Supplemental Appendix for

Model and Moment Selection in Factor Copula Models

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S.A.1 Additional Tables

Tables S.A.1 to S.A.4 provide simulation results concerning the selection frequencies using the SMM-BIC criterion for $T = 500, 2000$, which can be compared to the cases for $T = 1000$ in the main text. In addition to the main simulation results based on the SMM-BIC criterion in the paper, Tables S.A.5 to S.A.8 contain additional simulation results on the selection frequencies among all model/moment combinations according to the SMM-AIC and the SMM-HQIC criteria. Four factor copula models, *n-n*, *skewn-n*, *t-t* and *skewt-t*, and five sets of moments $m_i, i = 1, \ldots, 5$ constitute the candidate combinations used in the SMM estimation. The sets $m_i, i = 1, \ldots, 5$ are defined in Section 5. The marginal distributions of the simulated data follow AR(1)-GARCH(1,1) processes, see equation (14) in main text. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. Tables S.A.5 and S.A.7 provide the results for $N = 5$, whereas Tables S.A.6 and S.A.8 present the results for $N = 10$. Tables S.A.9 to S.A.12 contain the selection frequencies of models given predetermined sets of moments according to AIC and HQIC criteria.

The upper panels in Tables S.A.13 and S.A.14 list the following metrics: the average coverage rate of the VaR violations ($\overline{\text{Cov}}$) over $R = 100$ Monte Carlo replications, the average percentage bias ($\overline{\%\text{Bias}}$) and the average percentage RMSE ($\overline{\%\text{RMSE}}$) for the 5%-VaR and 1%-VaR predictions over the $T_s = 500$ forecasting horizon, and the average loss ($\overline{\text{Loss}}$) of the VaR forecasts over T_s and R Monte Carlo replications. The results of the Diebold-Mariano (DM) test for a pairwise comparisons between the models in terms of the VaR forecasting accuracy can be found in the lower panels in Tables S.A.13 and S.A.14. The left (right) number in each cell in the lower panel reports the number of times that the row model significantly outperforms (underperforms) the column model at the 5% significance level.

Table S.A.1: Simulation results: empirical selection frequencies among all model/moment combinations based on the BIC for $T = 500$ and $N = 5, 10$ dimensions

	$\hat{C}$ \ C_0	Panel A: $N = 5$ n-n	t-t	skewn-n	skewt-t	Panel B: $N = 10$ n-n	t-t	skewn-n	skewt-t
m_1	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_2	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_3	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_4	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_5	n-n	**0.970**	**0.690**	0.280	0.005	**0.995**	**0.755**	0.220	0.005
	t-t	0.000	0.215	0.000	0.000	0.000	0.160	0.000	0.000
	skewn-n	0.030	0.060	**0.715**	**0.525**	0.005	0.055	**0.780**	**0.525**
	skewt-t	0.000	0.035	0.005	0.470	0.000	0.030	0.000	0.470

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample length is $T = 500$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model/moment combination selected by the SMM-BIC selection procedure, C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.2: Simulation results: empirical selection frequencies among all model/moment combinations based on the BIC for $T = 2000$ and $N = 5, 10$ dimensions

	C_0 \ $\hat{C}$	Panel A: $N = 5$				Panel B: $N = 10$			
		n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_2	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_3	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_4	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_5	n-n	**0.995**	0.145	0.000	0.000	**1.000**	0.115	0.000	0.000
	t-t	0.000	**0.805**	0.000	0.000	0.000	**0.850**	0.000	0.000
	skewn-n	0.005	0.010	**1.000**	0.030	0.000	0.000	**1.000**	0.000
	skewt-t	0.000	0.040	0.000	**0.970**	0.000	0.035	0.000	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample length is $T = 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model/moment combination selected by the SMM-BIC selection procedure, C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.3: Simulation results: empirical selection frequencies of models given predetermined sets of moments based on the BIC for $T = 500$ and $N = 5, 10$ dimensions

		Panel A: $N = 5$				Panel B: $N = 10$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	**0.975**	**0.625**	**0.920**	0.410	**0.980**	**0.660**	**0.990**	0.425
	t-t	0.025	0.370	0.075	**0.570**	0.020	0.340	0.010	**0.575**
	skewn-n	0.000	0.005	0.005	0.005	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.015	0.000	0.000	0.000	0.000
m_2	n-n	**0.965**	**0.500**	**0.845**	0.065	**0.975**	**0.515**	**0.915**	0.045
	t-t	0.035	0.490	0.115	**0.630**	0.025	0.475	0.070	**0.685**
	skewn-n	0.000	0.010	0.040	0.120	0.000	0.010	0.015	0.130
	skewt-t	0.000	0.000	0.000	0.185	0.000	0.000	0.000	0.140
m_3	n-n	**0.975**	**0.730**	0.360	0.005	**1.000**	**0.820**	0.320	0.005
	t-t	0.000	0.175	0.000	0.000	0.000	0.095	0.000	0.000
	skewn-n	0.025	0.070	**0.640**	**0.630**	0.000	0.060	**0.680**	**0.610**
	skewt-t	0.000	0.025	0.000	0.365	0.000	0.025	0.000	0.385
m_4	n-n	**1.000**	**1.000**	**1.000**	**0.985**	**1.000**	**1.000**	**1.000**	**1.000**
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.015	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_5	n-n	**0.970**	**0.690**	0.280	0.005	**0.995**	**0.755**	0.220	0.005
	t-t	0.000	0.215	0.000	0.000	0.000	0.160	0.000	0.000
	skewn-n	0.030	0.060	**0.715**	**0.525**	0.005	0.055	**0.780**	**0.525**
	skewt-t	0.000	0.035	0.005	0.470	0.000	0.030	0.000	0.470

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample length is $T = 500$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model selected by the SMM-BIC procedure given $m_i, i = 1, \ldots, 5$, respectively. C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.4: Simulation results: empirical selection frequencies of models given predetermined sets of moments based on the BIC for $T = 2000$ and $N = 5, 10$ dimensions

		Panel A: $N = 5$				Panel B: $N = 10$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	**0.995**	0.270	**0.960**	0.035	**1.000**	0.270	**0.985**	0.020
	t-t	0.000	**0.730**	0.035	**0.940**	0.000	**0.730**	0.015	**0.950**
	skewn-n	0.005	0.000	0.005	0.010	0.000	0.000	0.000	0.010
	skewt-t	0.000	0.000	0.000	0.015	0.000	0.000	0.000	0.020
m_2	n-n	**0.990**	0.100	0.400	0.000	**0.995**	0.080	0.440	0.000
	t-t	0.000	**0.810**	0.115	0.460	0.005	**0.845**	0.075	0.425
	skewn-n	0.010	0.090	**0.485**	0.000	0.000	0.075	**0.485**	0.000
	skewt-t	0.000	0.000	0.000	**0.540**	0.000	0.000	0.000	**0.575**
m_3	n-n	**0.995**	0.205	0.000	0.000	**1.000**	0.165	0.000	0.000
	t-t	0.000	**0.750**	0.000	0.000	0.000	**0.795**	0.000	0.000
	skewn-n	0.005	0.010	**1.000**	0.025	0.000	0.000	**1.000**	0.005
	skewt-t	0.000	0.035	0.000	**0.975**	0.000	0.040	0.000	**0.995**
m_4	n-n	**1.000**	**1.000**	**0.995**	**0.695**	**1.000**	**1.000**	**0.995**	**0.745**
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.005	0.180	0.000	0.000	0.005	0.175
	skewt-t	0.000	0.000	0.000	0.125	0.000	0.000	0.000	0.080
m_5	n-n	**0.995**	0.145	0.000	0.000	**1.000**	0.115	0.000	0.000
	t-t	0.000	**0.805**	0.000	0.000	0.000	**0.850**	0.000	0.000
	skewn-n	0.005	0.010	**1.000**	0.030	0.000	0.000	**1.000**	0.000
	skewt-t	0.000	0.040	0.000	**0.970**	0.000	0.035	0.000	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample length is $T = 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model selected by the SMM-BIC procedure given $m_i, i = 1, \ldots, 5$, respectively. C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.5: Simulation results: empirical selection frequencies among all model/moment combinations based on the AIC for $N = 5$ dimensions

	$\hat{C}$ \ C_0	$T = 500$				$T = 1000$				$T = 2000$			
		n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_2	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_3	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_4	n-n	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.015
m_5	n-n	**0.745**	0.265	0.035	0.005	**0.760**	0.055	0.005	0.000	**0.790**	0.000	0.000	0.000
	t-t	0.020	**0.430**	0.000	0.000	0.025	**0.600**	0.000	0.000	0.015	**0.675**	0.000	0.000
	skewn-n	0.230	0.100	**0.920**	0.245	0.205	0.040	**0.970**	0.060	0.195	0.000	**0.960**	0.000
	skewt-t	0.005	0.200	0.045	**0.750**	0.010	0.305	0.025	**0.940**	0.000	0.325	0.040	**0.985**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model/moment combination selected by the SMM-AIC selection procedure, C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.6: Simulation results: empirical selection frequencies among all model/moment combinations based on the AIC for $N = 10$ dimensions

		$T=500$				$T=1000$				$T=2000$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_2	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_3	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_4	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_5	n-n	**0.875**	0.235	0.020	0.000	**0.895**	0.035	0.000	0.000	**0.910**	0.005	0.000	0.000
	t-t	0.015	**0.515**	0.000	0.000	0.010	**0.690**	0.000	0.000	0.005	**0.680**	0.000	0.000
	skewn-n	0.105	0.085	**0.980**	0.195	0.095	0.015	**0.990**	0.025	0.085	0.000	**0.995**	0.000
	skewt-t	0.005	0.165	0.000	**0.805**	0.000	0.260	0.010	**0.975**	0.000	0.315	0.005	**0.995**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model/moment combination selected by the SMM-AIC selection procedure, C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.7: Simulation results: empirical selection frequencies among all model/moment combinations based on the HQIC for $N = 5$ dimensions

		$T = 500$				$T = 1000$				$T = 2000$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_2	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_3	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_4	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_5	n-n	**0.890**	**0.480**	0.100	0.005	**0.910**	0.190	0.015	0.000	**0.935**	0.035	0.000	0.000
	t-t	0.005	0.355	0.000	0.000	0.005	**0.625**	0.000	0.000	0.005	**0.815**	0.000	0.000
	skewn-n	0.105	0.060	**0.885**	0.350	0.080	0.035	**0.975**	0.115	0.060	0.005	**1.000**	0.000
	skewt-t	0.000	0.105	0.015	**0.645**	0.005	0.150	0.010	**0.885**	0.000	0.145	0.000	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model/moment combination selected by the SMM-HQIC selection procedure, C_0 (columnwise) denotes the DGP. The constant $Q = 2.01$ is specified in SMM-HQIC criterion. The highest selection frequencies are marked in bold.

Table S.A.8: Simulation results: empirical selection frequencies among all model/moment combinations based on the HQIC for $N = 10$ dimensions

		$T = 500$				$T = 1000$				$T = 2000$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_2	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_3	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_4	n-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
m_5	n-n	**0.960**	**0.445**	0.045	0.005	**0.980**	0.165	0.000	0.000	**0.980**	0.025	0.000	0.000
	t-t	0.000	0.385	0.000	0.000	0.000	**0.670**	0.000	0.000	0.000	**0.850**	0.000	0.000
	skewn-n	0.040	0.095	**0.955**	0.295	0.020	0.020	**1.000**	0.075	0.020	0.000	**1.000**	0.000
	skewt-t	0.000	0.075	0.000	**0.700**	0.000	0.145	0.000	**0.925**	0.000	0.125	0.000	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model/moment combination selected by the SMM-HQIC selection procedure, C_0 (columnwise) denotes the DGP. The constant $Q = 2.01$ is specified in SMM-HQIC criterion. The highest selection frequencies are marked in bold.

Table S.A.9: Simulation results: empirical selection frequencies of models given predetermined sets of moments based on the AIC for $N = 5$ dimensions

		$T = 500$				$T = 1000$				$T = 2000$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	**0.830**	0.350	**0.685**	0.220	**0.755**	0.260	**0.660**	0.075	**0.770**	0.135	**0.665**	0.005
	t-t	0.105	**0.530**	0.135	**0.670**	0.145	**0.655**	0.150	**0.765**	0.085	**0.830**	0.175	**0.880**
	skewn-n	0.055	0.070	0.180	0.050	0.075	0.050	0.185	0.040	0.115	0.010	0.155	0.015
	skewt-t	0.010	0.050	0.000	0.060	0.025	0.035	0.005	0.120	0.030	0.025	0.005	0.100
m_2	n-n	**0.760**	0.240	**0.510**	0.015	**0.685**	0.130	0.330	0.000	**0.740**	0.015	0.060	0.000
	t-t	0.120	**0.615**	0.240	0.385	0.135	**0.695**	0.190	0.300	0.095	**0.805**	0.120	0.140
	skewn-n	0.095	0.125	0.250	0.150	0.165	0.155	**0.475**	0.050	0.160	0.125	**0.785**	0.000
	skewt-t	0.025	0.020	0.000	**0.450**	0.015	0.020	0.005	**0.650**	0.005	0.055	0.035	**0.860**
m_3	n-n	**0.770**	0.285	0.055	0.005	**0.780**	0.070	0.010	0.000	**0.820**	0.000	0.000	0.000
	t-t	0.005	**0.435**	0.000	0.000	0.025	**0.570**	0.000	0.000	0.005	**0.660**	0.000	0.000
	skewn-n	0.225	0.095	**0.920**	0.235	0.190	0.035	**0.980**	0.065	0.175	0.000	**0.980**	0.000
	skewt-t	0.000	0.185	0.025	**0.760**	0.005	0.325	0.010	**0.935**	0.000	0.340	0.020	**1.000**
m_4	n-n	**1.000**	**1.000**	**0.900**	**0.640**	**1.000**	**1.000**	**0.645**	0.085	**1.000**	**1.000**	0.185	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.100	0.155	0.000	0.000	0.340	0.180	0.000	0.000	**0.735**	0.180
	skewt-t	0.000	0.000	0.000	0.205	0.000	0.000	0.015	**0.735**	0.000	0.000	0.080	**0.820**
m_5	n-n	**0.745**	0.265	0.035	0.005	**0.760**	0.055	0.005	0.000	**0.790**	0.000	0.000	0.000
	t-t	0.020	**0.430**	0.000	0.000	0.025	**0.600**	0.000	0.000	0.015	**0.675**	0.000	0.000
	skewn-n	0.230	0.100	**0.920**	0.245	0.205	0.040	**0.970**	0.060	0.195	0.000	**0.960**	0.000
	skewt-t	0.005	0.205	0.045	**0.750**	0.010	0.305	0.025	**0.940**	0.000	0.325	0.040	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model selected by the SMM-AIC procedure given $m_i, i = 1, \ldots, 5$, respectively. C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.10: Simulation results: empirical selection frequencies of models given predetermined sets of moments based on the AIC for $N = 10$ dimensions

	$\hat{C}$ \ C_0	$T = 500$				$T = 1000$				$T = 2000$			
		n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	**0.945**	0.415	**0.810**	0.230	**0.885**	0.275	**0.830**	0.075	**0.920**	0.090	**0.825**	0.005
	t-t	0.040	**0.525**	0.100	**0.680**	0.080	**0.675**	0.095	**0.835**	0.045	**0.880**	0.120	**0.865**
	skewn-n	0.015	0.035	0.090	0.025	0.035	0.015	0.075	0.015	0.035	0.015	0.055	0.010
	skewt-t	0.000	0.025	0.000	0.065	0.000	0.035	0.000	0.075	0.000	0.015	0.000	0.120
m_2	n-n	**0.880**	0.245	**0.560**	0.005	**0.850**	0.095	0.290	0.000	**0.875**	0.010	0.055	0.000
	t-t	0.060	**0.605**	0.165	0.335	0.085	**0.740**	0.145	0.270	0.045	**0.800**	0.055	0.170
	skewn-n	0.045	0.130	0.270	0.130	0.060	0.140	**0.560**	0.050	0.080	0.115	**0.860**	0.000
	skewt-t	0.015	0.020	0.005	**0.530**	0.005	0.025	0.005	**0.680**	0.000	0.075	0.030	**0.830**
m_3	n-n	**0.905**	0.255	0.025	0.000	**0.910**	0.045	0.000	0.000	**0.925**	0.000	0.000	0.000
	t-t	0.000	**0.495**	0.000	0.000	0.005	**0.695**	0.000	0.000	0.000	**0.695**	0.000	0.000
	skewn-n	0.090	0.100	**0.970**	0.185	0.085	0.015	**1.000**	0.015	0.075	0.000	**1.000**	0.000
	skewt-t	0.005	0.150	0.005	**0.815**	0.000	0.245	0.000	**0.985**	0.000	0.305	0.000	**1.000**
m_4	n-n	**1.000**	**1.000**	**0.945**	**0.615**	**1.000**	**1.000**	**0.635**	0.055	**1.000**	**1.000**	0.150	0.000
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.055	0.240	0.000	0.000	0.365	0.235	0.000	0.000	**0.830**	0.175
	skewt-t	0.000	0.000	0.000	0.145	0.000	0.000	0.000	**0.710**	0.000	0.000	0.020	**0.825**
m_5	n-n	**0.875**	0.235	0.020	0.000	**0.895**	0.035	0.000	0.000	**0.910**	0.005	0.000	0.000
	t-t	0.015	**0.515**	0.000	0.000	0.010	**0.690**	0.000	0.000	0.005	**0.680**	0.000	0.000
	skewn-n	0.105	0.085	**0.980**	0.195	0.095	0.015	**0.990**	0.025	0.085	0.000	**0.995**	0.000
	skewt-t	0.005	0.165	0.000	**0.805**	0.000	0.260	0.010	**0.975**	0.000	0.315	0.005	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model selected by the SMM-AIC procedure given $m_i, i = 1, \ldots, 5$, respectively. C_0 (columnwise) denotes the DGP. The highest selection frequencies are marked in bold.

Table S.A.11: Simulation results: empirical selection frequencies of models given predetermined sets of moments based on the HQIC for $N = 5$ dimensions

		$T = 500$				$T = 1000$				$T = 2000$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	**0.945**	**0.485**	**0.855**	0.340	**0.895**	0.360	**0.830**	0.160	**0.935**	0.195	**0.870**	0.010
	t-t	0.045	0.460	0.100	**0.615**	0.085	**0.605**	0.100	**0.770**	0.025	**0.790**	0.075	**0.930**
	skewn-n	0.010	0.040	0.045	0.015	0.020	0.020	0.070	0.010	0.040	0.000	0.055	0.015
	skewt-t	0.000	0.015	0.000	0.030	0.000	0.015	0.000	0.060	0.000	0.015	0.000	0.045
m_2	n-n	**0.895**	0.325	**0.665**	0.020	**0.865**	0.170	**0.485**	0.000	**0.905**	0.025	0.170	0.000
	t-t	0.070	**0.595**	0.190	**0.480**	0.070	**0.695**	0.170	0.460	0.040	**0.825**	0.130	0.300
	skewn-n	0.025	0.075	0.145	0.145	0.065	0.130	0.340	0.060	0.055	0.145	**0.695**	0.000
	skewt-t	0.010	0.005	0.000	0.355	0.000	0.005	0.005	**0.480**	0.000	0.005	0.005	**0.700**
m_3	n-n	**0.895**	**0.530**	0.145	0.005	**0.920**	0.235	0.015	0.000	**0.945**	0.035	0.000	0.000
	t-t	0.000	0.290	0.000	0.000	0.005	**0.580**	0.000	0.000	0.005	**0.805**	0.000	0.000
	skewn-n	0.105	0.085	**0.845**	0.375	0.075	0.030	**0.985**	0.110	0.050	0.005	**1.000**	0.000
	skewt-t	0.000	0.095	0.010	**0.620**	0.000	0.155	0.000	**0.890**	0.000	0.155	0.000	**1.000**
m_4	n-n	**1.000**	**1.000**	**0.990**	**0.910**	**1.000**	**1.000**	**0.925**	**0.610**	**1.000**	**1.000**	**0.690**	0.035
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.010	0.085	0.000	0.000	0.075	0.165	0.000	0.000	0.310	0.180
	skewt-t	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.225	0.000	0.000	0.000	**0.785**
m_5	n-n	**0.890**	**0.480**	0.100	0.005	**0.910**	0.190	0.015	0.000	**0.935**	0.035	0.000	0.000
	t-t	0.005	0.355	0.000	0.000	0.005	**0.625**	0.000	0.000	0.005	**0.815**	0.000	0.000
	skewn-n	0.105	0.060	**0.885**	0.350	0.080	0.035	**0.975**	0.115	0.060	0.005	**1.000**	0.000
	skewt-t	0.000	0.105	0.015	**0.645**	0.005	0.150	0.010	**0.885**	0.000	0.145	0.000	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model selected by the SMM-HQIC procedure given $m_i, i = 1, \ldots, 5$, respectively. C_0 (columnwise) denotes the DGP. The constant $Q = 2.01$ is specified in SMM-HQIC criterion. The highest selection frequencies are marked in bold.

Table S.A.12: Simulation results: empirical selection frequencies of models given predetermined sets of moments based on the HQIC for $N = 10$ dimensions

		$T = 500$				$T = 1000$				$T = 2000$			
	$\hat{C}$ \ C_0	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t	n-n	t-t	skewn-n	skewt-t
m_1	n-n	**0.970**	**0.545**	**0.955**	0.320	**0.950**	0.380	**0.960**	0.135	**0.980**	0.190	**0.955**	0.015
	t-t	0.030	0.445	0.030	**0.645**	0.050	**0.605**	0.035	**0.825**	0.020	**0.810**	0.045	**0.925**
	skewn-n	0.000	0.000	0.015	0.020	0.000	0.005	0.005	0.000	0.000	0.000	0.000	0.010
	skewt-t	0.000	0.010	0.000	0.015	0.000	0.010	0.000	0.040	0.000	0.000	0.000	0.050
m_2	n-n	**0.975**	0.350	**0.795**	0.015	**0.945**	0.155	**0.520**	0.000	**0.955**	0.035	0.155	0.000
	t-t	0.025	**0.570**	0.120	**0.515**	0.055	**0.740**	0.120	0.415	0.030	**0.850**	0.075	0.310
	skewn-n	0.000	0.075	0.085	0.135	0.000	0.105	0.360	0.055	0.015	0.105	**0.765**	0.000
	skewt-t	0.000	0.005	0.000	0.335	0.000	0.000	0.000	**0.530**	0.000	0.010	0.005	**0.690**
m_3	n-n	**0.970**	**0.535**	0.085	0.005	**0.980**	0.230	0.000	0.000	**0.980**	0.025	0.000	0.000
	t-t	0.000	0.300	0.000	0.000	0.000	**0.605**	0.000	0.000	0.000	**0.855**	0.000	0.000
	skewn-n	0.030	0.100	**0.910**	0.380	0.020	0.030	**1.000**	0.085	0.020	0.000	**1.000**	0.000
	skewt-t	0.000	0.065	0.005	**0.615**	0.000	0.135	0.000	**0.915**	0.000	0.120	0.000	**1.000**
m_4	n-n	**1.000**	**1.000**	**1.000**	**0.870**	**1.000**	**1.000**	**0.975**	**0.640**	**1.000**	**1.000**	**0.710**	0.005
	t-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	skewn-n	0.000	0.000	0.000	0.130	0.000	0.000	0.025	0.225	0.000	0.000	0.290	0.175
	skewt-t	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.135	0.000	0.000	0.000	**0.820**
m_5	n-n	**0.960**	**0.445**	0.045	0.005	**0.980**	0.165	0.000	0.000	**0.980**	0.025	0.000	0.000
	t-t	0.000	0.385	0.000	0.000	0.000	**0.670**	0.000	0.000	0.000	**0.850**	0.000	0.000
	skewn-n	0.040	0.095	**0.955**	0.295	0.020	0.020	**1.000**	0.075	0.020	0.000	**1.000**	0.000
	skewt-t	0.000	0.075	0.000	**0.700**	0.000	0.145	0.000	**0.925**	0.000	0.125	0.000	**1.000**

Note: The vector of true model parameters is $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The element-wise product of $\boldsymbol{\theta}$ and the selection vector $\boldsymbol{b}$ gives one specific candidate model. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are candidate models used in SMM estimation. The candidate sets of moments are $m_i, i = 1, \ldots, 5$. The marginal distributions of simulated data follow AR(1)-GARCH(1,1) processes. The sample lengths are $T = 500, 1000, 2000$. The number of Monte Carlo replications is $R = 200$. $\hat{C}$ (rowwise) denotes the model selected by the SMM-HQIC procedure given $m_i, i = 1, \ldots, 5$, respectively. C_0 (columnwise) denotes the DGP. The constant $Q = 2.01$ is specified in SMM-HQIC criterion. The highest selection frequencies are marked in bold.

Table S.A.13: Simulation results: evaluation of 5%-VaR forecasts and backtesting for $N = 5$ dimensions

		n-n			skewn-n			t-t			skewt-t		
		m_1	m_2	m_5	m_1	m_2	m_5	m_1	m_2	m_5	m_1	m_2	m_5
		Panel A: Metrics											
	$\overline{\text{Cov}}$	0.041	0.048	0.067	0.043	0.047	0.048	0.050	0.057	0.070	0.050	0.054	0.051
	$\overline{\text{\%Bias}}$	-8.257	-1.874	12.605	-6.822	-2.937	-1.864	-0.078	5.092	14.347	0.288	3.093	0.901
	$\overline{\text{\%RMSE}}$	9.577	5.140	13.000	8.480	4.964	4.316	4.441	6.695	14.678	4.425	5.796	3.785
	$\overline{\text{Loss}}$	0.092	0.092	0.093	0.092	0.092	0.092	0.092	0.092	0.093	0.092	0.092	0.092
		Panel B: Diebold-Mariano test											
n-n	m_1	-	91|0	100|0	56|24	88|3	88|4	92|3	98|0	100|0	90|4	97|0	97|0
	m_2	-	-	100|0	5|82	30|63	52|42	67|27	100|0	100|0	71|23	88|8	80|12
	m_5	-	-	-	0|100	0|100	0|100	0|100	0|100	93|5	0|100	0|100	0|100
skewn-n	m_1	-	-	-	-	72|13	84|9	93|4	99|0	100|0	93|3	98|0	98|0
	m_2	-	-	-	-	-	60|35	77|20	98|0	100|0	76|21	95|3	92|7
	m_5	-	-	-	-	-	-	70|21	99|0	100|0	76|14	91|3	91|4
t-t	m_1	-	-	-	-	-	-	-	100|0	100|0	65|24	87|7	59|32
	m_2	-	-	-	-	-	-	-	-	100|0	1|99	10|90	5|95
	m_5	-	-	-	-	-	-	-	-	-	0|100	0|100	0|100
skewt-t	m_1	-	-	-	-	-	-	-	-	-	-	73|22	55|37
	m_2	-	-	-	-	-	-	-	-	-	-	-	27|69
	m_5	-	-	-	-	-	-	-	-	-	-	-	-

Note: The true DGP is skewt-t model with parameter vector $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are the candidate models. The candidate sets of moments are m_1, m_2 and m_5. The marginal distributions of the simulated data follow an AR(1)-GARCH(1,1) process, see equation (14) in main text. $\overline{\text{Cov}}$ denotes the average coverage rate of VaR violations in $R = 100$ Monte Carlo replications. $\overline{\text{\%Bias}}$ and $\overline{\text{\%RMSE}}$ are the average percentage bias and the average percentage RMSE of VaR forecasts over $T_s = 500$ forecasting horizon. $\overline{\text{Loss}}$ is the average loss of the VaR forecasts using equation (16) in main text over T_s and R Monte Carlo replications. The left (right) number in each cell in the lower panel reports the number of times that the row model significantly outperforms (underperforms) the column model, using a Diebold-Mariano test at the 5% significance level.

Table S.A.14: Simulation results: evaluation of 1%-VaR forecasts and backtesting for $N = 5$ dimensions

		n-n			skewn-n			t-t			skewt-t		
		m_1	m_2	m_5	m_1	m_2	m_5	m_1	m_2	m_5	m_1	m_2	m_5
		Panel A: Metrics											
	$\overline{\text{Cov}}$	0.012	0.015	0.024	0.011	0.014	0.017	0.011	0.011	0.018	0.011	0.010	0.010
	$\overline{\%\text{Bias}}$	3.659	8.633	20.023	2.638	6.919	11.949	2.284	3.013	13.556	2.340	1.786	0.988
	$\overline{\%\text{RMSE}}$	5.256	9.413	20.177	5.081	7.774	12.218	4.532	4.878	14.001	4.768	4.382	4.031
	$\overline{\text{Loss}}$	0.024	0.025	0.027	0.024	0.025	0.025	0.024	0.024	0.025	0.024	0.024	0.024
		Panel B: Diebold-Mariano test											
n-n	m_1	-	48|0	75|0	12|18	42|2	53|0	3|20	11|13	57|0	5|17	3|19	4|19
	m_2	-	-	83|0	0|43	9|42	50|8	0|42	0|40	61|2	0|42	0|40	0|39
	m_5	-	-	-	0|71	0|79	0|88	0|72	0|73	1|88	0|71	0|71	0|71
skewn-n	m_1	-	-	-	-	40|0	49|0	9|13	16|9	55|0	14|12	6|15	6|16
	m_2	-	-	-	-	-	58|1	0|39	0|36	64|1	0|39	0|38	2|31
	m_5	-	-	-	-	-	-	0|51	0|52	53|13	0|51	0|50	0|49
t-t	m_1	-	-	-	-	-	-	-	20|3	56|0	15|5	3|16	5|17
	m_2	-	-	-	-	-	-	-	-	57|0	7|17	0|27	5|18
	m_5	-	-	-	-	-	-	-	-	-	0|57	0|55	0|52
skewt-t	m_1	-	-	-	-	-	-	-	-	-	-	1|20	7|16
	m_2	-	-	-	-	-	-	-	-	-	-	-	6|13
	m_5	-	-	-	-	-	-	-	-	-	-	-	-

Note: The true DGP is skewt-t model with parameter vector $(\beta, \nu^{-1}, \lambda)' = (1, 0.25, -0.5)'$. The *n-n*, *skewn-n*, *t-t* and *skewt-t* factor copula models are the candidate models. The candidate sets of moments are m_1, m_2 and m_5. The marginal distributions of the simulated data follow an AR(1)-GARCH(1,1) process, see equation (14) in main text. $\overline{\text{Cov}}$ denotes the average coverage rate of VaR violations in $R = 100$ Monte Carlo replications. $\overline{\%\text{Bias}}$ and $\overline{\%\text{RMSE}}$ are the average percentage bias and the average percentage RMSE of VaR forecasts over $T_s = 500$ forecasting horizon. $\overline{\text{Loss}}$ is the average loss of the VaR forecasts using equation (16) in main text over T_s and R Monte Carlo replications. The left (right) number in each cell in the lower panel reports the number of times that the row model significantly outperforms (underperforms) the column model, using a Diebold-Mariano test at the 5% significance level.