Consistent Monitoring of Cointegrating Relationships

Supplementary Appendix A: Proofs

Martin Wagner & Dominik Wied

Proof of Lemmata 1 and 2

Both lemmata follow from the established convergence of the underlying estimators (Theorem 3.2 in Phillips and Hansen (1990) for FM-OLS, Theorem 4.1 in Saikkonen (1991) for D-OLS and Lemma 2 in Vogelsang and Wagner (2014) for IM-OLS), the assumption of consistent long-run variance estimation and the continuous mapping theorem.

Proof of Proposition 1:

For all three variants the limit $\mathcal{F}^m(s)$ of $\widehat{F}^m(s)$ is well defined and the same holds true for $\frac{\widehat{F}^m(s)}{g(s)}$ since $0 < g(s) < \infty$ is assumed to be continuous. This allows to find critical values for given $g(s)$, compare (21).

Proof of Proposition 2:

As the proposition in the main text, also the proof consists of two parts, consistency against fixed alternatives in part (a) and local asymptotic power in part (b).

We start with part (a) and again the limiting behavior of the partial sum process of the residuals is key. We distinguish again two cases FM-OLS and D-OLS on the one hand and IM-OLS on the other. FM-OLS and D-OLS lead to the same asymptotic behavior and need not be considered separately.

For FM-OLS, the partial sum process of the residuals is given by (again for $1 \geq s >$

$r \geq m$):

$$\begin{aligned}
\frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}\hat{u}^+_{t,m} &= \frac{1}{\sqrt{T}}\sum_{t=1}^{[rT]}\hat{u}^+_{t,m} + \frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]}\hat{u}^+_{t,m} \\
&= \frac{1}{\sqrt{T}}\sum_{t=1}^{[rT]}\hat{u}^+_{t,m} + \frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]}u_t - \frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]}v_t'\hat{\Omega}_{vv}^{-1}\hat{\Omega}_{vu} - \\
&\quad -\frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]}D_t'\left(\hat{\theta}_{D,m}-\theta_D\right) - \frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]}X_t'\left(\hat{\theta}_{X,m}-\theta_X\right).
\end{aligned} \tag{1}$$

The first term above converges to $\omega_{u\cdot v}\widehat{W}_{u\cdot v}(r)$, according to Lemma 1 and the second term diverges, since for $t \geq [rT]+1$ u_t is an I(1) process requiring $T^{-3/2}$ as scaling for weak convergence. The remaining three terms converge in distribution. Thus, the partial sum process is in this case $O_p(T)$. The argument is analogous for the IM-OLS partial sum process, i.e., for $\hat{S}^u_{t,m}$, based on Lemma 2 instead of Lemma 1.

For the second item of part (a) the partial sum process of the FM-OLS residuals, again for $1 \geq s > r \geq m$, can be written as:

$$\begin{aligned}
\frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}\hat{u}^+_{t,m} = {} & \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}u_t - \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}v_t'\hat{\Omega}_{vv}^{-1}\hat{\Omega}_{vu} \\
& - \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}D_t'\left(\hat{\theta}_{D,m}-\theta_D\right) - \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}X_t'\left(\hat{\theta}_{X,m}-\theta_X\right) \\
& - \frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]}D_t'\left(\theta_D-\theta_{D,1}\right) - \frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]}X_t'\left(\theta_X-\theta_{X,1}\right).
\end{aligned} \tag{2}$$

From this decomposition the result follows since at least one of the two last terms diverges (either deterministically or stochastically) whereas all other terms converge in distribution. For IM-OLS the result follows analogously, and for $1 \geq s > r \geq m$ we obtain:

$$\begin{aligned}
\frac{1}{\sqrt{T}}\hat{S}^u_{[sT],m} = {} & \frac{1}{\sqrt{T}}S^u_{[sT]} - \frac{1}{\sqrt{T}}S^{D\prime}_{[sT]}\left(\hat{\theta}_{D,m}-\theta_D\right) - \frac{1}{\sqrt{T}}S^{X\prime}_{[sT]}\left(\hat{\theta}_{X,m}-\theta_X\right) \\
& - \frac{1}{\sqrt{T}}X'_{[sT]}\hat{\varphi}_m - \frac{1}{\sqrt{T}}S^{D\prime}_{[sT]}\left(\theta_D-\theta_{D,1}\right) - \frac{1}{\sqrt{T}}S^{X\prime}_{[sT]}\left(\theta_X-\theta_{X,1}\right),
\end{aligned} \tag{3}$$

from which we see that again at least one of the last two terms diverges and the other terms converge in distribution.

We now turn to part (b) of the proposition. Here it follows for item (i) when considering FM-OLS that (for $1 \geq s > r \geq m$):

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}\hat{u}^+_{t,m} \;\Rightarrow\; \omega_{u\cdot v}\widehat{W}_{u\cdot v}(s) + \delta\omega_\xi\int_r^s (W_\xi(z)-W_\xi(r))dz. \tag{4}$$

For IM-OLS the partial sum limit is of similar form and given by:

$$\frac{1}{\sqrt{T}}\hat{S}^{u}_{[sT],m} \Rightarrow \omega_{u\cdot v}\tilde{P}_m(s) + \delta\omega_{\xi}\int_r^s (W_{\xi}(z) - W_{\xi}(r))dz. \tag{5}$$

We see for both cases that the (identical) second term in the limit can be made arbitrarily large (in mean square sense) by choosing δ sufficiently large.

For the second item of (b) we obtain the following partial sum limit under the considered local alternatives for FM-OLS for $1 \geq s > r \geq m$:

$$\begin{aligned}
\frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]} \hat{u}^{+}_{t,m} &= \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]} u_t - \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]} v_t'\hat{\Omega}_{vv}^{-1}\hat{\Omega}_{vu} \\
&\quad - \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]} D_t'\left(\hat{\theta}_{D,m} - \theta_D\right) - \frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]} X_t'\left(\hat{\theta}_{X,m} - \theta_X\right) \\
&\quad + \frac{1}{T^{3/2}}\sum_{t=[rT]+1}^{[sT]} X_t'\Delta_{\theta_X} \\
&\quad \Rightarrow \omega_{u\cdot v}\widehat{W}_{u\cdot v}(s) + \int_r^s W_v(z)'dz\Omega_{vv}^{1/2\prime}\Delta_{\theta_X}.
\end{aligned} \tag{6}$$

Similar calculations lead for IM-OLS for $1 \geq s > r \geq m$ to:

$$\frac{1}{\sqrt{T}}\hat{S}^{u}_{[sT],m} \Rightarrow \omega_{u\cdot v}\tilde{P}_m(s) + \int_0^s W_v(z)'dz\Omega_{vv}^{1/2\prime}\Delta_{\theta_X}. \tag{7}$$

In both cases the additional component that appears in the limit under the local alternatives considered, i.e., $\int_0^s W_v(z)'dz\Omega_{vv}^{1/2\prime}\Delta_{\theta_X}$, can be made arbitrarily large (in mean square sense) by choosing Δ_{θ_X} large enough.

It remains to consider the third item. Under the local alternatives considered now we obtain as additional term:

$$-\frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]} D_t'\left(\theta_D - \theta_{D,1}\right) = \frac{1}{\sqrt{T}}\sum_{t=[rT]+1}^{[sT]} D_t'G_D^{-1\prime}\Delta_{\theta_D}, \tag{8}$$

which converges in distribution to

$$\int_r^s D(z)'dz\Delta_{\theta_D}. \tag{9}$$

This shows the result as under the considered local alternatives it holds that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]} u^{+}_{t,m} \Rightarrow \omega_{u\cdot v}W_{u\cdot v}(s) + \int_r^s D(z)'dz\Delta_{\theta_D}. \tag{10}$$

For IM-OLS the corresponding result is given by:

$$\frac{1}{\sqrt{T}}\hat{S}^{u}_{[sT],m} \Rightarrow \omega_{u\cdot v}\tilde{P}_m(s) + \int_r^s D(z)'dz\Delta_{\theta_D}. \tag{11}$$

Thus, by choosing Δ_{θ_D} large enough the result follows similarly to above.

References

Phillips, P. C. B. and B. Hansen (1990): "Statistical Inference in Instrumental Variables Regression with I(1) Processes," *Review of Economic Studies*, 57, 99–125.

Saikkonen, P. (1991): "Asymptotically Efficient Estimation of Cointegrating Regressions," *Econometric Theory*, 7, 1–21.

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Consistent Monitoring of Cointegrating Relationships

Supplementary Appendix B: Additional Simulation Results

Martin Wagner & Dominik Wied

Local Asymptotic Power

We start by considering local asymptotic power (LAP). The results are based on 10,000 replications using time series of length 1,000 of i.i.d. standard normals to approximate the limiting distributions. The limiting distributions underlying LAP as discussed in Proposition 2(b) are based on the functional central limit theorems for the scaled partial sum residual processes under the local alternatives discussed in the main text and Supplementary Appendix A. In particular these FCLTs are given in:

- Local I(1) breaks: Equation (25) in the main document for FM-/D-OLS and equation (5) in the Supplementary Appendix A for IM-OLS.
- Local slope breaks: Equation (6) in the main document for FM-/D-OLS and equation (7) for IM-OLS in the Supplementary Appendix A.
- Local trend breaks: Equation (10) in the main document for FM-/D-OLS and equation (11) for IM-OLS in the Supplementary Appendix A.

As mentioned in the main document, in case of local I(1) breaks, LAP depends on the product of δ and the "signal-to-noise" ratio ω_ξ/ω. Similarly, LAP against slope breaks depends upon the variance of the regressors. We set the signal-to-noise ratios to one.

It is also clear that LAP depends upon the number of integrated regressors included. This is illustrated for the case of local I(1) breaks in Figure 1 for $m = r = 0.25$. As expected LAP decreases in the number of integrated regressors. In this figure the parameter δ varies on an equidistant grid with 21 values in the interval $[0, 100]$.[1] Keeping this observation in mind, all other LAP results displayed are for the single integrated regressor case for brevity.

Figure 2 considers the same values for δ and several combinations of m and r that are also considered in the main text. The upper two plots correspond to FM-/D-OLS and the lower two plots correspond to IM-OLS. Within each of these two graphs the

[1] The first value equal to zero corresponds, obviously, to the null hypothesis.

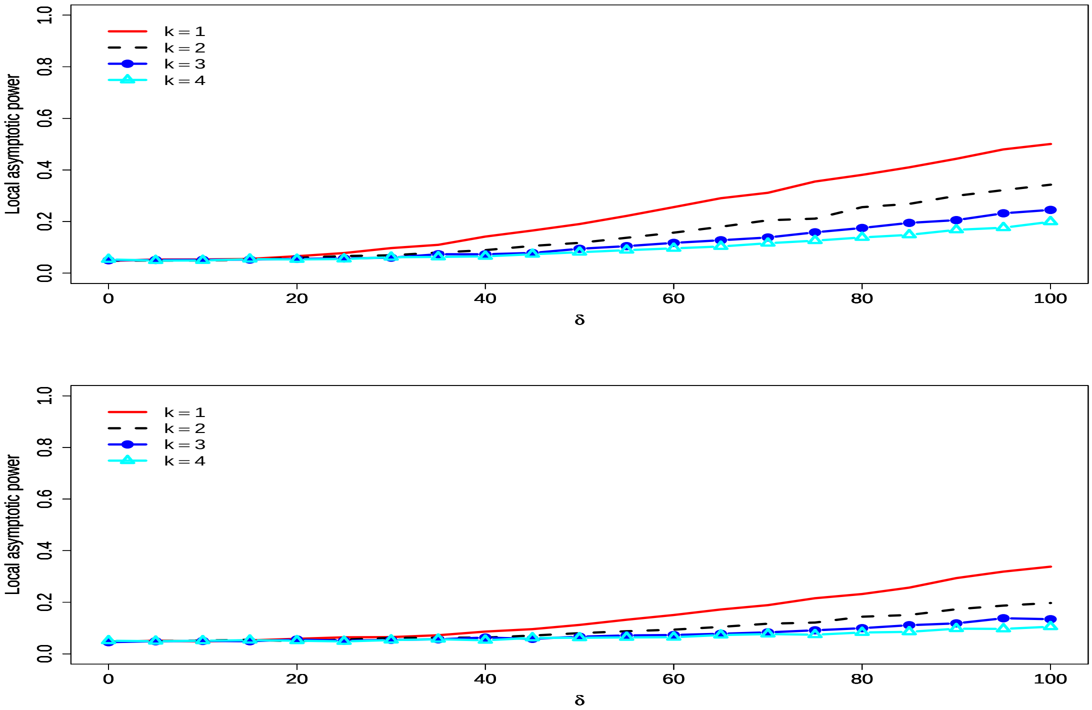

Figure 1: Local asymptotic power against I(1) breaks for $k = 1, \ldots, 4$ regressors. The upper plot corresponds to FM-OLS & D-OLS and the lower plot to IM-OLS. The plots show results for $m = r = 0.25$.

upper one displays the case $m = 0.25$ and $r = 0.25, 0.5, 0.75$, i.e., the situation with fixed calibration period and equal or later break-points; and the lower one displays the case $m = r = 0.25, 0.5, 0.75$, i.e., the case with increasing calibration period and break immediately at the end of the calibration period. The results show slightly lower LAP of IM-OLS compared to FM-/D-OLS. This is in line with the fact that FM-/D-OLS is conditionally more efficient than IM-OLS from a standard asymptotic theory point of view, as discussed in Vogelsang and Wagner (2014). The practical usefulness of IM-OLS stems from the fact that it exhibits lower finite sample size distortions than the other two variants, as illustrated in the main text. With respect to changing values of m and r the ranking is the same across methods.

In Figure 3 we display LAP against trend breaks with a similar structure as in Figure 2, where we now consider a break in the linear trend. In particular we consider 31 equally spaced values of Δ_θ in the interval $[0, 30]$, with Δ_θ here referring to a changing linear trend slope. The ranking is the same across combinations of m and r for both FM-/D-OLS and IM-OLS. Again, LAP of IM-OLS is slightly lower than for FM/D-OLS. The results in this figure indicate that a larger calibration period, with a correspondingly later break-point, leads to substantially higher LAP.

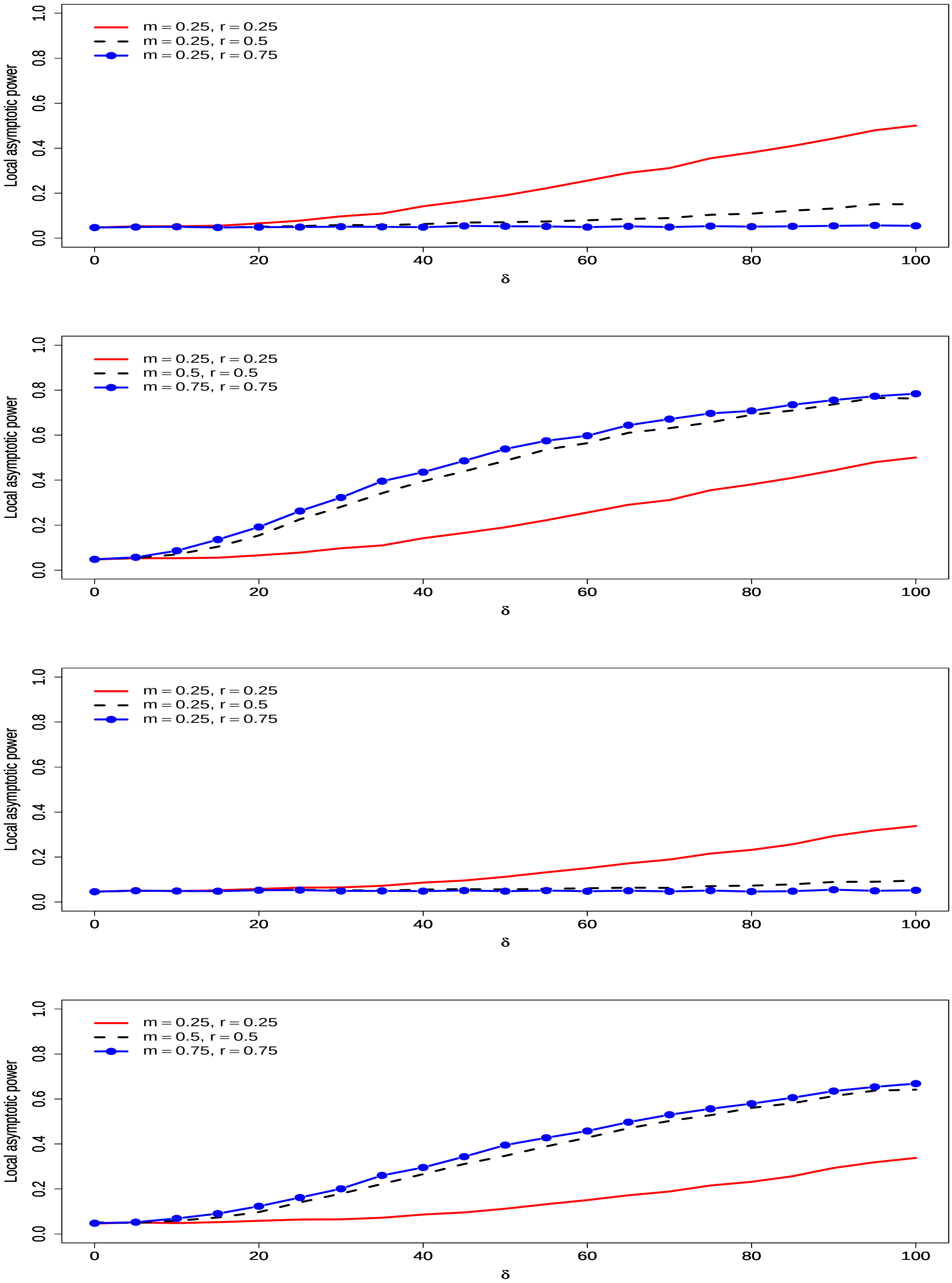

Figure 2: Local asymptotic power for different combinations of m and r. The upper two plots correspond to FM-OLS & D-OLS and the lower two plots to IM-OLS. The plots show results for different combinations of m and r.

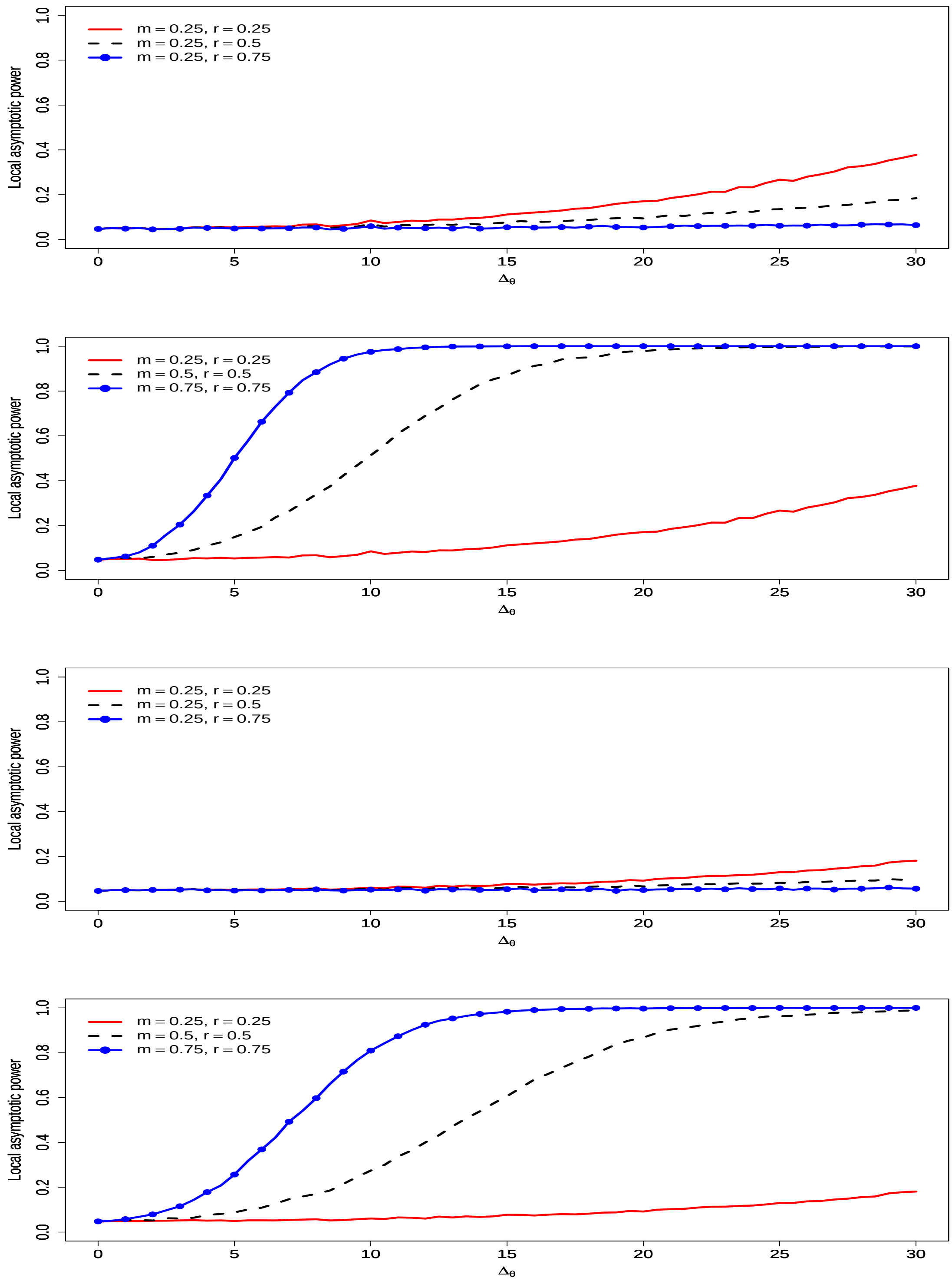

Figure 3: Local asymptotic power against trend breaks. The upper two plots correspond to FM-OLS & D-OLS and the lower two plots to IM-OLS. The plots show results for different combinations of m and r.

Finite Sample Performance: Power

We now turn to the finite sample *size-corrected* power against slope and trend breaks (for the setting discussed in the main text). We only report results for the case that $\rho_1, \rho_2 = 0.9$, i.e, we display the worst results from the set of ρ-values considered. All power figures have the same structure and display in the left column the results for $m = 0.25$ and $r = 0.25, 0.5, 0.75$ and in the right column the results for $m = r = 0.25, 0.5, 0.75$.

For slope breaks, with the results displayed in in Figures 4 for $T = 200$ and 5 for $T = 500$ we consider an equidistant grid for $\beta_1 = \beta_2$ from the interval $[1, 3]$, including the null value $\beta_1, \beta_2 = 1$. Three main observations emerge: First, power decreases with increasing r for given m (left column of the figures). Second, power increases with increasing $m = r$ (right column of the figures). Third, power is very similar for the three considered estimation methods. In some circumstances power of D-OLS is slightly higher and that of IM-OLS slightly lower. This, however, has to be seen in conjunction with the observations that D-OLS sometimes has the largest size distortions under the null and that IM-OLS very often hast the lowest size distortions under the null.

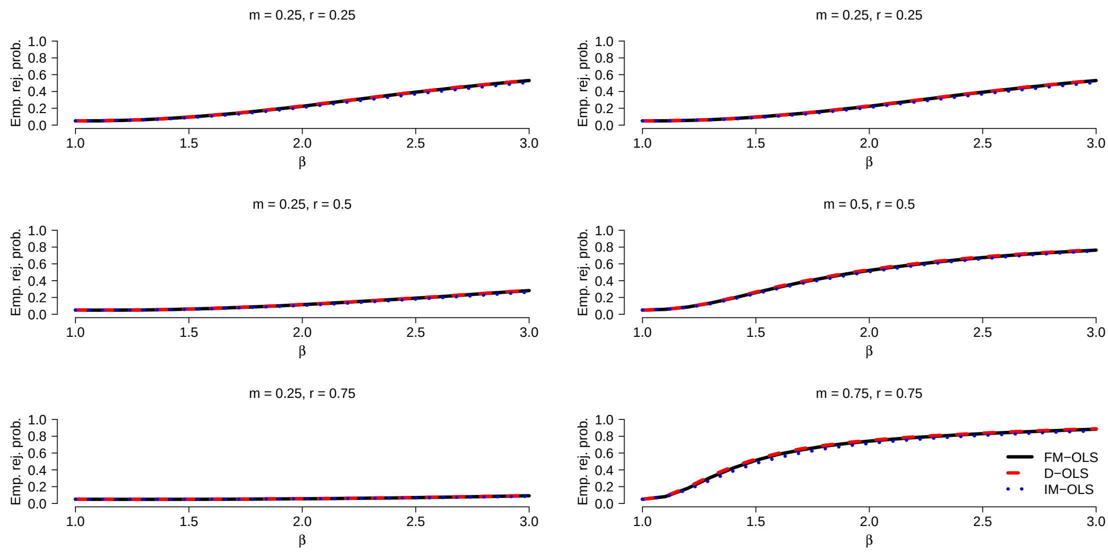

Figure 4: Size-corrected power against slope breaks for $T = 200$ and $\rho_1 = \rho_2 = 0.9$.

In Figures 6 ($T = 200$) and 7 ($T = 500$) we display power against breaks in the trend slope. Here we consider an equidistant grid with 21 values for the trend slope γ in the interval $[1, 1.05]$, including the null value $\gamma = 1$. The results are qualitatively very similar to the results for power against slope breaks just discussed.

Finite Sample Performance: Andrews-Kim

We now consider some results for the P test of Andrews and Kim (2006). We start with showing the differences of the null rejection probabilities of the Andrews-Kim test and our detector. Numbers below zero indicate smaller size distortions of the Andrews-Kim test, since – compare the figures in the main text – all tests are oversized. Depending upon

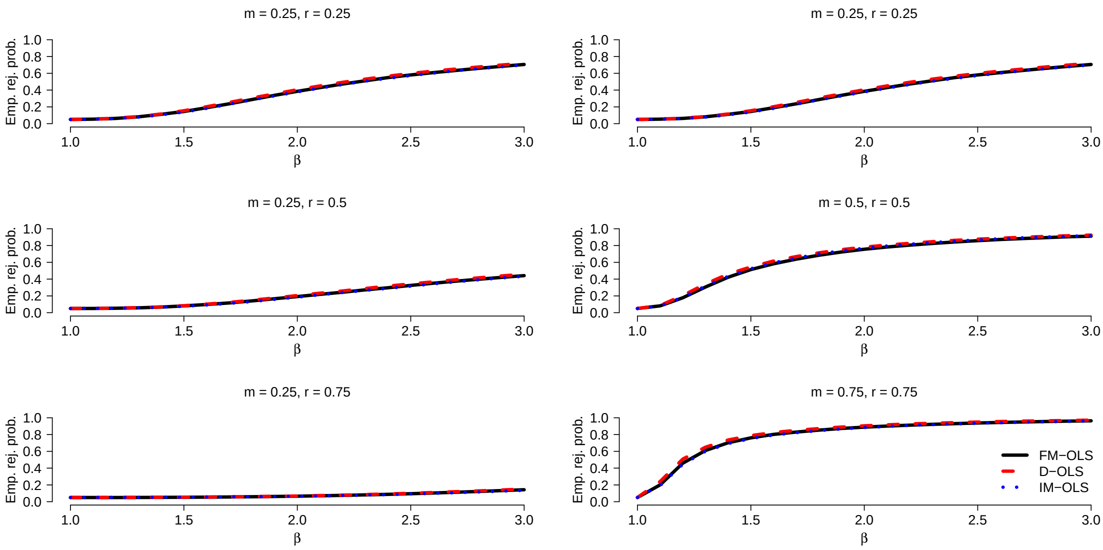

Figure 5: Size-corrected power against slope breaks for $T = 500$ and $\rho_1 = \rho_2 = 0.9$.

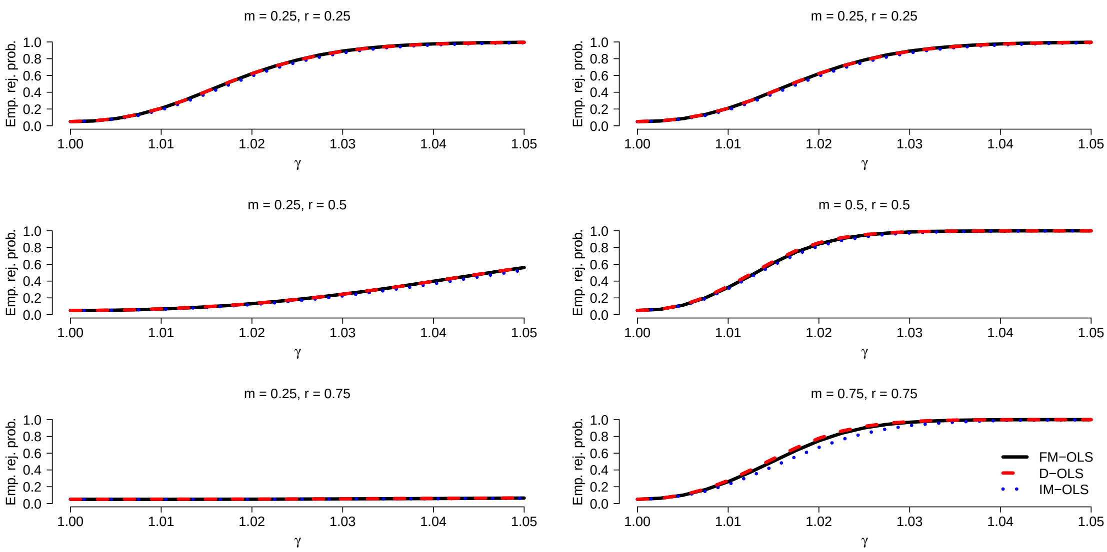

Figure 6: Size-corrected power against trend breaks for $T = 200$ and $\rho_1 = \rho_2 = 0.9$.

precise configuration, the Andrews-Kim test leads to lower size distortions for values of m larger than 0.7. For the smaller values of ρ_1, ρ_2 the Andrews-Kim does not lead to lower size distortions even for the largest considered value of $m = 0.9$.

In Tables 1 and 2 we consider raw and size corrected power of the Andrews-Kim test. Since for sub-sampled statistics size-corrections performed in the way described in the main text are not really representative, as under the alternative no sub-sampling is performed in the simulations, we also consider raw power. For standard unit root and cointegration tests, size corrected power is usually lower than raw power, because of the fact that the tests are oversized. This is different for sub-sampling statistics, compare

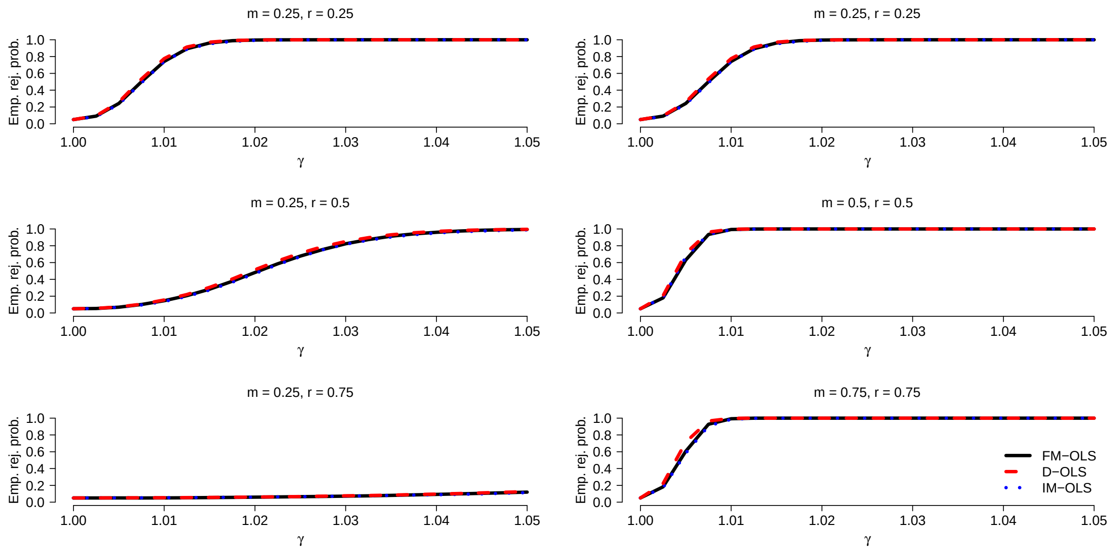

Figure 7: Size-corrected power against trend breaks for $T = 500$ and $\rho_1 = \rho_2 = 0.9$.

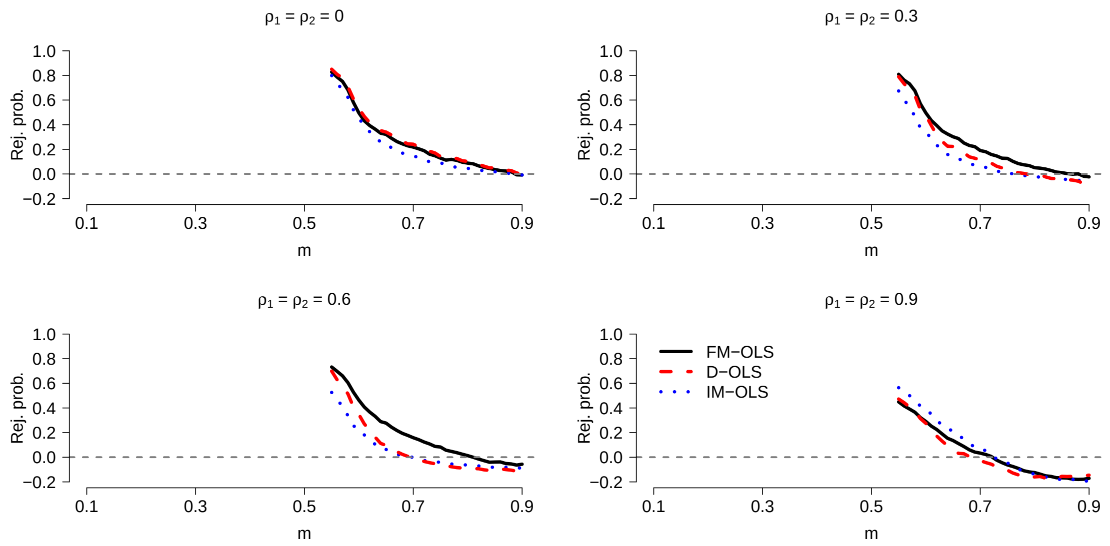

Figure 8: Differences in empirical null rejection probabilities between the Andrews-Kim test and our detector for $T = 200$.

the two tables, where raw power is seen to be in many cases lower than size corrected power despite the fact that the tests are oversized. This stems from the fact that for alternatives where $r < m$ raw-power is calculated from an empirical distribution of sub-sampled statistics that are calculated at least partly from integrated data. This leads to larger critical values compared to the null sub-sampling distribution used for size corrected power. Consequently, raw power is often lower than size corrected power for the Andrews-Kim test. This is not the case for our detectors, where raw power is larger than size corrected power.

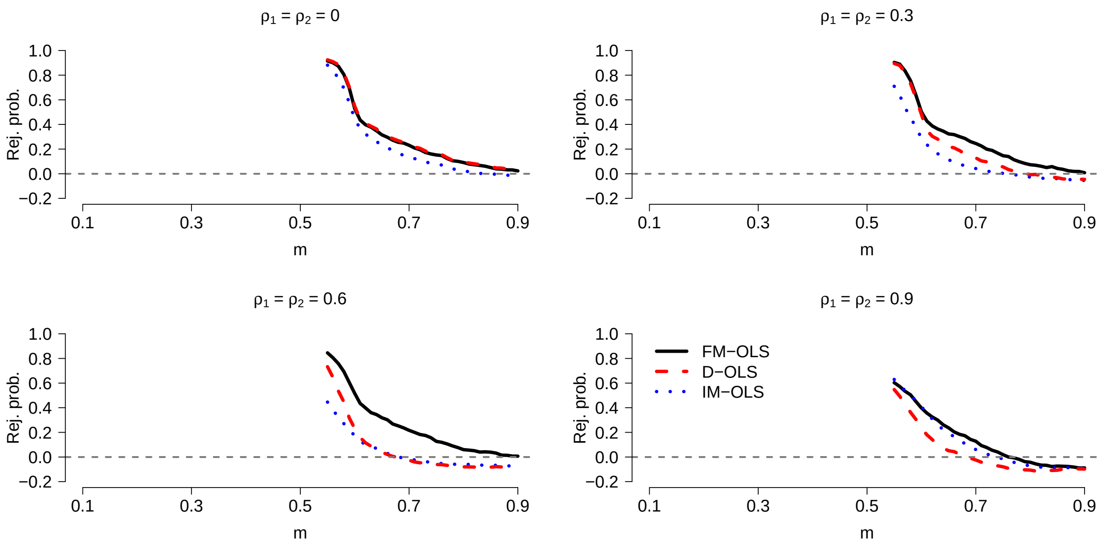

Figure 9: Differences in empirical null rejection probabilities between the Andrews-Kim test and our detector for $T = 500$.

This makes it hard to compare the power of the Andrews and Kim (2006) test with the power of our procedures. Comparing the power tables for our detectors in the main text with the power tables here lets us tentatively conclude, notwithstanding the complications just mentioned, that the behavior under I(1) breaks is not too different.

		$T = 200$			$T = 500$		
$\rho_1 = \rho_2$	r	0.25	0.5	0.75	0.25	0.5	0.75
0	FM	0.42	0.64	0.99	0.44	0.65	1.00
	D	0.44	0.64	0.99	0.46	0.65	1.00
	IM	0.42	0.61	0.98	0.36	0.54	1.00
0.3	FM	0.42	0.62	0.97	0.42	0.64	1.00
	D	0.45	0.62	0.96	0.44	0.64	1.00
	IM	0.38	0.51	0.85	0.34	0.52	1.00
0.6	FM	0.40	0.53	0.89	0.39	0.59	1.00
	D	0.43	0.55	0.94	0.42	0.61	1.00
	IM	0.97	0.95	0.89	0.32	0.48	0.98
0.9	FM	0.33	0.34	0.46	0.34	0.47	0.77
	D	0.37	0.37	0.56	0.38	0.50	0.87
	IM	0.35	0.36	0.41	0.30	0.37	0.52

Table 1: Andrews-Kim: *Raw* power against I(1) breaks for $m = 0.75$. Results for $T = 200$ in the left block-column and for $T = 500$ in the right block-column.

$\rho_1 = \rho_2$ \ r		$T = 200$: 0.25	$T = 200$: 0.5	$T = 200$: 0.75	$T = 500$: 0.25	$T = 500$: 0.5	$T = 500$: 0.75
	FM	1.00	0.99	0.98	1.00	1.00	1.00
0	D	0.99	0.98	0.97	1.00	1.00	1.00
	IM	1.00	1.00	0.98	1.00	1.00	1.00
	FM	0.97	0.97	0.97	1.00	1.00	1.00
0.3	D	0.98	0.98	0.96	1.00	1.00	1.00
	IM	1.00	0.99	0.97	1.00	1.00	1.00
	FM	0.83	0.83	0.83	1.00	1.00	1.00
0.6	D	0.94	0.94	0.93	1.00	1.00	1.00
	IM	0.97	0.95	0.89	1.00	1.00	1.00
	FM	0.16	0.18	0.20	0.50	0.56	0.60
0.9	D	0.32	0.34	0.35	0.84	0.94	0.88
	IM	0.28	0.22	0.18	0.62	0.59	0.42

Table 2: Andrews-Kim: *Size corrected* power against I(1) breaks for $m = 0.75$. Results for $T = 200$ in the left block-column and for $T = 500$ in the right block-column.

Finite Sample Performance: Detection Times

We close this document with some additional detection time figures, where we show the two "missing figures" displaying the results for $T = 200$ and $\rho_1, \rho_2 = 0$ in Figure 10 and for $T = 500$ and $\rho_1, \rho_2 = 0.9$ in Figure 11. The results are in line with the "boundary cases" shown in the main text.

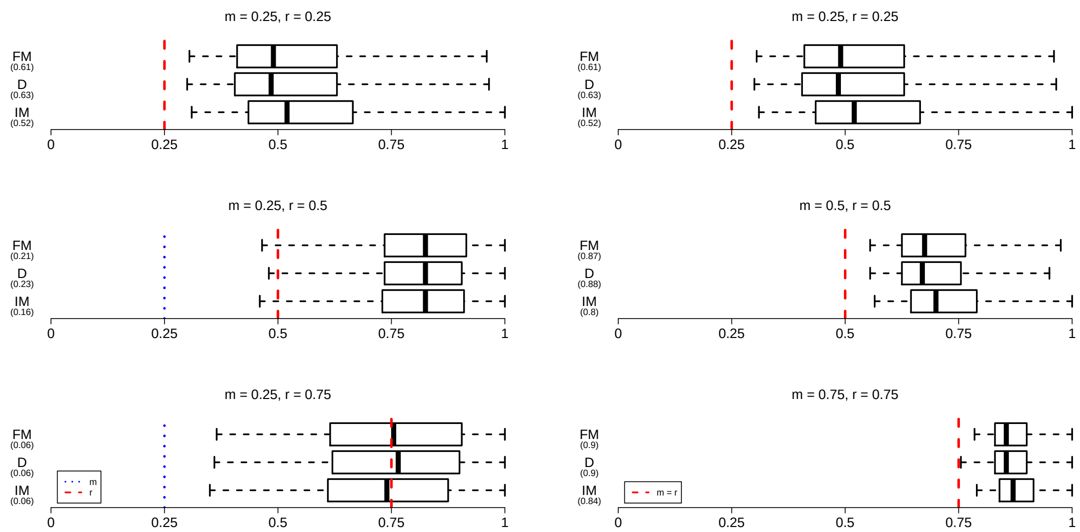

Figure 10: Detection times for I(1) breaks for $T = 200$ and $\rho_1 = \rho_2 = 0.0$.

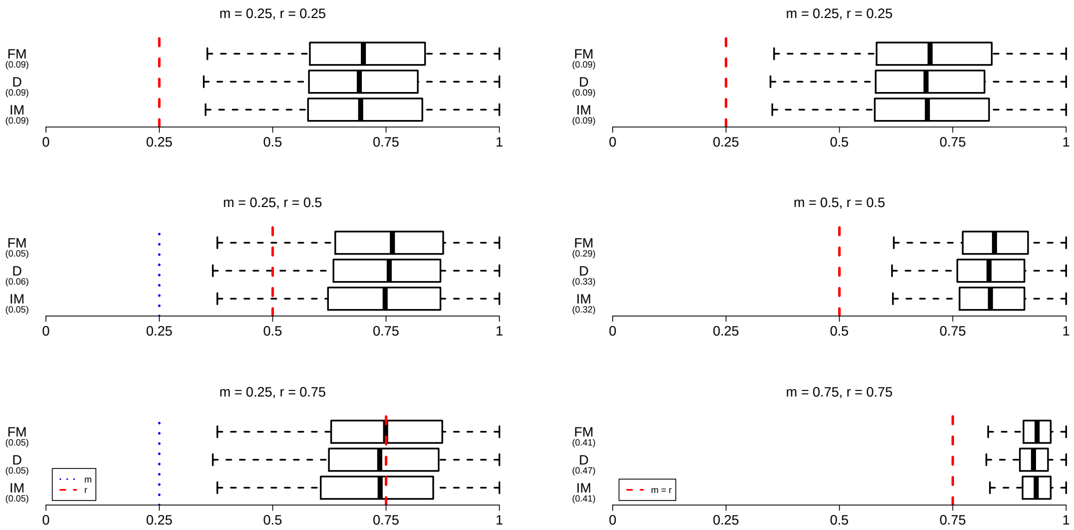

Figure 11: Detection times for I(1) breaks for $T = 500$ and $\rho_1 = \rho_2 = 0.9$.

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Supplementary Appendix C: Critical Values

Martin Wagner & Dominik Wied

Description

For the calculations the functionals of Brownian motions have been approximated with the corresponding functions of random walks with 1,000 observations generated from i.i.d. standard normal variables. The critical values are provided for a grid with mesh 0.01 for $m = 0.1, \dots, 0.9$. The number of replications in the simulations is 1,000,000. The tabulated critical values are used in the available R package `cointmonitoR`.

- Tables 1 and 2 provide the critical values for (OLS-based) stationarity monitoring, i.e., the case without stochastic regressors X_t, for the intercept only and the intercept and linear trend case.
- Tables 3 to 18 provide the critical values for FM- and D-OLS, and IM-OLS based monitoring for $k = 1, \dots, 4$ stochastic regressors for the intercept only and the intercept and linear trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	8.19	11.28	14.39	18.82	0.51	0.51	0.70	0.89	1.17
0.11	7.22	9.96	12.91	16.76	0.52	0.48	0.66	0.85	1.11
0.12	6.51	8.98	11.68	15.09	0.53	0.45	0.63	0.80	1.04
0.13	5.88	8.11	10.44	13.77	0.54	0.42	0.59	0.76	0.98
0.14	5.32	7.35	9.46	12.38	0.55	0.40	0.55	0.70	0.93
0.15	4.84	6.69	8.59	11.21	0.56	0.37	0.52	0.67	0.88
0.16	4.47	6.13	7.89	10.37	0.57	0.35	0.49	0.63	0.82
0.17	4.08	5.63	7.31	9.48	0.58	0.33	0.46	0.59	0.77
0.18	3.77	5.22	6.68	8.76	0.59	0.31	0.43	0.56	0.72
0.19	3.50	4.83	6.21	8.13	0.60	0.29	0.41	0.52	0.67
0.20	3.25	4.46	5.75	7.52	0.61	0.27	0.38	0.49	0.64
0.21	3.02	4.14	5.34	6.99	0.62	0.26	0.36	0.46	0.60
0.22	2.81	3.86	5.00	6.46	0.63	0.24	0.33	0.43	0.56
0.23	2.61	3.61	4.64	6.04	0.64	0.22	0.31	0.40	0.52
0.24	2.45	3.36	4.32	5.74	0.65	0.21	0.29	0.37	0.49
0.25	2.28	3.17	4.06	5.32	0.66	0.19	0.27	0.35	0.45
0.26	2.15	2.97	3.83	5.02	0.67	0.18	0.25	0.32	0.42
0.27	2.02	2.79	3.60	4.71	0.68	0.17	0.23	0.30	0.39
0.28	1.90	2.62	3.36	4.43	0.69	0.15	0.21	0.28	0.36
0.29	1.77	2.45	3.19	4.14	0.70	0.14	0.20	0.26	0.34
0.30	1.67	2.31	3.00	3.91	0.71	0.13	0.18	0.24	0.31
0.31	1.58	2.19	2.83	3.71	0.72	0.12	0.17	0.22	0.28
0.32	1.48	2.05	2.65	3.49	0.73	0.11	0.16	0.20	0.26
0.33	1.40	1.94	2.51	3.25	0.74	0.10	0.14	0.18	0.24
0.34	1.32	1.83	2.36	3.07	0.75	0.09	0.13	0.17	0.22
0.35	1.25	1.73	2.24	2.92	0.76	0.09	0.12	0.15	0.20
0.36	1.18	1.63	2.12	2.76	0.77	0.08	0.11	0.14	0.18
0.37	1.11	1.54	2.00	2.63	0.78	0.07	0.10	0.13	0.17
0.38	1.05	1.47	1.89	2.47	0.79	0.06	0.09	0.11	0.15
0.39	1.00	1.38	1.79	2.33	0.80	0.06	0.08	0.10	0.13
0.40	0.94	1.30	1.68	2.19	0.81	0.05	0.07	0.09	0.12
0.41	0.89	1.23	1.59	2.07	0.82	0.05	0.06	0.08	0.11
0.42	0.84	1.17	1.51	1.96	0.83	0.04	0.06	0.07	0.09
0.43	0.80	1.10	1.42	1.86	0.84	0.04	0.05	0.06	0.08
0.44	0.75	1.04	1.33	1.74	0.85	0.03	0.04	0.05	0.07
0.45	0.71	0.98	1.26	1.65	0.86	0.03	0.04	0.05	0.06
0.46	0.67	0.93	1.20	1.56	0.87	0.02	0.03	0.04	0.05
0.47	0.63	0.88	1.13	1.49	0.88	0.02	0.03	0.03	0.04
0.48	0.60	0.83	1.06	1.39	0.89	0.02	0.02	0.03	0.04
0.49	0.56	0.78	1.01	1.32	0.90	0.01	0.02	0.02	0.03
0.50	0.54	0.74	0.96	1.24					

Table 1: Critical values for OLS based stationarity monitoring for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	1254.13	1780.34	2318.80	3083.32	0.51	2.36	3.33	4.33	5.75
0.11	916.71	1302.85	1702.65	2258.45	0.52	2.13	3.01	3.90	5.23
0.12	685.20	977.02	1287.94	1699.01	0.53	1.90	2.70	3.52	4.68
0.13	524.56	746.17	980.01	1298.16	0.54	1.72	2.44	3.19	4.20
0.14	406.13	577.68	759.38	1000.11	0.55	1.55	2.20	2.87	3.78
0.15	322.05	457.74	601.89	793.79	0.56	1.41	1.99	2.58	3.39
0.16	259.31	364.20	477.85	631.34	0.57	1.27	1.80	2.35	3.09
0.17	209.57	296.19	385.92	510.08	0.58	1.14	1.61	2.10	2.74
0.18	171.68	242.53	316.59	417.40	0.59	1.04	1.46	1.91	2.50
0.19	140.97	199.38	261.27	343.65	0.60	0.94	1.32	1.72	2.25
0.20	116.50	164.97	215.91	287.08	0.61	0.85	1.19	1.56	2.05
0.21	98.04	139.20	182.90	239.53	0.62	0.77	1.07	1.40	1.85
0.22	83.05	118.16	154.43	203.82	0.63	0.69	0.97	1.26	1.67
0.23	70.16	100.72	132.67	173.26	0.64	0.62	0.88	1.14	1.51
0.24	60.44	85.78	112.69	150.03	0.65	0.56	0.79	1.03	1.37
0.25	51.72	73.43	96.03	127.01	0.66	0.51	0.71	0.93	1.23
0.26	44.38	63.04	82.80	109.40	0.67	0.45	0.63	0.83	1.10
0.27	38.69	54.85	71.99	95.23	0.68	0.41	0.57	0.75	0.99
0.28	33.47	47.83	62.52	82.46	0.69	0.36	0.51	0.67	0.88
0.29	29.31	41.79	54.34	71.65	0.70	0.32	0.46	0.59	0.78
0.30	25.61	36.36	47.58	63.09	0.71	0.29	0.41	0.53	0.70
0.31	22.49	32.05	41.95	54.90	0.72	0.26	0.37	0.48	0.63
0.32	19.85	28.20	37.16	48.89	0.73	0.23	0.32	0.43	0.57
0.33	17.55	24.83	32.62	43.17	0.74	0.21	0.29	0.38	0.50
0.34	15.46	21.86	28.61	37.95	0.75	0.18	0.26	0.33	0.44
0.35	13.64	19.43	25.67	34.02	0.76	0.16	0.23	0.29	0.39
0.36	12.10	17.30	22.65	30.04	0.77	0.14	0.20	0.26	0.34
0.37	10.74	15.34	20.13	26.68	0.78	0.12	0.17	0.23	0.30
0.38	9.56	13.50	17.76	23.56	0.79	0.11	0.15	0.20	0.26
0.39	8.56	12.11	15.93	20.92	0.80	0.10	0.13	0.17	0.23
0.40	7.61	10.79	14.11	18.88	0.81	0.08	0.12	0.15	0.20
0.41	6.77	9.62	12.58	16.79	0.82	0.07	0.10	0.13	0.17
0.42	6.07	8.61	11.24	15.12	0.83	0.06	0.08	0.11	0.14
0.43	5.47	7.73	10.10	13.27	0.84	0.05	0.07	0.10	0.12
0.44	4.90	6.93	9.03	11.91	0.85	0.04	0.06	0.08	0.10
0.45	4.42	6.19	8.10	10.76	0.86	0.04	0.05	0.07	0.09
0.46	3.98	5.61	7.31	9.69	0.87	0.03	0.04	0.06	0.07
0.47	3.58	5.07	6.61	8.71	0.88	0.03	0.04	0.05	0.06
0.48	3.22	4.57	6.00	7.85	0.89	0.02	0.03	0.04	0.05
0.49	2.90	4.13	5.41	7.05	0.90	0.02	0.02	0.03	0.04
0.50	2.61	3.71	4.86	6.37					

Table 2: Critical values for OLS based stationarity monitoring for the intercept and linear case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	155.57	298.22	502.32	922.87	0.51	1.31	2.15	3.29	5.28
0.11	125.67	239.17	405.79	735.77	0.52	1.20	1.96	2.99	4.83
0.12	101.89	193.25	325.10	584.60	0.53	1.11	1.80	2.72	4.36
0.13	83.90	159.81	266.50	467.33	0.54	1.02	1.65	2.48	3.94
0.14	70.75	133.45	221.67	390.66	0.55	0.94	1.50	2.26	3.61
0.15	59.50	112.53	185.12	326.11	0.56	0.87	1.37	2.05	3.25
0.16	50.12	94.25	157.92	278.06	0.57	0.79	1.25	1.85	2.92
0.17	42.82	80.44	134.56	238.18	0.58	0.73	1.14	1.68	2.63
0.18	37.02	69.09	115.10	205.25	0.59	0.67	1.05	1.52	2.39
0.19	32.16	59.81	100.26	178.37	0.60	0.61	0.95	1.38	2.15
0.20	28.24	52.28	87.38	157.69	0.61	0.56	0.87	1.26	1.97
0.21	24.67	45.35	76.29	134.14	0.62	0.51	0.79	1.14	1.76
0.22	21.68	39.88	66.96	117.66	0.63	0.47	0.72	1.03	1.58
0.23	19.00	35.20	59.13	102.11	0.64	0.43	0.65	0.93	1.44
0.24	16.82	31.26	52.14	90.29	0.65	0.39	0.59	0.85	1.29
0.25	14.92	27.46	45.35	79.80	0.66	0.35	0.54	0.77	1.15
0.26	13.28	24.35	39.84	69.89	0.67	0.32	0.49	0.69	1.05
0.27	11.90	21.80	35.67	62.23	0.68	0.29	0.44	0.62	0.92
0.28	10.79	19.57	31.91	54.98	0.69	0.27	0.40	0.56	0.83
0.29	9.73	17.41	28.78	48.85	0.70	0.24	0.36	0.51	0.76
0.30	8.80	15.77	25.87	44.10	0.71	0.22	0.33	0.46	0.67
0.31	7.97	14.24	22.99	39.40	0.72	0.20	0.30	0.41	0.61
0.32	7.17	12.91	20.75	35.05	0.73	0.18	0.26	0.37	0.53
0.33	6.47	11.52	18.59	31.55	0.74	0.16	0.24	0.33	0.47
0.34	5.81	10.36	16.77	28.13	0.75	0.14	0.21	0.29	0.43
0.35	5.32	9.44	15.18	25.53	0.76	0.13	0.19	0.26	0.37
0.36	4.80	8.59	13.70	22.81	0.77	0.12	0.17	0.23	0.33
0.37	4.41	7.78	12.51	20.73	0.78	0.10	0.15	0.20	0.29
0.38	4.05	7.05	11.34	18.94	0.79	0.09	0.13	0.18	0.25
0.39	3.70	6.42	10.22	16.95	0.80	0.08	0.12	0.16	0.22
0.40	3.38	5.81	9.22	15.20	0.81	0.07	0.10	0.14	0.19
0.41	3.11	5.27	8.32	13.71	0.82	0.06	0.09	0.12	0.16
0.42	2.84	4.82	7.64	12.61	0.83	0.05	0.08	0.10	0.14
0.43	2.58	4.37	6.90	11.52	0.84	0.05	0.07	0.09	0.12
0.44	2.38	4.02	6.21	10.20	0.85	0.04	0.06	0.07	0.10
0.45	2.16	3.63	5.66	9.36	0.86	0.03	0.05	0.06	0.08
0.46	1.99	3.33	5.16	8.59	0.87	0.03	0.04	0.05	0.07
0.47	1.82	3.05	4.72	7.81	0.88	0.02	0.03	0.04	0.06
0.48	1.68	2.83	4.39	7.01	0.89	0.02	0.03	0.03	0.05
0.49	1.56	2.58	3.93	6.34	0.90	0.02	0.02	0.03	0.04
0.50	1.43	2.35	3.63	5.83					

Table 3: Critical values for FM- and D-OLS with one regressor for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	302.99	620.08	1152.52	2167.19	0.51	2.52	4.49	7.30	12.99
0.11	245.02	496.40	894.34	1715.62	0.52	2.32	4.09	6.66	11.80
0.12	196.82	402.92	712.97	1382.54	0.53	2.13	3.75	6.07	10.73
0.13	164.23	330.14	591.61	1146.75	0.54	1.96	3.43	5.56	9.78
0.14	137.23	277.87	491.53	934.71	0.55	1.80	3.14	5.07	8.87
0.15	115.72	233.37	409.02	792.55	0.56	1.65	2.88	4.64	7.99
0.16	98.29	196.79	351.71	668.45	0.57	1.52	2.63	4.23	7.29
0.17	83.70	165.87	300.72	558.44	0.58	1.41	2.42	3.85	6.76
0.18	72.46	146.37	259.22	487.06	0.59	1.29	2.21	3.49	6.14
0.19	62.87	125.51	224.91	425.70	0.60	1.20	2.02	3.19	5.52
0.20	55.43	109.41	194.48	376.54	0.61	1.11	1.85	2.91	4.99
0.21	48.49	95.90	170.77	329.97	0.62	1.02	1.68	2.66	4.51
0.22	42.26	85.06	149.11	285.62	0.63	0.93	1.54	2.40	4.09
0.23	37.45	74.43	130.40	251.64	0.64	0.86	1.40	2.20	3.69
0.24	33.02	65.68	115.60	220.84	0.65	0.78	1.28	2.00	3.31
0.25	29.12	58.07	102.39	193.36	0.66	0.72	1.17	1.81	2.99
0.26	25.95	51.78	91.31	173.54	0.67	0.66	1.06	1.63	2.68
0.27	23.38	45.78	80.30	152.46	0.68	0.60	0.97	1.47	2.41
0.28	20.98	40.84	70.66	132.03	0.69	0.55	0.88	1.32	2.17
0.29	18.83	36.51	63.09	118.50	0.70	0.50	0.80	1.20	1.95
0.30	16.93	32.60	56.22	106.18	0.71	0.45	0.73	1.09	1.75
0.31	15.23	29.10	50.63	94.31	0.72	0.42	0.66	0.97	1.55
0.32	13.79	26.48	45.73	84.67	0.73	0.38	0.60	0.88	1.38
0.33	12.58	24.09	40.96	76.99	0.74	0.34	0.54	0.79	1.24
0.34	11.46	21.59	36.93	68.61	0.75	0.31	0.49	0.71	1.11
0.35	10.42	19.61	33.46	61.75	0.76	0.28	0.44	0.64	0.99
0.36	9.49	17.89	30.42	55.92	0.77	0.26	0.40	0.57	0.89
0.37	8.65	16.26	27.72	50.94	0.78	0.23	0.36	0.51	0.80
0.38	7.85	14.86	25.22	45.39	0.79	0.21	0.32	0.46	0.71
0.39	7.19	13.53	22.80	41.43	0.80	0.19	0.29	0.41	0.62
0.40	6.54	12.24	20.84	37.80	0.81	0.17	0.25	0.36	0.55
0.41	6.02	11.03	18.97	34.03	0.82	0.15	0.22	0.32	0.48
0.42	5.52	10.11	17.20	31.12	0.83	0.13	0.20	0.28	0.42
0.43	5.04	9.19	15.51	28.02	0.84	0.12	0.18	0.25	0.37
0.44	4.60	8.29	14.09	25.46	0.85	0.10	0.15	0.22	0.32
0.45	4.20	7.59	12.78	23.09	0.86	0.09	0.14	0.19	0.27
0.46	3.86	6.91	11.53	20.97	0.87	0.08	0.12	0.16	0.24
0.47	3.54	6.32	10.54	19.07	0.88	0.07	0.10	0.14	0.20
0.48	3.25	5.83	9.54	17.15	0.89	0.06	0.09	0.12	0.17
0.49	2.99	5.35	8.67	15.61	0.90	0.05	0.07	0.10	0.15
0.50	2.73	4.91	7.99	14.31					

Table 4: Critical values for IM-OLS with one regressor for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	3183.78	5229.90	8173.35	13345.41	0.51	4.47	6.91	9.87	15.05
0.11	2320.39	3809.84	5927.89	9753.53	0.52	3.97	6.16	8.82	13.30
0.12	1742.79	2909.00	4481.44	7356.00	0.53	3.54	5.47	7.94	11.78
0.13	1311.08	2185.45	3412.19	5525.22	0.54	3.18	4.88	7.00	10.32
0.14	1012.52	1678.29	2639.09	4434.55	0.55	2.83	4.33	6.18	9.41
0.15	809.22	1323.37	2052.22	3400.52	0.56	2.52	3.85	5.51	8.40
0.16	645.81	1058.46	1618.49	2672.58	0.57	2.25	3.44	4.91	7.40
0.17	517.02	847.52	1298.28	2125.79	0.58	2.00	3.06	4.35	6.55
0.18	425.16	690.29	1057.62	1702.40	0.59	1.79	2.73	3.85	5.79
0.19	346.93	568.36	859.21	1379.05	0.60	1.61	2.44	3.41	5.09
0.20	286.49	472.45	717.40	1159.44	0.61	1.43	2.17	3.03	4.52
0.21	240.90	391.67	595.80	965.35	0.62	1.27	1.91	2.70	3.98
0.22	201.33	328.52	500.40	797.16	0.63	1.13	1.70	2.39	3.53
0.23	168.21	275.55	421.62	673.57	0.64	1.00	1.52	2.12	3.14
0.24	143.28	231.76	354.03	569.02	0.65	0.90	1.35	1.89	2.78
0.25	122.20	196.12	298.44	479.46	0.66	0.79	1.19	1.68	2.46
0.26	103.52	169.04	255.19	412.50	0.67	0.71	1.05	1.47	2.18
0.27	89.23	144.61	217.17	348.63	0.68	0.62	0.94	1.31	1.92
0.28	76.72	125.40	188.78	298.66	0.69	0.56	0.83	1.16	1.69
0.29	65.90	107.36	162.83	258.57	0.70	0.49	0.73	1.02	1.48
0.30	57.44	93.26	141.68	225.12	0.71	0.44	0.65	0.90	1.29
0.31	49.96	80.63	123.04	193.88	0.72	0.39	0.57	0.78	1.13
0.32	43.61	69.61	106.02	168.30	0.73	0.34	0.50	0.68	0.99
0.33	38.10	60.68	91.81	148.48	0.74	0.30	0.44	0.60	0.85
0.34	33.33	53.03	80.40	129.29	0.75	0.26	0.38	0.53	0.75
0.35	29.19	46.35	70.71	112.06	0.76	0.23	0.33	0.45	0.65
0.36	25.58	40.60	61.03	97.30	0.77	0.20	0.29	0.39	0.55
0.37	22.67	36.19	53.83	87.03	0.78	0.17	0.25	0.34	0.47
0.38	20.05	32.19	47.63	77.57	0.79	0.15	0.22	0.29	0.40
0.39	17.75	28.48	42.75	67.71	0.80	0.13	0.18	0.25	0.35
0.40	15.59	24.94	37.43	60.27	0.81	0.11	0.16	0.21	0.29
0.41	13.89	22.10	33.09	52.29	0.82	0.09	0.13	0.18	0.25
0.42	12.42	19.70	29.20	46.20	0.83	0.08	0.11	0.15	0.21
0.43	11.05	17.26	25.76	41.03	0.84	0.07	0.09	0.13	0.17
0.44	9.82	15.50	22.90	36.31	0.85	0.06	0.08	0.11	0.14
0.45	8.74	13.78	20.47	32.30	0.86	0.05	0.07	0.09	0.12
0.46	7.84	12.22	18.13	28.62	0.87	0.04	0.05	0.07	0.10
0.47	6.96	10.85	16.07	25.00	0.88	0.03	0.04	0.06	0.08
0.48	6.24	9.70	14.22	22.04	0.89	0.02	0.03	0.05	0.06
0.49	5.54	8.63	12.67	19.78	0.90	0.02	0.03	0.04	0.05
0.50	5.01	7.74	11.34	17.27					

Table 5: Critical values for FM- and D-OLS with one regressor for the intercept and trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	5632.45	9908.99	16219.09	28763.00	0.51	7.93	13.04	20.14	33.78
0.11	4132.60	7281.00	11811.11	21166.32	0.52	7.10	11.59	17.80	29.53
0.12	3091.39	5434.09	8842.26	15505.62	0.53	6.33	10.37	15.82	26.43
0.13	2340.41	4176.42	6714.68	11561.93	0.54	5.66	9.30	14.15	23.47
0.14	1811.69	3225.57	5202.98	9098.73	0.55	5.09	8.28	12.68	20.83
0.15	1417.27	2521.66	4083.71	7344.18	0.56	4.55	7.39	11.33	18.18
0.16	1125.14	2003.04	3270.00	5751.32	0.57	4.08	6.64	10.04	16.21
0.17	902.14	1601.21	2631.74	4543.92	0.58	3.65	5.91	8.92	14.36
0.18	738.25	1294.98	2097.49	3700.03	0.59	3.26	5.28	7.98	12.77
0.19	604.81	1054.85	1731.58	3043.20	0.60	2.91	4.71	7.11	11.41
0.20	503.17	886.62	1435.66	2503.70	0.61	2.61	4.19	6.34	10.14
0.21	417.13	735.43	1202.80	2070.72	0.62	2.34	3.74	5.64	9.00
0.22	350.70	617.81	1004.13	1736.59	0.63	2.09	3.33	4.97	7.88
0.23	298.17	518.87	845.78	1471.74	0.64	1.87	2.96	4.38	6.99
0.24	253.22	440.15	711.55	1238.48	0.65	1.66	2.63	3.87	6.16
0.25	213.45	370.69	604.20	1034.17	0.66	1.48	2.35	3.46	5.48
0.26	182.30	317.66	511.64	894.24	0.67	1.32	2.09	3.05	4.82
0.27	155.88	272.29	444.21	766.92	0.68	1.17	1.85	2.70	4.26
0.28	134.48	234.19	379.58	641.08	0.69	1.04	1.64	2.39	3.76
0.29	116.49	202.09	325.23	554.62	0.70	0.93	1.45	2.11	3.28
0.30	100.47	173.70	278.44	479.00	0.71	0.82	1.28	1.86	2.89
0.31	87.44	150.82	242.44	418.75	0.72	0.73	1.14	1.65	2.55
0.32	76.13	131.32	210.83	362.66	0.73	0.65	1.00	1.44	2.22
0.33	66.32	113.96	183.14	318.38	0.74	0.58	0.88	1.27	1.93
0.34	58.80	99.70	159.53	275.65	0.75	0.51	0.77	1.12	1.68
0.35	51.48	87.20	139.31	243.39	0.76	0.45	0.68	0.97	1.46
0.36	45.11	76.37	123.33	211.67	0.77	0.40	0.60	0.85	1.27
0.37	39.86	67.68	108.79	185.56	0.78	0.35	0.52	0.74	1.09
0.38	35.34	59.84	95.97	166.09	0.79	0.30	0.45	0.64	0.93
0.39	31.35	53.15	85.54	147.08	0.80	0.27	0.39	0.55	0.81
0.40	27.77	47.27	75.95	127.78	0.81	0.23	0.34	0.48	0.69
0.41	24.70	41.90	66.09	112.05	0.82	0.20	0.30	0.41	0.59
0.42	21.79	37.11	58.47	98.82	0.83	0.17	0.25	0.35	0.49
0.43	19.46	32.89	51.56	87.26	0.84	0.15	0.22	0.30	0.42
0.44	17.36	29.09	45.80	78.67	0.85	0.13	0.19	0.25	0.36
0.45	15.41	25.73	40.82	69.87	0.86	0.11	0.16	0.21	0.30
0.46	13.70	22.93	36.57	62.29	0.87	0.09	0.13	0.18	0.25
0.47	12.24	20.54	32.48	55.35	0.88	0.08	0.11	0.15	0.21
0.48	10.97	18.26	28.57	48.87	0.89	0.06	0.09	0.12	0.17
0.49	9.84	16.30	25.33	43.43	0.90	0.05	0.08	0.10	0.14
0.50	8.81	14.50	22.64	38.39					

Table 6: Critical values for IM-OLS with one regressor for the intercept and trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	425.71	752.41	1210.70	2057.19	0.51	2.57	4.22	6.28	10.02
0.11	343.04	610.05	963.06	1640.70	0.52	2.35	3.80	5.69	9.01
0.12	279.36	489.78	786.99	1309.35	0.53	2.13	3.45	5.18	8.13
0.13	229.79	406.09	637.91	1060.49	0.54	1.95	3.11	4.67	7.29
0.14	190.66	336.55	535.24	898.65	0.55	1.75	2.84	4.24	6.64
0.15	158.86	280.88	449.01	745.46	0.56	1.60	2.54	3.74	5.83
0.16	133.79	240.20	382.60	620.53	0.57	1.46	2.30	3.36	5.24
0.17	114.22	202.96	324.00	526.85	0.58	1.31	2.07	3.05	4.81
0.18	97.56	171.55	275.60	455.63	0.59	1.18	1.87	2.76	4.25
0.19	84.46	147.71	235.97	392.45	0.60	1.08	1.71	2.45	3.78
0.20	73.58	128.98	204.17	340.34	0.61	0.97	1.53	2.20	3.38
0.21	64.10	111.87	177.16	297.98	0.62	0.88	1.37	2.00	3.03
0.22	55.41	97.85	153.37	255.84	0.63	0.79	1.24	1.79	2.72
0.23	48.91	85.81	134.39	221.07	0.64	0.71	1.11	1.60	2.41
0.24	43.17	75.23	117.87	193.17	0.65	0.65	0.98	1.43	2.16
0.25	38.14	66.38	103.62	169.64	0.66	0.58	0.90	1.27	1.91
0.26	33.87	58.45	92.09	150.99	0.67	0.52	0.81	1.15	1.71
0.27	30.12	51.68	81.82	133.42	0.68	0.47	0.72	1.04	1.52
0.28	26.97	46.34	71.96	119.87	0.69	0.42	0.64	0.91	1.34
0.29	23.91	41.18	64.72	106.29	0.70	0.37	0.58	0.81	1.19
0.30	21.36	36.86	57.89	93.66	0.71	0.33	0.51	0.72	1.05
0.31	19.06	33.07	51.82	83.42	0.72	0.30	0.46	0.64	0.94
0.32	17.11	29.62	46.13	75.51	0.73	0.27	0.40	0.56	0.82
0.33	15.43	26.24	41.01	66.67	0.74	0.24	0.35	0.49	0.71
0.34	13.82	23.57	36.87	59.98	0.75	0.21	0.31	0.43	0.63
0.35	12.50	21.21	33.06	53.42	0.76	0.18	0.27	0.38	0.54
0.36	11.26	19.00	29.33	48.96	0.77	0.16	0.24	0.33	0.47
0.37	10.23	17.22	26.66	43.38	0.78	0.14	0.21	0.29	0.41
0.38	9.23	15.61	23.94	38.47	0.79	0.12	0.18	0.25	0.36
0.39	8.37	14.14	21.62	34.55	0.80	0.11	0.16	0.22	0.30
0.40	7.58	12.71	19.35	31.61	0.81	0.09	0.14	0.19	0.26
0.41	6.83	11.45	17.45	28.06	0.82	0.08	0.12	0.16	0.22
0.42	6.17	10.35	15.87	25.47	0.83	0.07	0.10	0.13	0.18
0.43	5.61	9.42	14.37	22.91	0.84	0.06	0.08	0.11	0.16
0.44	5.13	8.54	12.83	20.47	0.85	0.05	0.07	0.09	0.13
0.45	4.62	7.69	11.65	18.48	0.86	0.04	0.06	0.08	0.11
0.46	4.22	7.02	10.53	16.63	0.87	0.03	0.05	0.06	0.09
0.47	3.79	6.29	9.53	14.78	0.88	0.03	0.04	0.05	0.07
0.48	3.46	5.71	8.62	13.33	0.89	0.02	0.03	0.04	0.06
0.49	3.12	5.14	7.72	12.12	0.90	0.02	0.03	0.03	0.05
0.50	2.86	4.65	6.94	11.09					

Table 7: Critical values for FM- and D-OLS with two regressors for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	867.12	1610.96	2726.92	4825.15	0.51	5.14	8.98	14.02	23.75
0.11	692.54	1290.06	2202.88	3905.77	0.52	4.67	8.15	12.74	21.49
0.12	561.33	1046.92	1748.13	3177.61	0.53	4.24	7.33	11.57	19.25
0.13	462.13	860.57	1449.39	2559.21	0.54	3.83	6.64	10.58	17.49
0.14	384.57	711.54	1184.01	2074.37	0.55	3.48	6.00	9.53	15.74
0.15	319.08	594.74	980.50	1738.88	0.56	3.15	5.41	8.61	14.25
0.16	270.31	499.08	821.42	1496.08	0.57	2.85	4.87	7.66	12.86
0.17	230.48	425.67	699.01	1265.20	0.58	2.59	4.40	6.86	11.50
0.18	196.92	364.69	608.49	1058.51	0.59	2.34	3.96	6.18	10.42
0.19	171.13	313.65	518.28	911.47	0.60	2.12	3.57	5.55	9.43
0.20	149.04	270.68	451.00	781.85	0.61	1.92	3.20	5.00	8.40
0.21	129.03	236.19	393.28	669.80	0.62	1.74	2.90	4.46	7.41
0.22	111.57	206.50	336.61	579.98	0.63	1.57	2.63	4.02	6.63
0.23	98.12	179.42	295.03	515.87	0.64	1.41	2.36	3.58	5.92
0.24	86.21	158.40	261.86	452.33	0.65	1.28	2.12	3.23	5.35
0.25	75.77	140.76	232.23	394.48	0.66	1.16	1.90	2.88	4.77
0.26	67.71	123.62	204.72	351.16	0.67	1.04	1.71	2.60	4.21
0.27	60.45	109.20	181.60	317.16	0.68	0.94	1.54	2.31	3.74
0.28	53.51	97.17	160.17	277.48	0.69	0.85	1.37	2.06	3.30
0.29	47.63	86.76	142.69	247.81	0.70	0.76	1.22	1.82	2.93
0.30	42.62	77.17	126.03	221.24	0.71	0.68	1.09	1.63	2.59
0.31	38.13	68.60	115.35	199.01	0.72	0.61	0.97	1.46	2.31
0.32	34.32	62.13	103.14	177.09	0.73	0.54	0.87	1.29	2.06
0.33	30.83	55.90	91.90	158.18	0.74	0.49	0.78	1.15	1.79
0.34	27.65	49.74	81.88	142.36	0.75	0.43	0.69	1.01	1.55
0.35	24.74	44.86	74.01	127.35	0.76	0.39	0.60	0.88	1.36
0.36	22.43	40.27	65.96	115.27	0.77	0.34	0.53	0.77	1.19
0.37	20.21	36.15	59.08	103.48	0.78	0.30	0.47	0.68	1.02
0.38	18.14	32.67	53.50	92.90	0.79	0.27	0.41	0.59	0.89
0.39	16.47	29.40	48.18	83.39	0.80	0.23	0.36	0.51	0.77
0.40	14.91	26.73	42.90	75.49	0.81	0.20	0.31	0.44	0.67
0.41	13.53	24.18	38.55	66.06	0.82	0.18	0.27	0.38	0.57
0.42	12.27	21.71	34.78	58.98	0.83	0.15	0.23	0.33	0.48
0.43	11.09	19.63	31.35	52.76	0.84	0.13	0.20	0.28	0.41
0.44	10.09	17.77	28.19	47.33	0.85	0.11	0.17	0.24	0.35
0.45	9.21	16.04	25.48	43.18	0.86	0.10	0.15	0.20	0.30
0.46	8.34	14.63	22.99	39.16	0.87	0.08	0.12	0.17	0.25
0.47	7.57	13.17	21.02	35.54	0.88	0.07	0.10	0.14	0.21
0.48	6.86	11.94	19.01	31.94	0.89	0.06	0.09	0.12	0.17
0.49	6.22	10.83	17.18	29.03	0.90	0.05	0.07	0.10	0.14
0.50	5.68	9.94	15.66	26.06					

Table 8: Critical values for IM-OLS with two regressors for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	6183.57	10121.63	15259.27	24189.16	0.51	7.25	11.63	16.93	26.29
0.11	4528.41	7431.06	11053.01	17557.08	0.52	6.42	10.21	15.02	22.95
0.12	3341.00	5522.08	8283.06	12978.83	0.53	5.74	9.07	13.23	20.28
0.13	2534.03	4184.15	6266.80	9854.03	0.54	5.13	8.08	11.89	17.93
0.14	1947.99	3196.54	4795.56	7447.72	0.55	4.53	7.18	10.46	15.94
0.15	1544.37	2521.22	3781.67	5877.13	0.56	4.00	6.28	9.19	14.02
0.16	1220.60	1984.04	2998.65	4652.60	0.57	3.57	5.58	8.09	12.16
0.17	974.44	1593.56	2408.14	3674.27	0.58	3.16	4.95	7.16	10.67
0.18	797.31	1303.87	1935.43	3014.69	0.59	2.81	4.33	6.26	9.46
0.19	648.62	1075.10	1594.54	2482.28	0.60	2.49	3.86	5.57	8.45
0.20	532.56	870.61	1308.84	2067.52	0.61	2.20	3.42	4.91	7.26
0.21	444.41	722.67	1088.20	1697.17	0.62	1.95	3.02	4.31	6.39
0.22	371.23	599.68	903.07	1423.13	0.63	1.71	2.66	3.78	5.65
0.23	308.51	504.09	759.19	1177.84	0.64	1.51	2.33	3.30	4.93
0.24	261.34	424.92	639.72	998.83	0.65	1.33	2.04	2.90	4.27
0.25	222.70	362.12	543.22	842.17	0.66	1.17	1.79	2.52	3.71
0.26	188.72	306.91	459.41	726.06	0.67	1.04	1.57	2.21	3.29
0.27	161.64	262.57	396.56	619.25	0.68	0.91	1.38	1.95	2.86
0.28	139.40	227.76	340.36	531.72	0.69	0.80	1.21	1.71	2.47
0.29	120.27	194.53	292.15	462.88	0.70	0.70	1.05	1.48	2.18
0.30	103.32	168.96	254.85	401.25	0.71	0.61	0.92	1.29	1.88
0.31	89.97	146.00	218.34	345.34	0.72	0.54	0.81	1.13	1.63
0.32	77.67	126.83	190.27	294.67	0.73	0.47	0.70	0.97	1.41
0.33	67.10	109.33	161.69	251.48	0.74	0.41	0.61	0.83	1.20
0.34	58.60	94.92	141.54	217.65	0.75	0.35	0.52	0.73	1.03
0.35	51.71	82.66	123.51	190.67	0.76	0.30	0.45	0.63	0.90
0.36	45.50	72.82	108.40	170.28	0.77	0.26	0.39	0.54	0.76
0.37	40.22	64.52	95.00	149.35	0.78	0.23	0.33	0.46	0.65
0.38	35.22	56.95	83.70	131.13	0.79	0.19	0.29	0.39	0.55
0.39	31.23	49.54	73.08	112.56	0.80	0.16	0.24	0.33	0.47
0.40	27.60	44.15	64.72	99.48	0.81	0.14	0.21	0.28	0.39
0.41	24.24	38.84	56.47	87.65	0.82	0.12	0.17	0.23	0.33
0.42	21.34	34.28	50.03	77.81	0.83	0.10	0.14	0.19	0.27
0.43	18.87	30.28	44.26	68.61	0.84	0.08	0.12	0.16	0.22
0.44	16.74	26.60	39.14	60.20	0.85	0.07	0.10	0.13	0.18
0.45	14.82	23.69	34.75	53.66	0.86	0.06	0.08	0.11	0.15
0.46	13.34	21.11	30.75	47.52	0.87	0.05	0.07	0.09	0.12
0.47	11.78	18.49	27.31	41.42	0.88	0.04	0.05	0.07	0.10
0.48	10.49	16.53	24.22	37.45	0.89	0.03	0.04	0.05	0.07
0.49	9.28	14.72	21.58	33.05	0.90	0.02	0.03	0.04	0.06
0.50	8.22	13.14	19.32	29.59					

Table 9: Critical values for FM- and D-OLS with two regressors for the intercept and trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	12006.99	20782.52	32267.34	53674.26	0.51	13.93	23.32	36.14	59.16
0.11	8705.92	14901.62	23232.17	39205.37	0.52	12.33	20.69	31.91	52.55
0.12	6461.49	11066.67	17298.36	29092.60	0.53	10.99	18.28	28.27	45.91
0.13	4847.07	8360.04	13240.50	22135.28	0.54	9.76	16.23	25.02	40.38
0.14	3743.22	6301.79	9930.39	16542.34	0.55	8.67	14.35	22.05	35.04
0.15	2912.02	5068.69	7887.18	12944.01	0.56	7.71	12.80	19.53	30.90
0.16	2311.62	3953.93	6228.15	10325.79	0.57	6.87	11.29	17.21	27.46
0.17	1848.95	3147.42	4975.74	8230.25	0.58	6.08	10.02	15.27	24.38
0.18	1507.82	2584.82	4060.59	6700.20	0.59	5.38	8.83	13.45	21.54
0.19	1236.35	2122.63	3341.44	5521.36	0.60	4.74	7.74	11.84	19.08
0.20	1019.39	1745.21	2759.56	4582.01	0.61	4.22	6.86	10.45	16.73
0.21	837.74	1458.18	2281.53	3851.10	0.62	3.76	6.06	9.18	14.60
0.22	701.47	1213.48	1906.34	3169.97	0.63	3.33	5.37	8.05	12.71
0.23	589.78	1017.32	1596.58	2672.51	0.64	2.96	4.72	7.08	11.10
0.24	500.45	860.28	1344.85	2232.60	0.65	2.62	4.16	6.23	9.79
0.25	422.78	717.41	1135.59	1888.01	0.66	2.30	3.68	5.44	8.62
0.26	360.71	612.16	960.53	1597.46	0.67	2.02	3.27	4.75	7.56
0.27	308.20	527.36	813.63	1379.33	0.68	1.78	2.86	4.22	6.56
0.28	265.00	457.38	709.37	1205.09	0.69	1.57	2.50	3.68	5.67
0.29	229.94	394.62	622.82	1034.72	0.70	1.38	2.19	3.22	4.91
0.30	200.56	341.82	537.10	878.12	0.71	1.22	1.90	2.82	4.29
0.31	173.01	297.17	469.99	778.77	0.72	1.06	1.66	2.44	3.70
0.32	150.76	258.65	405.17	679.96	0.73	0.93	1.45	2.10	3.25
0.33	131.02	222.93	347.57	590.29	0.74	0.81	1.27	1.83	2.81
0.34	113.46	194.10	301.82	500.66	0.75	0.70	1.10	1.59	2.44
0.35	99.13	168.24	264.57	434.07	0.76	0.61	0.95	1.37	2.08
0.36	86.42	147.54	230.18	380.56	0.77	0.53	0.82	1.17	1.78
0.37	75.65	129.08	201.14	326.90	0.78	0.46	0.70	1.00	1.51
0.38	66.84	113.73	175.30	283.85	0.79	0.40	0.60	0.86	1.29
0.39	58.87	100.43	154.74	250.59	0.80	0.34	0.52	0.73	1.10
0.40	52.01	88.10	136.86	222.62	0.81	0.29	0.44	0.62	0.93
0.41	46.15	77.88	119.92	198.17	0.82	0.25	0.38	0.53	0.78
0.42	40.78	68.09	106.94	172.90	0.83	0.21	0.32	0.44	0.65
0.43	36.13	60.59	94.46	155.94	0.84	0.18	0.26	0.37	0.54
0.44	32.02	53.95	83.40	135.61	0.85	0.15	0.22	0.31	0.45
0.45	28.46	48.22	74.42	119.66	0.86	0.12	0.18	0.26	0.37
0.46	25.23	42.67	65.68	106.20	0.87	0.10	0.15	0.21	0.30
0.47	22.34	37.50	57.83	94.56	0.88	0.08	0.12	0.17	0.24
0.48	19.83	33.12	51.36	84.07	0.89	0.07	0.10	0.14	0.19
0.49	17.57	29.55	45.72	75.61	0.90	0.05	0.08	0.11	0.15
0.50	15.59	26.26	40.61	66.50					

Table 10: Critical values for IM-OLS with two regressors for the intercept and trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	805.81	1365.45	2090.39	3348.35	0.51	4.29	6.90	10.09	15.68
0.11	644.83	1091.09	1683.04	2679.59	0.52	3.85	6.17	9.03	14.10
0.12	521.02	887.72	1356.76	2179.12	0.53	3.47	5.55	8.08	12.58
0.13	427.68	722.47	1119.66	1810.75	0.54	3.12	4.98	7.35	11.28
0.14	353.52	595.43	916.94	1482.28	0.55	2.81	4.48	6.64	10.10
0.15	296.54	499.70	765.69	1219.20	0.56	2.57	4.06	5.92	9.08
0.16	247.95	424.61	643.34	1012.75	0.57	2.29	3.64	5.31	8.09
0.17	212.25	357.75	553.31	878.86	0.58	2.07	3.24	4.70	7.11
0.18	180.05	305.60	470.91	754.65	0.59	1.85	2.91	4.22	6.53
0.19	155.36	260.79	397.51	637.04	0.60	1.68	2.63	3.81	5.79
0.20	134.68	225.26	347.03	557.45	0.61	1.49	2.37	3.42	5.26
0.21	117.58	197.26	302.80	488.58	0.62	1.35	2.14	3.06	4.62
0.22	103.13	172.71	265.39	426.26	0.63	1.20	1.89	2.73	4.12
0.23	90.51	152.44	231.62	371.22	0.64	1.08	1.69	2.44	3.64
0.24	80.06	134.32	204.32	318.61	0.65	0.97	1.50	2.15	3.20
0.25	70.45	118.10	179.83	283.01	0.66	0.86	1.33	1.92	2.88
0.26	61.76	103.42	158.79	251.56	0.67	0.76	1.19	1.71	2.57
0.27	54.68	91.66	139.67	221.31	0.68	0.68	1.05	1.51	2.24
0.28	48.77	81.21	124.06	196.44	0.69	0.60	0.93	1.34	1.99
0.29	43.38	72.92	109.71	174.26	0.70	0.53	0.83	1.18	1.74
0.30	38.30	64.80	97.49	154.08	0.71	0.47	0.72	1.03	1.53
0.31	34.32	57.53	87.90	135.36	0.72	0.42	0.64	0.91	1.34
0.32	30.71	51.56	78.52	120.04	0.73	0.37	0.57	0.80	1.18
0.33	27.56	46.38	69.69	107.98	0.74	0.33	0.49	0.70	1.00
0.34	24.65	41.54	62.77	98.04	0.75	0.29	0.43	0.60	0.87
0.35	22.23	37.09	56.85	88.29	0.76	0.25	0.38	0.52	0.75
0.36	19.94	33.35	50.66	77.44	0.77	0.22	0.32	0.45	0.65
0.37	17.83	29.88	45.00	69.61	0.78	0.19	0.28	0.39	0.55
0.38	16.13	26.61	40.40	61.45	0.79	0.16	0.24	0.33	0.47
0.39	14.45	23.96	36.29	55.94	0.80	0.14	0.20	0.28	0.40
0.40	13.07	21.59	32.51	49.53	0.81	0.12	0.18	0.24	0.34
0.41	11.78	19.32	28.36	44.80	0.82	0.10	0.15	0.21	0.29
0.42	10.58	17.49	25.70	39.65	0.83	0.09	0.13	0.17	0.24
0.43	9.62	15.70	23.15	35.96	0.84	0.07	0.11	0.14	0.20
0.44	8.62	14.13	20.84	32.48	0.85	0.06	0.09	0.12	0.16
0.45	7.80	12.73	18.82	29.50	0.86	0.05	0.07	0.10	0.13
0.46	7.08	11.43	16.99	26.29	0.87	0.04	0.06	0.08	0.11
0.47	6.41	10.37	15.54	23.61	0.88	0.03	0.05	0.06	0.08
0.48	5.81	9.45	13.86	21.51	0.89	0.03	0.04	0.05	0.07
0.49	5.25	8.51	12.51	19.55	0.90	0.02	0.03	0.04	0.05
0.50	4.75	7.71	11.28	17.33					

Table 11: Critical values for FM- and D-OLS with three regressors for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	1638.30	2866.79	4594.81	7845.45	0.51	8.50	14.29	22.26	36.18
0.11	1310.28	2300.25	3739.04	6377.04	0.52	7.67	12.87	19.94	32.53
0.12	1060.75	1862.37	2990.62	5183.62	0.53	6.92	11.67	17.83	29.32
0.13	864.11	1526.73	2437.26	4190.10	0.54	6.28	10.41	16.06	26.13
0.14	720.01	1262.20	2010.97	3449.51	0.55	5.64	9.31	14.38	23.20
0.15	603.15	1064.19	1687.46	2852.28	0.56	5.09	8.43	12.91	20.78
0.16	502.32	886.52	1413.71	2391.69	0.57	4.60	7.62	11.53	18.40
0.17	426.94	747.24	1199.71	2070.94	0.58	4.17	6.87	10.32	16.29
0.18	364.90	638.49	1012.02	1732.98	0.59	3.78	6.18	9.22	14.37
0.19	312.15	551.79	876.00	1477.31	0.60	3.37	5.56	8.33	12.73
0.20	269.16	477.67	763.59	1294.17	0.61	3.03	4.98	7.48	11.45
0.21	235.15	416.61	664.17	1122.11	0.62	2.71	4.47	6.67	10.35
0.22	205.96	367.40	586.55	964.99	0.63	2.42	3.99	5.95	9.27
0.23	180.95	319.14	508.23	834.18	0.64	2.17	3.56	5.31	8.31
0.24	157.25	277.92	444.00	738.02	0.65	1.94	3.17	4.76	7.51
0.25	139.89	242.65	390.23	636.69	0.66	1.73	2.82	4.22	6.71
0.26	123.46	214.57	341.01	567.66	0.67	1.54	2.50	3.74	5.96
0.27	108.60	188.35	301.27	504.40	0.68	1.37	2.24	3.33	5.25
0.28	96.84	168.54	266.55	449.21	0.69	1.22	1.98	2.96	4.62
0.29	86.34	149.53	236.26	400.93	0.70	1.08	1.75	2.59	4.07
0.30	77.40	133.08	212.64	364.34	0.71	0.96	1.54	2.29	3.59
0.31	69.28	120.51	189.78	323.80	0.72	0.85	1.35	2.01	3.15
0.32	61.67	108.15	169.92	285.35	0.73	0.75	1.19	1.75	2.70
0.33	55.44	95.26	150.14	255.11	0.74	0.66	1.05	1.53	2.32
0.34	49.89	85.51	134.61	225.62	0.75	0.58	0.92	1.34	2.02
0.35	44.62	77.24	120.07	198.32	0.76	0.51	0.80	1.16	1.76
0.36	39.78	69.00	107.98	180.52	0.77	0.45	0.70	1.01	1.53
0.37	35.92	61.65	98.05	162.84	0.78	0.39	0.61	0.88	1.31
0.38	32.25	55.66	87.74	144.58	0.79	0.34	0.53	0.75	1.12
0.39	29.08	49.89	78.46	130.55	0.80	0.29	0.45	0.64	0.96
0.40	26.03	45.07	70.19	117.70	0.81	0.25	0.39	0.55	0.81
0.41	23.33	40.55	63.02	103.53	0.82	0.22	0.33	0.46	0.68
0.42	21.14	36.38	56.80	94.36	0.83	0.18	0.28	0.39	0.57
0.43	18.97	32.77	50.92	85.52	0.84	0.16	0.24	0.33	0.48
0.44	17.22	29.49	46.04	76.91	0.85	0.13	0.20	0.27	0.40
0.45	15.57	26.52	41.74	69.70	0.86	0.11	0.17	0.23	0.33
0.46	13.93	24.01	37.53	62.42	0.87	0.09	0.14	0.19	0.27
0.47	12.70	21.75	33.81	55.41	0.88	0.08	0.11	0.16	0.22
0.48	11.54	19.67	30.45	50.21	0.89	0.06	0.09	0.13	0.18
0.49	10.50	17.64	27.48	44.59	0.90	0.05	0.07	0.10	0.14
0.50	9.42	15.90	24.67	39.83					

Table 12: Critical values for IM-OLS with three regressors for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	10067.93	16135.49	23848.88	36442.16	0.51	10.64	16.81	24.47	36.75
0.11	7228.22	11734.60	17464.88	26388.97	0.52	9.40	14.80	21.59	32.44
0.12	5368.67	8680.92	12938.62	20055.16	0.53	8.24	13.07	18.91	28.84
0.13	4021.04	6507.53	9816.86	14941.82	0.54	7.38	11.59	16.78	25.20
0.14	3095.62	4970.12	7454.85	11605.67	0.55	6.53	10.32	14.86	22.01
0.15	2427.35	3909.95	5838.45	9034.36	0.56	5.78	9.07	13.04	19.20
0.16	1928.08	3089.89	4606.20	7080.47	0.57	5.10	7.91	11.42	16.77
0.17	1549.16	2489.18	3711.88	5707.76	0.58	4.51	7.05	10.06	14.90
0.18	1255.54	2025.62	2995.24	4566.54	0.59	4.01	6.25	8.93	13.46
0.19	1026.44	1646.48	2446.07	3755.84	0.60	3.55	5.51	7.96	11.73
0.20	849.81	1359.85	2007.90	3082.60	0.61	3.14	4.86	6.96	10.19
0.21	701.19	1131.10	1659.96	2534.08	0.62	2.75	4.28	6.12	9.06
0.22	581.34	936.83	1378.28	2111.65	0.63	2.42	3.74	5.35	7.89
0.23	487.52	788.53	1173.78	1775.29	0.64	2.13	3.26	4.60	7.00
0.24	411.69	662.68	978.22	1490.40	0.65	1.87	2.85	4.07	6.05
0.25	348.10	562.31	826.84	1256.49	0.66	1.64	2.52	3.59	5.40
0.26	298.62	482.09	707.91	1089.30	0.67	1.43	2.21	3.12	4.64
0.27	254.32	408.56	606.23	954.64	0.68	1.25	1.92	2.75	4.10
0.28	218.47	350.94	519.89	812.69	0.69	1.10	1.69	2.40	3.52
0.29	187.97	301.68	444.21	690.19	0.70	0.97	1.46	2.09	3.04
0.30	161.96	261.01	381.69	579.21	0.71	0.84	1.27	1.78	2.63
0.31	140.36	224.93	328.89	506.24	0.72	0.73	1.10	1.56	2.30
0.32	122.23	193.79	282.31	436.87	0.73	0.63	0.96	1.35	1.96
0.33	105.69	167.78	249.98	374.26	0.74	0.55	0.83	1.15	1.66
0.34	92.48	148.00	217.37	330.37	0.75	0.47	0.71	1.00	1.43
0.35	80.94	129.08	190.17	286.04	0.76	0.40	0.61	0.84	1.22
0.36	70.64	113.10	166.59	252.61	0.77	0.34	0.51	0.71	1.02
0.37	61.40	99.04	145.53	220.12	0.78	0.29	0.43	0.60	0.84
0.38	54.28	86.84	126.63	193.12	0.79	0.25	0.36	0.50	0.71
0.39	47.48	75.59	111.24	169.12	0.80	0.21	0.31	0.42	0.60
0.40	41.47	66.59	98.96	149.47	0.81	0.17	0.26	0.35	0.50
0.41	36.80	58.74	86.39	129.87	0.82	0.15	0.22	0.29	0.40
0.42	32.37	51.45	75.00	115.61	0.83	0.12	0.18	0.24	0.33
0.43	28.58	45.67	65.83	100.84	0.84	0.10	0.15	0.20	0.27
0.44	25.33	40.51	58.65	89.06	0.85	0.08	0.12	0.16	0.22
0.45	22.38	35.44	51.65	77.06	0.86	0.07	0.10	0.13	0.17
0.46	19.78	31.33	45.39	68.31	0.87	0.05	0.08	0.10	0.14
0.47	17.45	27.95	40.44	60.25	0.88	0.04	0.06	0.08	0.11
0.48	15.47	24.64	35.64	53.27	0.89	0.03	0.05	0.06	0.08
0.49	13.68	21.72	31.32	46.62	0.90	0.03	0.04	0.05	0.06
0.50	12.14	19.19	27.45	41.47					

Table 13: Critical values for FM- and D-OLS with three regressors for the intercept and trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	19918.69	33486.88	51716.00	84344.08	0.51	21.05	35.05	52.67	83.63
0.11	14467.23	24066.03	37064.49	60223.30	0.52	18.66	30.58	46.26	73.95
0.12	10599.37	17695.04	27484.82	43542.25	0.53	16.54	26.99	40.84	64.40
0.13	7993.82	13330.24	20812.92	33783.17	0.54	14.66	23.70	35.91	56.32
0.14	6170.53	10382.00	15851.59	25112.72	0.55	12.90	20.94	31.42	49.86
0.15	4819.26	8047.75	12230.87	19599.88	0.56	11.42	18.58	27.59	43.38
0.16	3800.56	6410.34	9792.88	15790.44	0.57	10.11	16.40	24.32	37.89
0.17	3057.22	5138.04	7933.15	12602.45	0.58	8.94	14.46	21.36	33.28
0.18	2464.31	4144.56	6310.09	10253.24	0.59	7.86	12.75	18.81	29.41
0.19	2022.58	3363.18	5214.12	8413.32	0.60	6.97	11.34	16.59	25.60
0.20	1666.19	2768.93	4257.47	6845.03	0.61	6.21	9.95	14.65	22.56
0.21	1388.31	2304.33	3508.77	5605.31	0.62	5.49	8.75	13.07	20.11
0.22	1152.20	1916.98	2911.75	4686.64	0.63	4.79	7.74	11.53	17.79
0.23	966.75	1608.58	2446.35	3887.24	0.64	4.22	6.81	10.12	15.75
0.24	808.21	1355.97	2070.47	3285.50	0.65	3.73	5.99	8.90	13.75
0.25	684.30	1151.54	1730.61	2824.77	0.66	3.28	5.20	7.84	11.89
0.26	582.59	976.54	1495.80	2381.84	0.67	2.87	4.55	6.85	10.32
0.27	497.65	833.61	1257.92	2039.68	0.68	2.50	3.99	5.93	9.06
0.28	423.76	712.47	1107.95	1758.07	0.69	2.19	3.49	5.15	7.87
0.29	364.97	610.22	950.75	1547.06	0.70	1.91	3.04	4.45	6.77
0.30	317.66	528.99	817.42	1315.78	0.71	1.66	2.64	3.85	5.88
0.31	273.74	459.91	705.64	1138.36	0.72	1.44	2.29	3.34	5.05
0.32	240.34	397.94	610.86	967.64	0.73	1.25	1.98	2.87	4.42
0.33	206.69	346.77	532.63	833.80	0.74	1.08	1.70	2.47	3.78
0.34	180.08	302.75	461.46	733.79	0.75	0.94	1.47	2.12	3.24
0.35	157.73	263.92	404.42	638.79	0.76	0.81	1.25	1.82	2.77
0.36	138.62	231.98	354.31	562.19	0.77	0.69	1.08	1.55	2.36
0.37	121.67	204.27	311.07	493.00	0.78	0.59	0.91	1.33	1.96
0.38	106.08	178.57	272.43	433.05	0.79	0.51	0.77	1.12	1.64
0.39	93.12	156.48	239.48	375.63	0.80	0.43	0.65	0.93	1.38
0.40	81.80	137.45	209.17	334.90	0.81	0.36	0.55	0.77	1.14
0.41	72.51	120.68	184.07	293.05	0.82	0.30	0.46	0.64	0.94
0.42	63.91	106.85	161.59	254.46	0.83	0.25	0.38	0.54	0.77
0.43	56.68	93.85	141.54	223.58	0.84	0.21	0.32	0.44	0.63
0.44	50.16	82.74	124.81	196.87	0.85	0.18	0.26	0.36	0.52
0.45	44.42	72.84	109.33	172.30	0.86	0.14	0.21	0.30	0.43
0.46	39.36	64.79	97.19	152.40	0.87	0.12	0.17	0.24	0.34
0.47	34.70	57.42	86.61	136.16	0.88	0.09	0.14	0.19	0.27
0.48	30.50	50.74	76.42	120.88	0.89	0.08	0.11	0.15	0.21
0.49	26.78	44.58	67.80	107.10	0.90	0.06	0.09	0.12	0.17
0.50	23.74	39.43	59.73	95.25					

Table 14: Critical values for IM-OLS with three regressors for the intercept and trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	1302.41	2141.44	3251.78	5116.30	0.51	6.25	10.04	14.56	21.93
0.11	1045.41	1715.40	2606.17	4146.84	0.52	5.61	9.02	13.05	19.85
0.12	844.98	1387.10	2113.31	3308.90	0.53	5.10	8.06	11.70	17.65
0.13	693.47	1135.82	1723.18	2753.05	0.54	4.55	7.23	10.51	16.00
0.14	568.51	941.39	1412.00	2246.18	0.55	4.11	6.47	9.48	14.48
0.15	480.36	797.23	1188.12	1842.28	0.56	3.69	5.81	8.52	12.95
0.16	404.73	666.70	984.32	1558.39	0.57	3.32	5.26	7.66	11.52
0.17	344.72	567.88	852.44	1315.57	0.58	2.98	4.73	6.80	10.25
0.18	294.21	482.56	720.29	1132.96	0.59	2.68	4.23	6.07	9.28
0.19	249.42	412.19	614.13	963.33	0.60	2.40	3.76	5.43	8.20
0.20	215.31	357.20	534.06	818.33	0.61	2.13	3.34	4.85	7.31
0.21	187.17	305.90	453.37	715.01	0.62	1.91	2.99	4.32	6.45
0.22	162.07	266.13	397.46	616.47	0.63	1.70	2.66	3.85	5.75
0.23	141.55	232.27	350.25	538.77	0.64	1.52	2.37	3.43	4.99
0.24	124.15	204.40	305.69	479.36	0.65	1.34	2.10	3.01	4.43
0.25	108.94	178.97	268.37	422.64	0.66	1.18	1.84	2.64	3.86
0.26	95.23	156.22	233.95	365.96	0.67	1.06	1.63	2.33	3.42
0.27	84.21	137.49	206.43	325.06	0.68	0.93	1.43	2.04	3.00
0.28	74.91	122.33	182.25	285.95	0.69	0.83	1.26	1.79	2.64
0.29	67.34	109.67	162.90	254.41	0.70	0.72	1.10	1.57	2.32
0.30	59.83	98.00	145.51	226.99	0.71	0.64	0.98	1.37	2.00
0.31	53.64	86.70	128.78	200.74	0.72	0.56	0.85	1.20	1.76
0.32	48.41	77.64	114.79	176.59	0.73	0.49	0.75	1.05	1.55
0.33	42.94	69.59	103.54	159.52	0.74	0.43	0.65	0.91	1.32
0.34	38.69	62.84	92.11	144.17	0.75	0.37	0.56	0.79	1.14
0.35	34.21	56.20	83.24	128.39	0.76	0.32	0.49	0.68	0.96
0.36	30.60	50.33	74.59	114.47	0.77	0.28	0.42	0.58	0.82
0.37	27.42	44.87	66.97	103.52	0.78	0.24	0.36	0.49	0.70
0.38	24.60	40.22	60.75	92.05	0.79	0.21	0.30	0.42	0.59
0.39	22.02	36.02	53.12	82.60	0.80	0.17	0.26	0.36	0.51
0.40	19.78	32.49	47.83	74.54	0.81	0.15	0.22	0.30	0.43
0.41	17.89	29.07	42.34	66.14	0.82	0.13	0.18	0.25	0.35
0.42	16.04	25.97	38.22	59.07	0.83	0.11	0.15	0.21	0.29
0.43	14.38	23.22	34.52	53.12	0.84	0.09	0.13	0.17	0.24
0.44	13.07	21.06	30.94	47.79	0.85	0.07	0.11	0.14	0.20
0.45	11.72	18.78	27.81	42.60	0.86	0.06	0.09	0.11	0.16
0.46	10.58	16.86	25.05	38.65	0.87	0.05	0.07	0.09	0.13
0.47	9.53	15.17	22.52	34.42	0.88	0.04	0.06	0.07	0.10
0.48	8.54	13.70	20.32	31.27	0.89	0.03	0.04	0.06	0.08
0.49	7.77	12.38	18.32	28.11	0.90	0.02	0.03	0.05	0.06
0.50	6.97	11.03	16.19	24.75					

Table 15: Critical values for FM- and D-OLS with four regressors for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	2683.25	4597.94	7174.26	11948.80	0.51	12.46	20.44	30.78	49.25
0.11	2138.49	3676.00	5767.77	9340.19	0.52	11.19	18.40	27.64	44.80
0.12	1741.54	2959.64	4618.38	7506.44	0.53	10.08	16.55	24.87	40.05
0.13	1415.40	2417.13	3785.79	6090.38	0.54	9.10	14.90	22.40	35.83
0.14	1173.04	2005.95	3111.68	5022.80	0.55	8.18	13.46	20.31	32.10
0.15	968.91	1676.85	2593.22	4222.50	0.56	7.33	12.12	18.36	28.29
0.16	813.63	1396.19	2168.42	3501.48	0.57	6.62	10.84	16.34	25.27
0.17	687.02	1177.47	1808.17	2990.60	0.58	5.97	9.65	14.61	22.86
0.18	590.45	998.69	1565.72	2509.32	0.59	5.36	8.67	13.08	20.72
0.19	502.65	856.59	1332.23	2151.59	0.60	4.80	7.75	11.75	18.32
0.20	429.80	740.35	1141.93	1829.48	0.61	4.31	6.93	10.42	16.38
0.21	374.53	640.09	997.34	1575.91	0.62	3.83	6.24	9.20	14.60
0.22	326.33	557.08	856.73	1389.84	0.63	3.43	5.55	8.23	12.76
0.23	285.19	488.34	751.84	1201.73	0.64	3.05	4.93	7.27	11.34
0.24	249.28	425.70	656.73	1064.38	0.65	2.71	4.36	6.47	10.08
0.25	221.72	373.01	578.40	926.80	0.66	2.39	3.87	5.74	8.87
0.26	195.35	325.58	507.63	816.15	0.67	2.10	3.41	5.02	7.87
0.27	170.77	287.14	444.83	709.38	0.68	1.86	3.01	4.41	6.87
0.28	151.55	255.11	391.04	645.80	0.69	1.65	2.64	3.87	6.02
0.29	134.79	225.75	345.56	570.13	0.70	1.45	2.31	3.41	5.26
0.30	120.44	200.42	310.18	509.61	0.71	1.28	2.04	2.99	4.67
0.31	107.01	180.07	278.44	449.81	0.72	1.13	1.78	2.59	3.99
0.32	96.13	161.10	250.49	404.46	0.73	0.99	1.54	2.24	3.44
0.33	85.52	144.36	223.49	362.06	0.74	0.87	1.36	1.96	2.97
0.34	76.58	128.66	197.25	322.11	0.75	0.76	1.18	1.70	2.57
0.35	68.51	115.71	176.62	292.61	0.76	0.66	1.03	1.47	2.19
0.36	61.40	103.52	158.87	258.01	0.77	0.57	0.89	1.27	1.89
0.37	54.98	92.47	141.29	228.63	0.78	0.49	0.76	1.09	1.62
0.38	49.23	82.95	127.66	202.84	0.79	0.42	0.65	0.93	1.38
0.39	44.31	74.21	112.91	183.31	0.80	0.36	0.56	0.79	1.18
0.40	39.86	66.39	101.69	164.31	0.81	0.31	0.47	0.67	0.99
0.41	35.65	59.05	90.74	143.40	0.82	0.26	0.40	0.56	0.83
0.42	32.02	53.25	80.51	128.47	0.83	0.22	0.34	0.47	0.69
0.43	28.74	47.62	72.13	114.20	0.84	0.18	0.28	0.39	0.57
0.44	25.78	42.99	64.95	101.46	0.85	0.15	0.23	0.32	0.47
0.45	23.20	38.58	58.60	92.12	0.86	0.13	0.19	0.26	0.38
0.46	20.85	34.49	53.03	82.45	0.87	0.10	0.15	0.21	0.31
0.47	18.77	30.91	47.36	75.02	0.88	0.08	0.13	0.17	0.25
0.48	16.93	27.81	42.45	68.08	0.89	0.07	0.10	0.14	0.19
0.49	15.19	24.99	38.31	61.48	0.90	0.05	0.08	0.11	0.15
0.50	13.75	22.58	34.40	54.76					

Table 16: Critical values for IM-OLS with four regressors for the intercept case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	14436.42	22848.84	33424.93	50312.79	0.51	14.89	23.11	33.39	49.63
0.11	10446.94	16580.32	24045.97	36144.68	0.52	13.02	20.66	29.41	43.95
0.12	7807.50	12369.77	17798.66	26915.88	0.53	11.62	18.07	25.94	38.23
0.13	5923.69	9360.86	13639.20	20377.49	0.54	10.24	16.05	22.90	34.04
0.14	4524.18	7243.54	10634.38	16070.11	0.55	9.04	14.23	20.46	30.13
0.15	3553.03	5723.04	8363.28	12643.53	0.56	8.00	12.46	17.93	26.55
0.16	2794.70	4488.41	6569.75	9893.80	0.57	7.00	10.94	15.73	23.27
0.17	2245.21	3572.90	5217.76	7968.26	0.58	6.19	9.60	13.84	20.50
0.18	1812.66	2902.60	4255.35	6406.07	0.59	5.48	8.54	12.07	18.05
0.19	1477.45	2353.83	3443.99	5239.86	0.60	4.80	7.49	10.65	15.71
0.20	1207.33	1913.92	2798.86	4250.50	0.61	4.22	6.54	9.29	13.47
0.21	999.75	1581.41	2321.36	3526.78	0.62	3.70	5.76	8.11	11.69
0.22	829.17	1328.68	1940.16	2926.35	0.63	3.25	5.02	7.10	10.30
0.23	702.03	1107.86	1618.18	2486.06	0.64	2.85	4.38	6.22	8.99
0.24	585.25	937.83	1372.10	2068.40	0.65	2.48	3.80	5.42	7.83
0.25	499.61	783.12	1150.62	1744.46	0.66	2.17	3.31	4.71	6.83
0.26	426.31	673.42	989.44	1509.52	0.67	1.88	2.88	4.07	5.89
0.27	363.44	580.19	853.35	1283.44	0.68	1.64	2.48	3.51	5.07
0.28	311.19	498.61	724.99	1098.96	0.69	1.41	2.15	3.00	4.38
0.29	267.70	424.61	619.87	934.33	0.70	1.24	1.87	2.59	3.80
0.30	231.92	367.09	538.93	802.18	0.71	1.07	1.61	2.24	3.27
0.31	200.71	316.86	461.91	698.38	0.72	0.92	1.40	1.94	2.80
0.32	173.51	277.22	403.93	606.78	0.73	0.79	1.20	1.69	2.38
0.33	151.92	241.19	351.12	528.53	0.74	0.68	1.03	1.44	2.05
0.34	131.91	209.67	304.46	462.13	0.75	0.58	0.88	1.22	1.76
0.35	115.66	182.51	267.81	403.55	0.76	0.50	0.74	1.04	1.48
0.36	100.47	160.23	234.23	353.96	0.77	0.43	0.63	0.88	1.25
0.37	88.01	138.76	204.02	309.69	0.78	0.36	0.54	0.74	1.05
0.38	76.84	121.40	178.89	267.83	0.79	0.30	0.45	0.62	0.87
0.39	67.28	106.42	155.63	231.33	0.80	0.26	0.38	0.52	0.74
0.40	58.76	92.66	135.96	203.29	0.81	0.21	0.31	0.43	0.62
0.41	51.49	81.70	119.56	180.43	0.82	0.18	0.26	0.36	0.49
0.42	45.37	72.15	105.18	155.94	0.83	0.14	0.21	0.29	0.41
0.43	40.35	63.48	92.38	138.72	0.84	0.12	0.18	0.24	0.33
0.44	35.51	56.07	81.42	122.47	0.85	0.10	0.14	0.19	0.26
0.45	31.15	49.44	71.91	106.25	0.86	0.08	0.11	0.15	0.21
0.46	27.74	43.36	63.38	94.09	0.87	0.06	0.09	0.12	0.16
0.47	24.31	38.57	55.20	82.46	0.88	0.05	0.07	0.09	0.13
0.48	21.44	34.03	49.07	72.61	0.89	0.04	0.05	0.07	0.10
0.49	19.01	30.06	43.71	64.57	0.90	0.03	0.04	0.05	0.08
0.50	16.79	26.37	38.04	56.28					

Table 17: Critical values for FM- and D-OLS with four regressors for the intercept and trend case.

m	90 %	95 %	97.5 %	99 %	m	90 %	95 %	97.5 %	99 %
0.10	29421.89	47938.32	71611.40	113353.36	0.51	29.14	47.32	71.14	109.41
0.11	21136.79	34412.71	52052.91	82774.12	0.52	25.73	41.84	63.00	96.32
0.12	15669.76	25684.15	38639.66	61559.73	0.53	22.74	36.87	55.52	86.11
0.13	11763.77	19205.76	29041.44	46247.16	0.54	20.12	32.83	48.56	75.71
0.14	9086.50	14930.42	22421.31	35945.64	0.55	17.83	28.98	42.80	67.19
0.15	7080.86	11667.89	17569.17	28034.48	0.56	15.75	25.46	37.65	58.68
0.16	5587.23	9218.42	13904.85	22076.08	0.57	13.86	22.28	33.21	51.80
0.17	4458.83	7353.06	11250.71	17695.55	0.58	12.16	19.67	29.07	44.98
0.18	3588.96	5935.58	9037.89	14092.32	0.59	10.75	17.38	25.38	39.67
0.19	2919.77	4827.68	7359.77	11411.82	0.60	9.49	15.28	22.48	34.45
0.20	2391.82	3941.08	5912.48	9353.56	0.61	8.33	13.49	19.82	30.23
0.21	1968.28	3244.44	4856.99	7732.02	0.62	7.34	11.77	17.28	26.33
0.22	1644.55	2699.81	4105.70	6358.21	0.63	6.45	10.27	15.08	22.85
0.23	1381.91	2259.29	3411.40	5356.33	0.64	5.63	9.02	13.11	19.75
0.24	1162.09	1925.00	2889.81	4545.57	0.65	4.90	7.88	11.47	17.20
0.25	987.58	1621.88	2455.54	3802.57	0.66	4.29	6.83	9.96	14.93
0.26	840.53	1382.35	2101.43	3279.46	0.67	3.76	5.94	8.71	13.17
0.27	722.09	1191.15	1777.05	2792.56	0.68	3.27	5.13	7.57	11.43
0.28	620.45	1014.78	1547.23	2429.85	0.69	2.83	4.45	6.54	10.00
0.29	529.84	881.26	1325.46	2066.29	0.70	2.45	3.87	5.64	8.60
0.30	459.09	751.26	1132.32	1785.03	0.71	2.12	3.34	4.88	7.45
0.31	399.24	656.85	979.13	1508.62	0.72	1.83	2.88	4.20	6.34
0.32	345.00	570.75	853.01	1325.70	0.73	1.58	2.46	3.59	5.34
0.33	300.33	495.17	739.48	1146.30	0.74	1.36	2.11	3.06	4.52
0.34	262.33	427.06	644.66	997.94	0.75	1.17	1.81	2.59	3.84
0.35	229.16	374.62	560.92	882.03	0.76	1.00	1.54	2.21	3.28
0.36	199.82	326.82	492.48	769.58	0.77	0.86	1.32	1.88	2.78
0.37	174.99	284.67	431.04	684.87	0.78	0.72	1.12	1.58	2.37
0.38	152.35	250.96	377.24	597.43	0.79	0.62	0.94	1.35	1.98
0.39	133.16	220.05	329.95	524.67	0.80	0.52	0.79	1.13	1.63
0.40	116.46	191.28	287.56	456.36	0.81	0.44	0.66	0.94	1.34
0.41	102.44	166.26	250.33	396.66	0.82	0.36	0.55	0.77	1.13
0.42	90.33	146.23	219.50	343.58	0.83	0.30	0.45	0.63	0.92
0.43	79.12	127.91	190.43	298.21	0.84	0.25	0.37	0.52	0.76
0.44	69.57	112.35	168.56	264.38	0.85	0.20	0.30	0.42	0.62
0.45	61.49	99.90	148.63	233.38	0.86	0.16	0.24	0.34	0.49
0.46	54.14	87.29	131.18	208.51	0.87	0.13	0.20	0.27	0.39
0.47	47.98	77.95	115.66	184.41	0.88	0.10	0.16	0.21	0.30
0.48	42.38	69.61	102.92	161.35	0.89	0.08	0.12	0.17	0.23
0.49	37.63	61.07	90.38	143.21	0.90	0.06	0.09	0.13	0.18
0.50	33.32	53.80	80.44	125.62					

Table 18: Critical values for IM-OLS with four regressors for the intercept and trend case.