left(E[Y_{1,t}|\mathcal{F}_{t-1}] + r_{12,t}\frac{\sqrt{h_{11,t}}}{\sqrt{h_{22,t}}}(y_{2,t} - E[Y_{2,t}|\mathcal{F}_{t-1}]), h_{11,t}(1 - r_{12,t}^2) \right) . \tag{5.4}$$

The relation in equation (5.4) enables the electricity supplier to construct forecast intervals for $Y_{1,t}$ if he can predict how $Y_{2,t}$ evolves at t. In Section 3, we have seen that the electricity supplier can model $Y_{2,t}$ fairly well by means of a time series model.

For example, she may expect the temperature to be 6.1° on a certain day t. Using the bivariate GARCH model, she immediately obtains that with probability α, temperature $Y_{1,t}$ will lie in the interval $I_{1,t,\alpha} = [Y_{1,t,low}; Y_{1,t,high}]$. Henceforth, she can estimate the relation between temperature realizations $y_{2,t}$ at the non-traded locations and HDDs or CDDs based on temperature in Berlin on a daily scale.

More precisely, let us consider a conditional HDD based on $Y_{1,t}$ and conditioned on $Y_{2,t} = y_{2,t}$. Moreover, let $Z_t^* = 18.33^\circ - \{Y_{1,t}|Y_{2,t} = y_{2,t}\}$. In addition, we denote,

$$Z_t = \begin{cases} 0 & \text{if } Z_t^* \leq 0 \,, \\ Z_t^* & \text{if } Z_t^* > 0 \,. \end{cases} \tag{5.5}$$

Obviously, Z_t corresponds to the value of a daily HDD(t_1, t_1). Moreover, we obtain

$$Prob(Z_t = 0) = Prob(\{Y_{1,t}|Y_{2,t} = y_{2,t}\} \geq 18.33°) = 1 - \Phi_{Norm}(18.33°)\,. \quad (5.6)$$

$$Prob(0 \leq Z_t \leq z_t) = Prob(18.33° - z_t \leq \{Y_{1,t}|Y_{2,t} = y_{2,t}\} \leq 18.33°) = \Phi_{Norm}(18.33°) - \Phi_{Norm}(18.33° - z_t)\,. \quad (5.7)$$

Note that $\Phi_{Norm}(x)$ is the value at quantile x of the cumulative conditional normal distribution given in equation (5.4).
Hence, equations (5.6) and (5.7) directly provide probabilities for the daily HDD(t_1, t_1) given a temperature realization $y_{2,t}$ at a non-traded location.
In addition, the tick size, which is the amount attached to each HDD or CDD, has to be fixed for each contract. The electricity supplier may know that temperature at Echterdingen is on average 4.1° in winter. Moreover, she may also know that every additional degree above this average temperature is accompanied by a loss of 5000 Euro on average.
Unfortunately, she has to compute a tick size with respect to the temperature dynamics in Berlin. Tick sizes are determined by a least square regression of load on temperature to examine how temperature on average affects demand for electricity. As a result, we obtain a relation between load and temperature which enables us to fix a tick size. The tick size for load at Echterdingen and temperature in Berlin can be determined, analogously.
Although to the best of our knowledge constant tick sizes are typical of temperature contracts, we think that a time-varying tick size may be more realistic. Demand and therefore load exhibit different patterns of seasonality such as interdaily, weekly and yearly seasonality. As a result, unexpected temperature values can have a very different impact on electricity demand depending on the hour or the type of day, for example. Time-varying tick sizes, however, allow more accurate hedging and can be very easily included in the bivariate GARCH framework. Additionally, daily load information could be brought into play to determine time-varying tick sizes.

6 Summary

Volumetric risk has become a crucial issue in competitive electricity markets. Especially in the USA, energy companies seek to hedge their volumetric risk. Weather derivatives are attractive instruments which allow to protect from volumetric risk due to unforeseen weather conditions.
In this article, we focus on temperature derivatives since over 90 % of weather contracts are struck on heating degree days or cooling degree days that are transformations of daily average temperature. Exchange-based trading mainly takes

place at the Chicago Mercantile Exchange, abbreviated CME. To ensure liquidity, contracts at the CME can only be negotiated on temperature from few selected cities. Consequently, market participants who wish to hedge their volumetric risk at non-traded locations cannot buy tailor- made contracts. Hence, they have to correlate their risk with the risk at tradeable cities. Therefore , the correlation between temperature time series from traded locations with temperature from non-traded locations is of special interest.
After a thorough analysis, we have found dynamic conditional correlation models, DCC, to be most appealing among the plethora of competing multivariate GARCH models for our purposes.
Our main challenge is to integrate seasonality into bivariate GARCH models. DCC models allow for an utmost flexibility in modelling the conditional variance and conditional correlation dynamics, respectively. In addition, the flexible dynamic correlation models even allow to model yearly seasonality of conditional correlations. Finally, we discuss how our presented methodology may be used by an investor to construct a hedge for a non-traded location.
We think that further research with correlation dynamics should concentrate on the DCC model class, with a special focus on seasonality. The univariate study has revealed that a simple ARMA-GARCH cannot completely capture temperature dynamics. Therefore, in further research the ability of more sophisticated models which explicitly allow to model skewness should be investigated.

References

[1] Baba Y, Engle RF, Kraft D, Kroner KF(1991): Multivariate simultaneous GARCH.*Working paper*, , San Diego, Department of Economics.

[2] Baur D.(2006): A flexible dynamic correlation model.*Advances in Econometrics (Part A): Econometric analysis of financial and economic time series*,20 , Elsevier.

[3] Bauwens L, Laurent S, Rombouts JVK(2006): Multivariate GARCH models : A survey.*Journal of Applied Econometrics*, 21, 79-109.

[4] Berndt EK, Hall RE, Hausman JA (1974): Estimation and inference in nonlinear structural models. *Annals of Economic and Social Measurement*, 3, 653-665.

[5] Bollerslev, T (1986): Generalized conditional heteroskedasticity. *Journal of Econometrics*, 81, 307-327.

[6] Bollerslev T(1990): Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model.*Review of Economics and Statistics*, 72, 498-505.

[7] Brody DC, Syroke J, Zervos M(2000): Dynamic pricing of weather derivatives.*Quantitative Finance*,2, 189-198.

[8] Bollerslev T, Engle RF, Wooldridge JM(1988): A capital asset pricing model with time-varying covariances.*Journal of Political Economy*, 96, 116-131.

[9] Campbell, S.& Diebold Francis X.(2005):Weather Forecasting for Weather Derivatives , *Journal of the American Statistical Association*, 100 , No. 469, 6-16.

[10] Cao M, Wei J(2000): Pricing the weather.*Risk*, 67, 67-70.

[11] Cao & Wei (2004):Weather Derivatives valuation and market price of weather risk *The Journal of Futures Markets*,24 , No. 11, 1065-1089.

[12] Cao M, Li A, Wei J(2004): Watching the weather report. *Canadian Investment Review*,17(2), 27-33.

[13] Cappiello L, Engle RF, Sheppard K(2003): Asymmetric Dynamics in the correlations of global equity and bond returns.*European Central Bank, Working Paper No. 204.*

[14] Christodoulakis GA, Satchell SE(2002): Correlated ARCH: Modelling the time-varying correlation between financial asset returns.*European Journal of Operations Research*, 139, 351-370.

[15] Davis, M.(2001):Pricing Weather Derivatives by marginal value *Quantitative Finance*,1,1-4.

[16] Dischel B(1998): Black-Scholes won't do.*Risk*, 8-9.

[17] Diebold FX, Hahn J, Tay AS(1999): Multivariate density forecast evaluation and calibration in financial risk management: High-frequency returns on foreign exchange.*The Review of Economics and Statistics*,81(4),661-673.

[18] Engle RF (1982): Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50, 987-1007.

[19] Engle RF(2002): Dynamic conditional correlation - A simple class of multivariate Garch models. *Journal of Business and Economic Statistics*, 20, 339-350.

[20] Engle RF, Kroner KF(1995): Multivariate simultaneous generalized arch.*Econometric Theory*, 11, 122-150.

[21] Engle RF, Mezrich J(1996): Garch for groups.*RISK*, 9, 36-40.

[22] Engle RF, Sheppard K(2001): Theoretical and empirical properties of dynamic conditional correlation multivariate Garch.*UCSD, Discussion Paper No. 2001-15.*

[23] Eviews 5.1. Quantitative Micro Software, LLC. *Irvine California*.1994-2004.

[24] Franses PH, van Dijk D(1996): Forecasting stock market volatility using nonlinear GARCH models.*Journal of Forecasting*, 15 , 229-235.

[25] Franses PH, van Dijk D(2000): Non-linear time series models in empirical finance.*Cambridge University press*, Cambridge.

[26] Franses PH, Neele J, van Dijk D(2001): Modeling asymmetric volatility in weekly Dutch temperature data.*Environmental Modelling & Software* , 16 , 131-137.

[27] Glosten LR, Jagannathan R, Runkle D(1993): On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48, 1779-1801.

[28] Hafner CM, Franses PH(2003): A generalized dynamic conditional correlation model for many assets. *Econometric Institute Report EI 2003-18*. Erasmus University Rotterdam.

[29] Hafner CM, van Dijk D, Franses PH(2006): Semi-Parametric modelling of correlation dynamics.*Advances in Econometrics (Part A): Econometric analysis of financial and economic time series*,20, Elsevier.

[30] Jarque CM, Bera AK(1987): A Test for normality of Observations and regression residuals. *International Statistical Review* , 55(2), 163-172.

[31] Jondeau E, Rockinger M(2001): The copula-GARCH model of conditional dependencies: An international stock-market application.*Journal of International Money and Finance*.

[32] Kroner KF, Ng VK(1998): Modeling asymmetric comovements of asset returns.*The Review of Financial Studies*, 11(4), 817-844.

[33] Patton A(1999):Modelling time-varying exchange rate dependence using the conditional copula. University of California, San Diego, Discussion Paper 01-09.

[34] Pelletier D(2006): Regime switching for dynamic correlations.*Journal of Econometrics*, 131 , 445-473.

[35] Rosenblatt M(1952): Remarks on a multivariate transformation.*Annals of Mathematical Statistics*, 23, 470-472.

[36] Sentana E(1995): Quadratic ARCH models.*Review of Economic Studies*, 62 , 639-661.

[37] Taylor JW, Buizza R(2004): Comparing Temperature Density Forecasts from GARCH and Atmospheric Models.*Journal of Forecasting*, 23, 337-355.

[38] Taylor JW, Buizza R(2006): Density forecasting for weather derivatives pricing.*International Journal of Forecasting*,22, 29-42.

[39] Silvennoinen A, Teräsvirta T(2005): Multivariate autoregressive conditional heteroskedasticity with smooth transitions in conditional correlations. *SSE/EFI Working Paper Series in Economics and Finance No. 577*, Stockholm School of Economics.

[40] Tol RJS(1996): Autoregressive conditional heteroscedasticity in daily temperature measurements.*Environmetrics*,7, 67-75.

[41] Torro H, Meneu V, Valor E(2003): Single factor stochastic models with seasonality applied to underlying weather derivatives variables. *Journal of Risk Finance*.

[42] Tse YK, Tsui AKC(2002): A multivariate GARCH model with timevarying correlations.*Journal of Business and Economic Statistics*, 20, 351-362.

[43] van der Weide R(2002): GO-GARCH: A multivariate generalized orthogonal GARCH model.*Journal of Applied Econometrics*, 17, 549-564.