Improved GMM estimation of the spatial autoregressive error model

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Abstract

We suggest an improved GMM estimator for the autoregressive parameter of a spatial autoregressive error model by taking into account that unobservable regression disturbances are different from observable regression residuals.
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1 Introduction and Summary

Disturbances of regression models are typically not observable, so inference on the disturbances must rely on the regression residuals. It is well known that under general conditions, the residuals converge to the disturbances when the sample size increases, see e.g. Rao and Toutenburg (1995). However, the statistical properties of the disturbances and the residuals are different in finite samples.

This paper considers a linear regression model where the disturbances are generated by a spatial autoregressive model introduced by Cliff and Ord (1973) and where the parameter of main interest is the spatial autoregressive parameter.

Since the calculation of the maximum likelihood estimator can be computationally expensive, Kelejian and Prucha (1999) suggest a generalized method of moments (GMM) estimator, which uses three theoretical moments of the disturbances and equates them to

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the corresponding empirical moments of the residuals. This estimator has been applied to industrial specialization by Tingvall (2004), to microlevel data by Bell and Bockstael (2000) and to agricultural data by Schlenker et al. (2006) and Anselin et al. (2004). It has also been extended in several ways, for example to panel data by Druska and Horrace (2004) and to systems of simultaneous equations by Kelejian and Prucha (2004).

We suggest a variation of this estimator that is motivated by the following argument: If the empirical moments must rely on the residuals, the theoretical moments should be calculated in terms of the residuals, too. The computational costs are of the same order of magnitude for both estimators. Although both estimators coincide as sample size increases, our version is superior in finite samples, both in terms of bias and mean squared error. As a consequence, significance tests for the regression coefficients are less distorted because estimation of the corresponding covariance matrix is more accurate.

An empirical example illustrates our results. For a data set of Indonesian rice farms previously analyzed by Erwidodo (1990) and Druska and Horrace (2004), statistically significant effects of some of the covariates disappear if we implement the proposed modification.

In the following, we restrict ourselves to ordinary least squares regression in order to keep notation as simple as possible. The main idea however also applies to generalized least squares or nonlinear regression.

2 The Model and the Main Result

We consider a linear regression model

$$y = X\beta + u, \tag{1}$$

where y is the $(n \times 1)$-vector of observations on the dependent variable, X is the non-stochastic $(n \times k)$-matrix on the explanatory variables and β is the $(k \times 1)$-vector of unknown model parameters. We assume that u, the $(n \times 1)$-vector of disturbances, is generated by a spatial autoregressive model,

$$u = \rho W u + \varepsilon, \tag{2}$$

where W $(n \times n)$ is a weighting matrix of known constants, ρ is a scalar parameter and ε is an $(n \times 1)$-vector of innovations. We impose the following assumptions.

Assumption 1. *(a) All diagonal elements of W are zero. (b) The row sums of W are equal to one, $\sum_{j=1}^{n} w_{ij} = 1 \; \forall \; i = 1, \ldots, n$. (c) $|\rho| < 1$.*

Assumption 2. *The innovations $\varepsilon_1, \ldots, \varepsilon_n$ are independently and identically distributed with zero mean and variance σ^2, where the variance is bounded by some positive constant b, $0 < \sigma^2 < b < \infty$. Additionally, $E(\varepsilon_i^4) < \infty \; \forall \; i \in \{1, \ldots, n\}$.*

Assumption 3. *The elements of X are nonstochastic and bounded in absolute value by some $0 < c_X < \infty$. Further, X has full column rank, and the matrix $Q_X = \lim_{n\to\infty} \frac{1}{n} X'X$ is finite and nonsingular.*

Assumption 1 ensures that the matrix $I - \rho W$ is nonsingular so that we have $u = (I - \rho W)^{-1}\varepsilon$ and

$$\mathrm{Cov}(u) = \sigma^2 (I - \rho W)^{-1}(I - \rho W')^{-1}. \tag{3}$$

Since u is not observable, estimation of ρ and σ^2 must rely on $\hat{u}$, the vector of regression residuals. For the case of OLS-regression, $\hat{u}$ is given by $\hat{u} = y - X\hat{\beta} = Mu$, where $M = I - X(X'X)^{-1}X'$, and $\hat{\beta} = (X'X)^{-1}X'y$ is the OLS-estimator of β.

In this situation, Kelejian and Prucha (1999) suggest a GMM estimator for ρ and σ^2 that uses three moments of ε, namely

$$\mathrm{E}\left(\frac{1}{n}\varepsilon'\varepsilon\right) = \sigma^2, \quad \mathrm{E}\left(\frac{1}{n}\varepsilon' W' W \varepsilon\right) = \frac{\sigma^2}{n}\mathrm{tr}\left(W'W\right), \quad \mathrm{E}\left(\frac{1}{n}\varepsilon' W' \varepsilon\right) = 0. \tag{4}$$

With the help of equation (2), the sample counterpart of (4) can be written as

$$G\left(\rho, \rho^2, \sigma^2\right)' - g = v\left(\rho, \sigma^2\right),$$

where

$$G = \begin{pmatrix} \frac{2}{n}\hat{u}'W\hat{u} & -\frac{1}{n}\hat{u}'W'W\hat{u} & 1 \\ \frac{2}{n}\hat{u}'W'WW\hat{u} & -\frac{1}{n}\hat{u}'W'W'WW\hat{u} & \frac{1}{n}\mathrm{tr}\left(W'W\right) \\ \frac{1}{n}\hat{u}'\left[W + W'\right]W\hat{u} & -\frac{1}{n}\hat{u}'W'WW\hat{u} & 0 \end{pmatrix}$$

and

$$g = \left(\frac{1}{n}\hat{u}'\hat{u},\ \frac{1}{n}\hat{u}'W'W\hat{u},\ \frac{1}{n}\hat{u}'W\hat{u}\right)'.$$

The nonlinear least squares estimator of Kelejian and Prucha (1999) for ρ and σ^2 is defined as

$$\left(\widehat{\rho}_{KP}, \widehat{\sigma}^2_{KP}\right) = \mathrm{argmin}\left\{v\left(\rho, \sigma^2\right)' v\left(\rho, \sigma^2\right) : \rho \in [-a, a], \sigma^2 \in [0, b]\right\}, \tag{5}$$

where $a \geq 1$ and $b < \infty$.

Our version proceeds as follows: If the unobservable disturbances u have to be replaced by the regression residuals $\hat{u}$, why not calculate the moment conditions (4) also in terms of $\hat{\varepsilon} = M\varepsilon = Mu - \rho MWu$ instead of ε? Therefore, we suggest an estimator that is based on the moments of $M\varepsilon$ corresponding to (4):

$$\mathrm{E}\left(\frac{1}{n}(M\varepsilon)'M\varepsilon\right) = \frac{\sigma^2}{n}\mathrm{tr}(M), \tag{6}$$

$$\mathrm{E}\left(\frac{1}{n}(WM\varepsilon)'WM\varepsilon\right) = \frac{\sigma^2}{n}\mathrm{tr}\left(MW'W\right), \tag{7}$$

$$\mathrm{E}\left(\frac{1}{n}(WM\varepsilon)'M\varepsilon\right) = \frac{\sigma^2}{n}\mathrm{tr}(WM), \tag{8}$$

where we use the fact that M is an orthogonal projector. If we multiply (2) by M and WM, respectively, we get

$$M\varepsilon = Mu - \rho MWu, \tag{9}$$

$$WM\varepsilon = WMu - \rho WMWu. \tag{10}$$

Plugging equations (9) and (10) into the moment conditions (6)-(8) yields

$$\begin{aligned} \frac{1}{n}\mathrm{E}\left(u'Mu\right) - \frac{2\rho}{n}\mathrm{E}\left(u'MWu\right) & \\ + \frac{\rho^2}{n}\mathrm{E}\left(u'W'MWu\right) &= \frac{\sigma^2}{n}\mathrm{tr}(M), \\ \frac{1}{n}\mathrm{E}\left(u'MW'WMu\right) - \frac{2\rho}{n}\mathrm{E}\left(u'W'WMWMu\right) & \\ + \frac{\rho^2}{n}\mathrm{E}\left(u'W'MW'WMWu\right) &= \frac{\sigma^2}{n}\mathrm{tr}\left(MW'W\right), \\ \frac{1}{n}\mathrm{E}\left(u'MW'Mu\right) - \frac{\rho}{n}\mathrm{E}\left(u'M\left[W+W'\right]MWu\right) & \\ + \frac{\rho^2}{n}\mathrm{E}\left(u'W'MWMWu\right) &= \frac{\sigma^2}{n}\mathrm{tr}(WM). \end{aligned}$$

Finally, for every $(n \times n)$-matrix A, the theoretical moments $\mathrm{E}\left(u'Au\right)$ are replaced by their sample counterparts $\hat{u}'A\hat{u}$. Since $Mu = \hat{u}$ and $\mathrm{tr}(M) = \frac{n-k}{n}$, the sample counterpart to (6) - (8) can be written as

$$H\left(\rho, \rho^2, \sigma^2\right)' - h = w\left(\rho, \sigma^2\right),$$

where

$$H = \begin{pmatrix} \frac{2}{n}\hat{u}'W\hat{u} & -\frac{1}{n}\hat{u}'W'MW\hat{u} & \frac{n-k}{n} \\ \frac{2}{n}\hat{u}'W'WMW\hat{u} & -\frac{1}{n}\hat{u}'W'MW'WMW\hat{u} & \frac{1}{n}\mathrm{tr}\left(MW'W\right) \\ \frac{1}{n}\hat{u}'\left[W+W'\right]MW\hat{u} & -\frac{1}{n}\hat{u}'W'MWMW\hat{u} & \frac{1}{n}\mathrm{tr}(WM) \end{pmatrix}$$

and $h = g$. Our nonlinear least squares estimator for ρ and σ^2 is defined as

$$\left(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB}\right) = \mathrm{argmin}\left\{w\left(\rho, \sigma^2\right)' w\left(\rho, \sigma^2\right) : \rho \in [-a, a], \sigma^2 \in [0, b]\right\}, \tag{11}$$

where $a \geq 1$ and $b < \infty$.

The following theorem states the asymptotic equivalence of $(\widehat{\rho}_{KP}, \widehat{\sigma}^2_{KP})$ and $(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB})$.

Theorem. *Under assumptions 1-3, for $n \to \infty$*

$$\left(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB}\right) - \left(\widehat{\rho}_{KP}, \widehat{\sigma}^2_{KP}\right) \xrightarrow{P} 0.$$

Proof. Because of assumption 3, for large n the elements of $X\left(X'X\right)^{-1}X'$ are bounded by the corresponding elements of $\frac{kc_X^2}{n}Q_X^{-1} \overset{n\to\infty}{\longrightarrow} 0$ so that $M = I - X\left(X'X\right)^{-1}X' \overset{n\to\infty}{\longrightarrow} I$ and thus $H \xrightarrow{P} G$ as $n \to \infty$. Since $g = h$, $w(\rho, \sigma^2) \xrightarrow{P} v(\rho, \sigma^2)$, so that the minimization problems (11) and (5) coincide for $n \to \infty$ because $w\left(\rho, \sigma^2\right)$ and $v\left(\rho, \sigma^2\right)$ are continuous functions of ρ and σ^2. □

As a consequence of our theorem, $(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB})$ shares the asymptotic properties of $(\widehat{\rho}_{KP}, \widehat{\sigma}^2_{KP})$ given in theorems 1 and 2 of Kelejian and Prucha (1999): $(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB})$ is a consistent estimator for (ρ, σ^2), the feasible GLS estimator $\hat{\beta}^{FG}$ is a consistent estimator for β and the asymptotic covariance matrix of $\hat{\beta}^{FG}$ can be estimated consistently, too.

However, the finite properties of both estimators are different since $(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB})$ accounts for the difference between disturbances u and residuals $\hat{u}$. To illustrate this effect, we solve (5) with G and g replaced by their expectations, respectively. Let $A :=\mathrm{E}\big(G'\big)\mathrm{E}(G)$ and $a := \mathrm{E}\big(G'\big)\mathrm{E}(g)$, then

$$\begin{aligned}
&\Big([\mathrm{E}(G)]\left(\rho, \rho^2, \sigma^2\right)' - \mathrm{E}(g)\Big)' \Big([\mathrm{E}(G)]\left(\rho, \rho^2, \sigma^2\right)' - \mathrm{E}(g)\Big)\\
&\quad = a_{22}\rho^4 + (a_{12} + a_{21})\rho^3 + (a_{11} - 2a_2)\rho^2 + (a_{23} + a_{32})\rho^2\sigma^2\\
&\qquad +(a_{13} + a_{31})\rho\sigma^2 - 2a_1\rho + a_{33}\sigma^4 - 2a_3\sigma^2, \qquad (12)
\end{aligned}$$

where a_{ij} and a_i, $i, j = 1, 2, 3$, are the elements of the matrix A and the vector a, respectively. In general, the formulas for the minimizing values of ρ and σ^2 in (12) are complicated, but for $\rho = 0$, (12) reduces to $a_{33}\sigma^4 - 2a_3\sigma^2$. In this case, the minimizing value for σ^2 is given by

$$\tilde{\sigma}^2 = \frac{a_3}{a_{33}} = \sigma^2 \cdot \frac{n(n-k) + \mathrm{tr}\left(W'W\right)\mathrm{tr}\left(W'WM\right)}{n^2 + \mathrm{tr}\left(W'W\right)^2} < \sigma^2.$$

Carrying out the corresponding calculations for the new estimator with $B :=\mathrm{E}\big(H'\big)\mathrm{E}(H)$ and $b := \mathrm{E}\big(H'\big)\mathrm{E}(h)$, we find that the objective function in (11) with H and h replaced by their expectations is minimized by

$$\tilde{\sigma}^2 = \frac{b_3}{b_{33}} = \frac{\frac{\sigma^2}{n^2}\left([\mathrm{tr}(M)]^2 + \mathrm{tr}\left(MW'W\right)\mathrm{tr}\left(MW'WM\right) + \mathrm{tr}(WM)\mathrm{tr}\left(MW'M\right)\right)}{\frac{1}{n^2}\left([\mathrm{tr}(M)]^2 + [\mathrm{tr}\left(MW'W\right)]^2 + [\mathrm{tr}(WM)]^2\right)} = \sigma^2,$$

where we made use of the fact that the trace of a matrix product is invariant against cyclic permutations of the matrices.

The next section compares the finite sample performance of $(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB})$ and $(\widehat{\rho}_{KP}, \widehat{\sigma}^2_{KP})$ by way of Monte Carlo simulation.

3 Some finite sample Monte Carlo evidence

We compare the finite sample properties of $(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB})$ and $(\widehat{\rho}_{KP}, \widehat{\sigma}^2_{KP})$ for $n = 20$, 100, 400, $\rho = -0.5, 0, 0.5$ and $\sigma^2 = 1$. The matrix W is specified such that each element of u_i is directly related to the three elements immediately after and immediately before it. For the first three and the last three elements of u, we imply a circular setting such that for example u_1 is directly related to $u_2, u_3, u_4, u_{n-2}, u_{n-1}$ and u_n. The row sums of W

are standardized to one. Thus, in each row of W, six elements are equal to $\frac{1}{6}$ and the other elements are equal to zero. With respect to the regression model (1), we consider

$$X = \begin{pmatrix} 1 & \dots & \dots & \dots & \dots & \dots & \dots & 1 \\ 1 & \dots & \dots & 1 & 0 & \dots & \dots & 0 \\ 1 & 0 & \dots & \dots & \dots & \dots & 1 & 0 \end{pmatrix}',$$

the model matrix of a regression on an intercept and two binary regressors. For each combination of n and ρ, we generated $10,000$ realizations of the disturbance vector u corresponding to the spatial autoregressive model (2), where the components of ε are i.i.d. $N(0,1)$.

- Table 1 here -

The left part of Table 1 shows the simulated bias and MSE of both estimators for ρ. For all sample sizes, the bias of $\widehat{\rho}_{RB}$ is $65-80\%$ smaller than the bias of $\widehat{\rho}_{KP}$. As a consequence, the MSE can also be reduced by around 45% for $n=20$, 20% for $n=100$ and 5% for $n=400$ if we use $\widehat{\rho}_{RB}$ instead of $\widehat{\rho}_{KP}$ to estimate ρ.

With respect to the estimators for σ^2, the middle part of Table 1 shows the same positive effect for the bias and the MSE. The bias reductions are in line with analytical findings because the „expected“ objective function (12) is minimized by the true parameters.

The last three columns show empirical null rejection probabilities for significance tests on the regression coefficients. To perform such tests, the disturbance covariance matrix is estimated by plugging in an estimator for ρ and σ^2 in (3). Consequently, a less biased estimation of ρ and σ^2 results in a more accurate estimation of (3). The size distortion of these tests can be reduced by 50% if we use $(\widehat{\rho}_{RB}, \widehat{\sigma}^2_{RB})$ instead of $(\widehat{\rho}_{KP}, \widehat{\sigma}^2_{KP})$. We conclude that finite sample properties can be improved if we account for the differences between residuals and disturbances: The bias of the estimators for ρ and σ^2 can be reduced by approximately 75% and half of the size distortion of significance tests can be avoided, and these effects stay stable even for larger sample sizes.

Kelejian and Prucha (1999) use this kind of spatial weighting matrix W labeled „3 ahead and 3 behind“ among others for their simulations. We also ran simulations for other matrices W called „1 ahead and 1 behind“ or „5 ahead and 5 behind“. The results of these simulations agree with the ones presented in table 1 with respect to our modification.

4 Application to Indonesian rice farming

We illustrate our results with an empirical analysis of Indonesian rice farming data. It comprises data of 171 rice farms over six growing seasons. The farms are located in six different villages. We use a standard fixed effects model to regress the output (ln(rice)) on the covariates seed, urea, phosphate (TSP), labor and land as well as dummies for pesticides (DP), high yield varieties (DV1), mixed varieties (DV2) and wet growing seasons (DSS). For a detailed description of the data see Erwidodo (1990). The disturbances are assumed to be spatially correlated across cross-sectional units where the typical element w_{ij} of the spatial weighting matrix W is positive if observations i and j belong to (a) farms located in the same village and (b) the same growing season. The row sums of W are standardized to one.

First, we only use data of the three wet growing seasons so that $n = 513$. We estimate ρ and σ^2 in two ways, once following Kelejian and Prucha (1999) and once by our new residual based approach. For each case, the regression results of the corresponding feasible GLS procedure are given. Secondly, we repeat this analysis by using the total of $n = 1026$ data points.

- Table 2 here -

Table 2 shows that for both models, the residual based estimates of ρ and σ^2 are larger than in the standard approach. This corresponds to the simulation findings that downward bias is reduced. In consequence, the resulting t-statistics are smaller when we account for the differences between residuals and disturbances. Although the sample sizes are even larger than in our simulations, the results implied are essentially different: For the wet seasons, the use of phosphate (TSP) is no longer statistically significant on a 5%-level if we use residual based estimates. The same holds true for the whole data set on a 1%-level with respect to the effects of high yielding varieties (DV1) and TSP. Again, this corresponds to our simulation findings that overrejection probabilities can be reduced by half if one accounts for the differences between residuals and disturbances.

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Table 1: Bias, MSE and empirical null rejection probabilities, $\alpha = 0.05$ nominal level

n	ρ		Bias	MSE		Bias	MSE	t_{β_1}	t_{β_2}	t_{β_3}
20	-0.5	$\widehat{\rho}_{KP}$	-0.607	1.039	$\widehat{\sigma}^2_{KP}$	-0.276	0.159	0.048	0.089	0.042
20	-0.5	$\widehat{\rho}_{RB}$	-0.151	0.547	$\widehat{\sigma}^2_{RB}$	-0.094	0.137	0.034	0.058	0.051
20	0	$\widehat{\rho}_{KP}$	-0.683	1.107	$\widehat{\sigma}^2_{KP}$	-0.273	0.154	0.084	0.144	0.042
20	0	$\widehat{\rho}_{RB}$	-0.181	0.588	$\widehat{\sigma}^2_{RB}$	-0.100	0.129	0.053	0.084	0.054
20	0.5	$\widehat{\rho}_{KP}$	-0.650	0.917	$\widehat{\sigma}^2_{KP}$	-0.224	0.138	0.196	0.232	0.041
20	0.5	$\widehat{\rho}_{RB}$	-0.147	0.507	$\widehat{\sigma}^2_{RB}$	-0.069	0.129	0.111	0.118	0.053
100	-0.5	$\widehat{\rho}_{KP}$	-0.101	0.063	$\widehat{\sigma}^2_{KP}$	-0.060	0.024	0.054	0.074	0.050
100	-0.5	$\widehat{\rho}_{RB}$	-0.030	0.052	$\widehat{\sigma}^2_{RB}$	-0.019	0.022	0.049	0.060	0.052
100	0	$\widehat{\rho}_{KP}$	-0.095	0.049	$\widehat{\sigma}^2_{KP}$	-0.049	0.021	0.065	0.076	0.050
100	0	$\widehat{\rho}_{RB}$	-0.030	0.038	$\widehat{\sigma}^2_{RB}$	-0.017	0.020	0.054	0.061	0.051
100	0.5	$\widehat{\rho}_{KP}$	-0.073	0.025	$\widehat{\sigma}^2_{KP}$	-0.032	0.021	0.086	0.099	0.052
100	0.5	$\widehat{\rho}_{RB}$	-0.026	0.019	$\widehat{\sigma}^2_{RB}$	-0.009	0.021	0.065	0.075	0.053
400	-0.5	$\widehat{\rho}_{KP}$	-0.023	0.012	$\widehat{\sigma}^2_{KP}$	-0.016	0.005	0.051	0.055	0.054
400	-0.5	$\widehat{\rho}_{RB}$	-0.006	0.012	$\widehat{\sigma}^2_{RB}$	-0.005	0.005	0.050	0.052	0.054
400	0	$\widehat{\rho}_{KP}$	-0.022	0.009	$\widehat{\sigma}^2_{KP}$	-0.012	0.005	0.057	0.055	0.048
400	0	$\widehat{\rho}_{RB}$	-0.007	0.008	$\widehat{\sigma}^2_{RB}$	-0.005	0.005	0.054	0.050	0.048
400	0.5	$\widehat{\rho}_{KP}$	-0.014	0.004	$\widehat{\sigma}^2_{KP}$	-0.009	0.005	0.057	0.056	0.048
400	0.5	$\widehat{\rho}_{RB}$	-0.004	0.004	$\widehat{\sigma}^2_{RB}$	-0.003	0.005	0.052	0.052	0.048

Table 2: Regression results with standard deviations in paranthesis and t-statistics below the estimates

	wet seasons ($n = 513$)		all seasons ($n = 1026$)	
	KP	RB	KP	RB
$\hat{\rho}$	0.7388	0.8743	0.7161	0.8358
$\hat{\sigma}^2$	0.0556	0.0634	0.0650	0.0757
Seed	0.1383 (0.041)	0.1366 (0.044)	0.1024 (0.023)	0.1006 (0.025)
	3.360	3.096	4.405	4.022
Urea	0.0908 (0.028)	0.0916 (0.030)	0.0907 (0.017)	0.0904 (0.018)
	3.272	3.081	5.476	5.066
TSP	0.0363 (0.017)	0.0315 (0.018)	0.0323 (0.011)	0.0268 (0.012)
	2.153	1.738	2.898	2.215
Labor	0.2248 (0.039)	0.2225 (0.042)	0.2383 (0.027)	0.2379 (0.029)
	5.714	5.262	8.907	8.232
Land	0.4664 (0.044)	0.4712 (0.047)	0.4877 (0.028)	0.4913 (0.031)
	10.685	10.066	17.141	16.016
DP	0.0241 (0.037)	0.0165 (0.040)	-0.0222 (0.026)	-0.0276 (0.028)
	0.643	0.409	-0.844	-0.971
DV1	0.1042 (0.061)	0.0965 (0.066)	0.1064 (0.036)	0.0975 (0.039)
	1.697	1.454	2.982	2.523
DV2	0.0530 (0.064)	0.0447 (0.069)	0.1042 (0.045)	0.0980 (0.049)
	0.859	0.651	2.298	2.006
DSS	-	-	0.0805 (0.056)	0.0831 (0.105)
	-	-	1.428	0.793