Noname manuscript No.
(will be inserted by the editor)

On the application of new tests for structural changes on global minimum-variance portfolios

Dominik Wied · Daniel Ziggel · Tobias Berens

27.01.2014 - accepted by Statistical Papers

Abstract We investigate if portfolios can be improved if the classical Markowitz mean-variance portfolio theory is combined with recently proposed change point tests for dependence measures. Taking into account that the dependence structure of financial assets typically cannot be assumed to be constant over longer periods of time, we estimate the covariance matrix of the assets, which is used to construct global minimum-variance portfolios, by respecting potential change points. It is seen that a recently proposed test for changes in the whole covariance matrix is indeed partially useful whereas pairwise tests for variances and correlations are not suitable for these applications without further adjustments.

Keywords: Fluctuation test; Markowitz; Portfolio optimization; Structural break

2010 Mathematics Subject Classification: 62P05, 91G10

Dominik Wied, corresponding author
TU Dortmund, 44221 Dortmund
Phone: +49/231/755 5419, Fax: +49/231/755 5284
E-Mail: wied@statistik.tu-dortmund.de

Daniel Ziggel
quasol GmbH, Marktallee 8, 48165 Münster
daniel.ziggel@quasol.de

Tobias Berens
TU Dortmund, 44221 Dortmund
tobias.berens@tu-dortmund.de

1. Introduction

The mean-variance approach proposed by Markowitz (1952) has become the foundation of large parts of modern finance theory. Its simplicity and intuitive arrangement cause its common use in both industry and science. In the beginning it was usually supposed that the parameters of interest, i.e. expected returns, volatilities and correlations can be estimated accurately. Nowadays, this assumption is, at least, questionable. As shown in various works, it is not an appropriate simplification for expected returns in most practical situations (Chopra and Ziemba, 1993, Kempf and Memmel, 2002, Merton, 1980). Moreover, it is well known, in particular in empirical finance, that variances and correlations among many time series cannot be assumed to remain constant over longer periods of time (Krishan et al., 2009). A good example is the recent financial crisis, in which capital market volatilities and correlations raised quite dramatically. As a consequence, risk figures increased significantly as diversification effects were overestimated (Bissantz et al., 2011a, Bissantz et al., 2011b).

It is well known that the expected return is the most important parameter within the Markowitz model, cf. e.g. Gohout and Specht (2007). Chopra and Ziemba (1993) demonstrate that, for mean tolerated risk levels, wrong return estimators have an eleven times larger impact than wrong risk estimators. Best and Grauer (1991) investigate the sensitivity of mean-variance-efficient portfolios to changes in the means of individual assets. The results state that portfolio weights are extremely sensitive to changes in asset means and a surprisingly small increase in the mean of a single asset drives half the securities from the portfolio.

One possible solution to overcome the problem of portfolio weights, which seem overly sensitive to estimation errors of returns, is the usage of the global minimum-variance portfolio at the left-most tip of the mean-variance efficient frontier. This portfolio has the unique property that portfolio weights are independent of the forecasted or expected returns on the individual assets as risk is minimized without an expected return input. Besides the advantage that no returns have to be estimated, the global minimum-variance portfolio allows the investor a risk optimal strategy. This is of special interest as passive (equity) investing has gained popularity (Clarke et al., 2006). Moreover, the covariance matrix can usually be estimated and forecasted much more reliable, resulting in an advantage for both practical and theoretical problems (Golosnoy et al., 2011).

In this context, several studies find that mean-variance optimization does not appear to result in a meaningful diversification improvement over non-optimized portfolios, despite the added complexity. This finding is consistent with the extensive literature documenting the puzzling under-performance of global minimum-variance approaches (Chow et al., 2011). Nevertheless, using historical backtests, Haugen and Baker (1991) and Clarke et al. (2006) demonstrate that minimum-variance strategies improve upon their cap-weighted counterparts by supplying better returns with reduced volatility, suggesting a latent potential of this approach. In order to open up this potential, the remaining market parameters (i.e. correlations and volatilities) have to be modelled time-dependent and flexible.

One of such methods is the regime switching model. This model allows the market parameters to depend on the market mode which switches among a finite number of states. In the simplest form, the market could be divided as "bullish" and "bearish" with very different market parameters. Of course, it is useful to define more intermediate states between these extremes, cf. e.g. Garcia and Perron (1996). Zhou and Yin (2003) develop a continuous-time version mean-variance portfolio selection model with regime switching and attempt to derive closed-form solutions for both, efficient portfolios and efficient frontier. Although this approach is promising, the question arises how to define the states properly. Moreover, market parameters change frequently and are complexly interwoven. Hence, it might be useful to address this kind of problem with tests for structural changes in the market parameters.

Recently, several fluctuation tests were proposed in this context. For example, Aue et al. (2009) and Wied et al. (2012b) propose formal completely nonparametric tests for unconditional dependence measures. They do not build upon prior knowledge as to the timing of potential shifts. More precisely, Aue et al. (2009) propose a test to detect changes in the (multivariate) covariance structure, while Wied et al. (2012b) present a method to test for changes in the correlation structure between two assets. They are based on cumulated sums of second order empirical cross moments (in the vein of Ploberger et al., 1989) and reject the null of constant covariance or correlation structure if these cumulated sums fluctuate too much. A similar fluctuation test for univariate variances is introduced and applied to financial time series by Wied et al. (2012a).

In this paper, we investigate if a combination of these new fluctuation tests and the classical minimum-variance approach improves global minimum-variance portfolios. To this end, we perform an empirical out-of-sample study and compare the performance of the resulting portfolios. Moreover, we investigate the resulting portfolio shiftings as a further quality measure. To the best of our knowledge, we are the first ones in the literature to provide such an analysis.

The remainder of the paper is organized as follows. In Section 2 we present a summary of the required theory and introduce the investigated tests for structural breaks. A description of the empirical analysis is given in Section 3, while the results can be found in Section 4. Finally, we end with a discussion of the results in Section 5 and a conclusion in Section 6.

2. Methods

In this section, we briefly present the mean-variance approach proposed by Markowitz (1952) for constructing a portfolio with minimal variance. To this end, we assume that there are d risky assets with random returns, $R^1, \dots, R^d$, such that $R := (R^1, \dots, R^d)$. Let μ be the vector of expectations and Σ the covariance matrix of R which is assumed to be positively definite (such that there is no risk-free asset). The vector μ and the matrix Σ are both assumed to be constant over time.

A portfolio is a mixture of the n assets with portfolio weights $\mathbf{a} = (a_1, \dots, a_d) \in \mathbb{R}^d$ such that $\mathbf{a}'1_d = 1$. In the mean-variance theory we want to solve the optimization problem

$$\min_{\mathbf{a} \in \mathbb{R}^d} \; \mathbf{a}'\Sigma\mathbf{a} \text{ s.t. } \mathbf{a}'\mu = \mu_{\mathsf{P}}, \mathbf{a}'1_d = 1, \tag{1}$$

where μ_{P} is a constant chosen by the analyst.

In general, the solution of this problem depends on the value of μ_{P}. However, it is also possible to solve the problem globally with the weighting vector

$$\mathbf{a}_{min} = \frac{1}{1_d'\Sigma^{-1}1_d}\Sigma^{-1}1_d$$

which yields the lowest possible variance $(1_d'\Sigma^{-1}1_d)^{-1}$. In practice, some more assumptions on $\mathbf{a}$ are often imposed, e.g. the entries of $\mathbf{a}$ have to be nonnegative (such that no short sales are allowed) or have to be bounded (such that we have maximal limits). In these cases, the optimization problem (1) still has a well-defined solution which can be calculated or approximated with numerical optimization.

The global minimum-variance portfolio has the unique property that portfolio weights are independent of the expected returns, which are very difficult to estimate. Hence, this portfolio relies solely on the covariance matrix which can usually be estimated more accurately.

To get a feasible solution of (1) in practice, it is necessary to estimate Σ based on realizations of R. If we assume Σ to be constant over time, it is useful to use the largest quantity of data available for estimation. If we, however, make the more realistic assumption that Σ is subject to structural changes, we have to take these changes into account. In this paper, we compare two nonparametric approaches for testing for the presence of structural breaks: The first one by Aue et al. (2009) tests for changes in the complete matrix. Since they assume throughout their paper that the vector of expectations is equal to 0, the whole test and the test statistic base on the second-order cross moments. Complementarily to this, the tests proposed by Wied et al. (2012b) and Wied et al. (2012a) separately test for changes in correlations and variances. Since the covariance matrix Σ can be written as

$$\Sigma(i,j) = \sqrt{\mathsf{Var}(R^i)} \cdot \sqrt{\mathsf{Var}(R^j)} \cdot \mathsf{Cor}(R^i, R^j), \; i,j \in \{1, \dots, d\}, \tag{2}$$

we can thus steer each entry of Σ separately.

Basically, all three nonparametric tests work in a similar way: Given the null hypothesis of constant covariance matrix, correlation or variance and T realizations of R these fluctuation tests compare the successively estimated covariance matrix (transformed into a vector with the vec-operator), (pairwise) correlation coefficient or (element-wise) variance with the respective value calculated from all T observations. The null hypothesis is rejected whenever these differences become too large over time. To be more precisely, the test statistic is a functional, for example the maximum - functional, of the series

$$P(j) := \frac{j^2}{T}(\hat{q}_j - \hat{q}_T)' \hat{\Sigma}_q^{-1} (\hat{q}_j - \hat{q}_T),$$

where $\hat{q}_j$ is the quantity of interest calculated from the first j observations, $\hat{q}_T$ is the quantity of interest calculated from the first T observations and $\hat{\Sigma}_q$ is an estimator (from all T observations) for the asymptotic covariance matrix of $\hat{q}_j$ under the null. Both expressions $\frac{j^2}{T}$ and $\hat{\Sigma}_q^{-1}$ serve for standardization. In particular, with $\frac{j^2}{T}$ less weight is laid on the differences at the beginning, where the parameters cannot be well estimated. The expression $\hat{\Sigma}_q^{-1}$ captures serial dependence and fluctuations of the time series. The process $P(j)$ converges against a Gaussian process and thus, in practice we compare the functionals of $P(j)$ with the respective quantiles of this functional. In the correlation case, we get

$$\max_{2 \le j \le T} |\sqrt{P(j)}| \to_d \sup_{0 \le z \le 1} |B(z)|$$

and in the variance case, we get

$$\max_{1 \le j \le T} |\sqrt{P(j)}| \to_d \sup_{0 \le z \le 1} |B(z)|.$$

In these cases, $B(z)$ is a one-dimensional Brownian bridge with quantiles 1.358 (95%) and 1.628 (99%). In the case of a covariance matrix, we have

$$\max_{1 \le j \le T} |P(j)| \to_d \sup_{0 \le z \le 1} \sum_{i=1}^{d} (B_i(z))^2,$$

where d is the number of upper-diagonal elements in the covariance matrix and $B_i(\cdot)$ are independent Brownian bridges. We approximate the quantiles of this limit distribution by simulating Brownian bridges on a fine grid. For this, the representation of a Brownian bridge as a limit of a random walk is used. Note that we do not use the "second" approximation for growing d, which is discussed in Remark 2.1 in Aue et al. (2009), as this does not seem to be appropriate here (cf. Aue et al., 2009, p. 4064). Based on these simulations, we obtain 53.583 (95%) and 56.961 (99%) as quantiles for 18 assets.

The tests are basically applicable to financial time series with its specific characteristics such as serial dependence and missing normality. For example, all tests can be applied if the returns can be modeled by a GARCH process. An important property is the fact that the location of the possible change points need not be specified a priori. In general, these fluctuation tests are sufficiently powerful and Aue et al. (2009) prove consistency of the covariance matrix test against fixed alternatives while Wied et al. (2012b) and Wied et al. (2012a) obtain local power results against smooth alternatives characterized by a continuous function g.

Once the presence of a parameter change is detected, a suitable estimate of its location can be obtained by the statistic proposed in Galeano and Wied (2012) (the original idea goes back at least to Vostrikova, 1981), i.e. by the point at which $P(j)$ (or a transformation of $P(j)$) takes its maximum. For example, in the correlation case we get $\widehat{k} = \arg\max_{2 \leq j \leq T} \sqrt{P(j)}$. Since we use these break point estimators in our study, we have decided to focus on the maximum-functional instead of considering for example the Cramér-von Mises functional as e.g. do Aue et al. (2009), equation (2.8).

3. Empirical investigation

3.1. Data In order to investigate if a combination of the above mentioned fluctuation tests and the classical minimum-variance optimization yields reasonable results, we perform an out-of-sample study and compare the results with several alternative methods. We use two different data sets. More precisely, we use daily log-returns based on final quotes of 18 sector subindices based on the STOXX EUROPE 600 (total return indices) and log-returns based on final quotes of 18 stocks (treated as total return indices), which were listed on the DAX 30 for the period between 01.01.1973 and 30.06.2011 (10044 data points). For the subindices, data are available for the time span 01.01.1992 - 30.06.2011, which equates to 5087 data points. All data sets are obtained from *Thomson Reuters Datastream.*

3.2. Parameter Estimation As already mentioned, for a fixed point in time, calculation of the global minimum-variance portfolio depends only on the estimated covariance matrix. Hence, we compare the results of several estimation procedures. First, we use the empirical covariance matrix given by the last 250/500/1000 data points. For sake of simplicity, we denote combinations of these empirical estimators and the minimum-variance optimization as plain Markowitz optimizations. In addition to that, we use the new fluctuation tests. Here, the estimation procedure is as follows:

1. Initialize $i = 1$, $k = 1000$ and $m =$ number of observed returns.
2. Perform the fluctuation test for the data $\{x_i, ..., x_k\}$.
3. If the test rejects the null, define $i = l$, where l maximizes the corresponding functional of $P(j)$ and go back to step 2. Otherwise use the data $\{x_p, ..., x_k\}$ in order to calculate the empirical estimator of the respective parameter, where $p = min\{i, k - 19\}$.
4. Set $k = k + n$, where n is the number of days between two optimizations. If $k > m$, stop. Otherwise go back to step 2.

Note that the modification $p = min\{i, k - 19\}$ ensures that at least 20 data points are used for parameter estimation. This proceeding is in line with Wied et al. (2012a). As mentioned above, we use the fluctuation tests in two different ways. On the one hand, we use the test of Aue et al. (2009). Hence, the procedure provides the covariance matrix directly. On the other hand, we separately apply the tests proposed by Wied et al. (2012b) and Wied et al. (2012a). The resulting covariance matrix is then given by (2).

We choose $\alpha = 1\%$ and $\alpha = 5\%$ as significance levels for the fluctuation tests. The choice of $\alpha = 1\%$ is due to the fact that in this case the number of possible false signals should be relatively small. Nevertheless, several applications show that $\alpha = 5\%$ yields convincing results in practice (Ziggel and Wied, 2012).

3.3. Optimization In addition to parameter estimation, there are several adjusting screws concerning the optimization which have an impact on the results. First, we have to define an interval for re-optimizations. To this end, we define $n = 21, 63$ and 252, respectively. These choices correspond to monthly, quarterly and yearly re-optimization. The same intervals will also be used in order to perform a re-balancing of the equally weighted portfolio which serves as a benchmark. These frequencies allow us to neglect the problem of sequential testing. Nevertheless, it would be worthwhile to implement a theoretical analysis for smaller frequencies about this issue using ideas of Chu et al. (1996) and Wied and Galeano (2013), but this lies beyond the scope of the present paper.
Aside from the interval for re-optimizations, we analyze the impact of some additional constraints to the portfolio weights. In a first run, we define $|a_i| \leq 1, \forall i$ which particularly allows for short selling. In the next step, we exclude short selling by requiring $0 \leq a_i \leq 1, \forall i$.

3.4. Miscellaneous We use several quality criteria in order to judge the performance. Of course, we investigate the resulting variance. Nevertheless, we also compare the resulting returns and Sharpe-ratios. To this end, we assume 1.1% as risk free return for the latter. This corresponds to the average return of German government bonds with less than 3 years to maturity in the year 2011. Besides, we measure the portfolio turnover in order to draw conclusions for a usage in practice. In line with DeMiguel et al. (2009), we define the average absolute change in the weights as

$$Turnover(R) = \frac{1}{TD - 1} \sum_{i=1}^{TD-1} \sum_{j=1}^{d} |a_{i+1,j} - a_{i^+,j}|,$$

where TD is the number of the trading days and d the number of assets. Besides, $a_{i^+,j}$ is the portfolio weight of asset j before a rebalancing or re-optimization at time $i + 1$. In addition, we call $Turnover(A)$ the absolute amount of changes, that means $Turnover(A) = Turnover(R) \cdot (TD - 1)$.

To evaluate the impact of diverging turnovers, we compute adjusted returns and Sharpe-ratios by including transaction costs. Therefore, we assume a constant relative bid-ask spread s_c (bid-ask spread divided by bid-ask midpoint) which diminishes the return R. To quantify the spread, we have analyzed daily bid and ask quotes of the 18 stocks listed on the DAX 30 and for all stocks listed on the STOXX EUROPE 600 for the time span 01.07.2010-30.06.2011. The average relative spread of the analyzed stocks amounts to 0.15% (DAX) and 0.22% (STOXX). As a simple approximation, we determine s_c to be 0.2% in both cases. The loss of return due to transaction costs is calculated by $Turnover(A) \cdot \frac{s_c}{2}$.

MATLAB R2009b is used for all computations. While the global optimization problem can be solved analytically, numerical optimization methods are necessarily under additional conditions on the weighting vector $\mathbf{a}$ (see Section 2). We perform these methods with the "fmincon" function included in the "Optimization Toolbox". Since the usage of just one starting point in the optimization can lead to a local minimum, we use multiple starting points. More precisely, we use starting points which lie on the boundary of the feasible region, the equally weighted portfolio and some random starting points. However, the optimizations have proceeded stable and the starting points have had only minor impact on the results.

4. Results

In this section we present the results of our out-of-sample study which can be found in Tables 1 to 8. We start with the dataset including 18 sector subindices based on the STOXX EUROPE 600. As described in Section 3, the equally weighted portfolios serve as a benchmark. It is noticeable that the interval for re-balancings has only a negligible effect on these results. In all cases the p.a. volatility is around 19.2%, while the average p.a. return is slightly above 8%. Besides, the portfolio turnover is very low and has no relevant impact on adjusted returns and Sharpe-ratios.

- Insert Table 1 about here -

The Markowitz optimizations based on the empirical covariance matrix improve upon the equally weighted portfolios. The average volatility decreases by 3.99% to 5.78%, while the return simultaneously increases by 0.1% to 3.04%. Nevertheless, the portfolio turnover increases by about ten times on average leading to return losses of 0.03% to 0.59%.

With respect to the setup options of the plain Markowitz optimizations, the choice of the days of data history as well as the re-optimization interval has only little impact to the volatility results. Nevertheless, in terms of returns, turnover and the Sharpe-ratio, the choice of 1,000 days as the data point history seems to be preferable. Besides, the choice of the weight limits has a marked effect on the results. The allowance for short selling reduces the volatility by more than 1% on average.

- Insert Table 2 about here -

By using the test of Aue et al. (2009) in order to estimate the covariance matrix, the resulting level of volatilities is slightly higher than those of the empirical covariance matrix. Nevertheless, the returns increase by about 2% on average which leads to superior Sharpe-ratios. Besides, the turnover is much lower and is reduced by about two thirds compared to the plain Markowitz optimization approach. Transaction costs reduce the returns by only 0.07% on average.
The application of different significance levels (5% and 1%) makes no notable difference in the results. Considering the volatility, the choice of 21 days for the re-optimization interval lowers the volatility by about 1% compared to 252 days. Astonishingly, no clear statement can be made with regards to the limits of asset weights because the results are quite inconclusive. On average, the differences between both options are negligible. In terms of the returns, the results of the different test and optimization options are comparable to each other and can be located between 10.88% and 11.11%.

- Insert Table 3 about here -

An application of the tests of Wied et al. (2012b) and Wied et al. (2012a) yields favorable results compared to the benchmark of equally weighted portfolios. The results are however considerably worse with respect to each measured performance indicator compared to the plain Markowitz optimization as well as to the optimization including the test of Aue et al. (2009). High turnovers lead to a substantial loss of returns by 0.35% on average. Nevertheless, it should be noted that the choice of the weight limits has a considerable impact on the resulting volatility. Surprisingly, the more restrictive option of $0 \leq a_i \leq 1, \forall i$ shows lower volatilities.

- Insert Table 4 about here -

We continue with the second dataset including the returns of 18 stocks, which were listed on the DAX 30 for the time span 01.01.1973 - 30.06.2011. The benchmark of equally weighted portfolios shows that the re-balancing interval has only very little effect on the volatility as well as on the return. The volatility amounts to about 18.8%, while the returns are around 11.4%. Transaction costs are negligible.

- Insert Table 5 about here -

Compared to the equally weighted portfolios, the results of the plain Markowitz optimizations show an improvement again. The average volatility decreases by 2.21%, while the average return increases by 0.48%. Consequently, the average Sharpe-ratio increases by about 0.10 points. The portfolio turnover is about six times higher, while transaction costs decrease the returns by averaged 0.14%. Concerning the setup options, a lower re-optimization interval is accompanied by lower volatilities and returns, while the influence to the Sharpe-ratio is inconclusive. Furthermore, the results show only a little impact of the choice of the data history and surprisingly the weight limits.

- Insert Table 6 about here -

The extension by the test of Aue et al. (2009) outperforms the plain Markowitz optimization and results to small improvements of the average returns and Sharpe-ratios. But in contrast to the application to the subindices dataset, the volatility remains unchanged. Additionally, the portfolio turnover and hence transaction costs are much lower. There are just minor changes of the performance measures due to the choice of the setup options except the re-optimization interval, where the return increases with larger gaps.

- Insert Table 7 about here -

Employing the tests of Wied et al. (2012b) and Wied et al. (2012a), the results show a slight decrease of the Sharpe-ratio compared to the equally weighted benchmark portfolio which is caused by a small improvement of the average return and an increase of the volatility. However, this approach does not achieve the convincing results of the remaining optimization methods. This goes along with the highest portfolio turnover and transaction costs of all alternatives. In line with the corresponding optimization on the basis of the subindices data, the allowance for short sales leads to a substantial higher volatility on average.

- Insert Table 8 about here -

5. Discussion

In line with previous works of Haugen and Baker (1991) and Clarke et al. (2006), our empirical study supports the finding that plain Markowitz optimized portfolios deliver superior results in terms of portfolio variance as well as portfolio returns compared to equally weighted portfolios. On the basis of two different datasets, we show that equally weighted portfolios are clearly outperformed by this optimization strategy. Moreover, the benefit of lower volatilities and higher returns is only marginally offset by increasing transaction costs due to considerable higher portfolio turnovers.
The extension of the plain Markowitz optimization by the test of Aue et al. (2009) leads to inconclusive results. With respect to the two used datasets, the results show increased returns and volatilities on average. However, it is remarkable that the portfolio turnover is much lower compared to the classical optimization. Basically, this is reasoned by the fact that the test yields only a few rejections of the null hypothesis of a constant covariance matrix. For example, a portfolio optimization including the DAX 30 dataset under the option setup of a 5% significance level and a re-optimization interval of 21 days leads to only four rejections within 10,043 data points.
The small number of rejections might be the result of a lack of accuracy of the critical values in connection to the setup of our study. The critical values are approximated by simulating Brownian bridges on a fine grid as described in Section 2. However, an additional simulation study, whose results are available from the authors upon request, indicates that this approximation does not perform well if the sample size

is small. We simulated the actual critical values for $d = 18$ by generating standard normal distributed values and calculating the respective test statistic. For example, for a sample of $1,000$ data points the 0.95-quantile is 24.20 while the asymptotic critical value is 53.58. Probably, a suitable derivation of finite sample critical values is a non-trivial task because in practice the underlying distribution of the asset returns is unclear; especially the assumption of the standard normal distribution is doubtful. Nevertheless, we used this procedure to show the effect of using critical values that are to some degree more suitable for the finite samples of our dataset. As a simple and rough approximation we concern a sample size of $\lceil \frac{5{,}087}{2} \rceil = 2,544$ for the STOXX EUROPE 600 subindices dataset and $\lceil \frac{10{,}043}{2} \rceil = 5,022$ with respect to the DAX 30 dataset. The actual critical values for the 0.95-quantile (0.99-quantile) are estimated to 34.77 (36.43) and 41.32 (43.40). Applying the test with the modified critical values leads to a higher number of rejections, e.g. seven instead of four considering the example above (DAX 30, 5% significance level, 21 days interval). Compared to Table 3, the improved results of Table 9 show exemplary that the adjustment of our very simple approach is a step in the right direction. A more sophisticated procedure for calculating critical values may perform even better.

- Insert Table 9 about here -

Certainly, the dates at which the null is rejected are of interest. Returning to the example mentioned above (DAX 30, 5% significance level, 21 days interval) these dates are 26.01.1983, 25.07.1989, 05.11.1996, and 19.02.2001 for the critical values based on the asymptotic analysis. In contrast to that, the dates at which the null is rejected are 28.08.1975, 03.02.1981, 10.10.1986, 13.11.1990, 25.08.1995, 15.02.1999 and 03.12.2001 for the modified critical values based on a sample size of 5,022. Most of these dates seem to be reasonable. The Latin American debt crisis of the early 1980s in combination with the savings and loan crisis of the 1980s in the United States explain some rejection dates. Besides, in each case one rejection date corresponds to the German reunification. Finally, in both cases the last rejection date can be explained by the burst of the dot-com bubble. Nevertheless, in both cases no change point is detected during the market turmoils of the financial crisis at the end of the last decade or the current European sovereign-debt crisis. Hence, it is very likely that the accurate number of changes in the covariance matrix is somewhat higher.

As described in Section 4, the results of the optimization in combination with the tests proposed by Wied et al. (2012b) and Wied et al. (2012a) are relatively poor compared to the remaining optimization approaches. This could be the result of the special character of these statistical tests. In contrast to the test for changes in the entire covariance matrix, the test for changes of variances is applied to each of the $d = 18$ different time series, whereas the test for changes in correlations is applied to each of the $18(18-1)/2 = 153$ upper-diagonal elements of the correlation matrix. Due to the high number of statistical tests, it is very likely that after every re-optimization interval one or more tests (wrongly) reject the null hypothesis. For example, Table 10 and 11 show the number of rejections of the tests including the DAX 30 dataset under the option setup of a 1% significance level and a re-optimization interval of 21 days.

- Insert Table 10 about here -

- Insert Table 11 about here -

In summary, this setup leads to 161 rejections of the volatility test and $1,001$ rejections of the correlation test within 431 test intervals in total or 2.7 rejections at each test interval on average. As a consequence, the data history changes after each interval which might lead to substantial fluctuations within the covariance matrix and hence an increased portfolio turnover. Apparently, these large shifts have negative effects on the performance of the model.

In order to remedy this drawback, it may be advantageous to leave out the test for changes in the bivariate correlations. Bissantz et al. (2011b) show that the impact of fluctuations and estimation errors is ten times larger for volatilities than for correlations. Consequently, the detection of change points of volatilities is obviously much more important than the correlation based test. By omitting that test, the number of tests for each interval is reduced to $d = 18$. First studies show an improvement into the desired direction. On average, volatility is reduced by 1.29% (STOXX EUROPE 600) and by 2.68% (DAX 30). But the benchmark volatility and Sharpe-ratio levels of the plain Markowitz optimizations are still not attained. More details are available from the authors upon request. Further studies of the suggested solution may provide a deeper analysis.

In addition to the investigated strategies, it might be possible to pursue a further strategy, i.e. to let the fluctuation tests themselves determine reasonable dates for a re-optimization. To be more precisely, a re-optimization of the portfolio would only be performed if a fluctuation test rejects the null hypothesis. However, in this paper we refused this further strategy for two different reasons. With respect to the test of Aue et al. (2009), this strategy suffers from the seldom rejections of the null hypothesis. We would then re-optimize very infrequently which is not useful in practice. Regarding the tests proposed by Wied et al. (2012b) and Wied et al. (2012a), the opposite problem arises, namely the problem of multiple testing and undesired frequent re-optimizations. Consequently, this kind of application would require different theoretical adjustments of the procedures.

Surprisingly, the allowance for short selling does not lead to lower volatilities in all cases (e.g., see Table 6). Although it is not intuitive that imposing the constraint of non-negative portfolio weights leads to an improved efficiency, this finding is in line with the empirical study of Jagannathan and Ma (2003). These authors argue that constraints for portfolio weights increase specification error, but can also reduce sampling error. The gain or loss in efficiency depends on the trade-off between both error types.

Since we employ portfolios consisting of very liquid German and European blue chip stocks, transaction costs are marginal due to the small bid-ask spread of 0.2%. For example, assuming $Turnover(A)$ to be 100 for the portfolio consisting of the dataset of 18 stocks listed on the DAX 30, the loss of annual log-returns amounts only to 0.25%. However, the impact of high turnovers may be significantly higher when datasets of less liquid assets are used. It would be worthwhile for further research to address a more detailed analysis of the trade-off between improved volatility and return of an optimized portfolio on the one side and costs relating to increased portfolio turnover on the other side.

6. Conclusion

The aim of this paper is to investigate whether a classical Markowitz mean-variance portfolio can be improved by the use of change point tests for dependence measures. To the best of our knowledge, we are the first to apply, on the one hand, the recently proposed test of Aue et al. (2009) for a constant covariance matrix and, on the other hand, the tests of Wied et al. (2012b) and Wied et al. (2012a) for constant variances and correlations to a minimum-variance optimization. We find out that portfolio optimizations considering change points of the covariance matrix yield considerable results and outperform plain Markowitz optimizations in several cases. In conducting the empirical study, we gain interesting insights in the behavior of these tests in combination with a portfolio optimization. This allows us to carve out the benefits as well as some challenging drawbacks of these new approaches. Moreover, we make some notes which might be helpful to future works.

Acknowledgements:

Financial support by Deutsche Forschungsgemeinschaft (SFB 823, Statistik nichtlinearer dynamischer Prozesse, project A1) is gratefully acknowledged. We would like to thank two unknown referees for their helpful comments, which led to a substantial improvement of an earlier version of this paper.

References

Aue, A., S. Hörmann, L. Horvàth, and M. Reimherr (2009): "Break detection in the covariance structure of multivariate time series models," *Annals of Statistics*, 37(6B), 4046–4087.

Best, M. and R. Grauer (1991): "On the Sensitivity of Mean-Variance Portfolios to Changes in Asset Means: Some Analytical and Computational Results," *The Review of Financial Studies*, 4(2), 315–342.

Bissantz, N., V. Steinorth, and D. Ziggel (2011a): "Stabilität von Diversifikationseffekten im Markowitz-Modell," *AStA Wirtschafts- und Sozialstatistisches Archiv*, 5(2), 145–157.

Bissantz, N., D. Ziggel, and K. Bissantz (2011b): "An empirical study of correlation and volatility changes of stock indices and their impact on risk figures," *Acta Universitatis Danubius (Œconomica)*, 7(4), 127–141.

Chopra, V. and W. Ziemba (1993): "The effect of errors in means, variances and covariances on optimal portfolio choice," *The Journal of Portfolio Management*, 19(2), 6–11.

Chow, T., J. Hsu, V. Kalesnik, and B. Little (2011): "A Survey of Alternative Equity Index Strategies," *Financial Analysts Journal*, 67(5), 37–57.

Chu, C.-S. J., M. Stinchcombe, and H. White (1996): "Monitoring structural change," *Econometrica*, 64(5), 1045–1065.

Clarke, R., H. de Silva, and S. Thorley (2006): "Minimum-Variance Portfolios in the U.S. Equity Market," *Journal of Portfolio Management*, 33(1), 10–24.

DeMiguel, V., L. Garlappi, and R. Uppal (2009): "Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?" *Review of Financial Studies*, 22(5), 1915–1953.

Galeano, P. and D. Wied (2012): "Multiple change point detection in the correlation structure of random variables," *arXiv:1206.5367v1*.

Garcia, R. and P. Perron (1996): "An Analysis of the Real Interest Rate under Regime Shifts," *Review of Economics and Statistics*, 78(1), 111–125.

Gohout, W. and K. Specht (2007): "Mean-variance portfolios using Bayesian vector-autoregressive forecasts," *Statistical Papers*, 48(3), 403–418.

Golosnoy, V., S. Ragulin, and W. Schmid (2011): "CUSUM control charts for monitoring optimal portfolio weights," *Computational Statistics and Data Analysis*, 55(11), 2991–3009.

Haugen, R. and N. Baker (1991): "The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios," *Journal of Portfolio Management*, 17(3), 35–40.

Jagannathan, R. and T. Ma (2003): "Risk Reduction in Large Portfolios: Why Imposing the Wrong Constrains Helps," *The Journal of Finance*, 58(4), 1651–1684.

Kempf, A. and C. Memmel (2002): "Schätzrisiken in der Portfoliotheorie," in *Handbuch Portfoliomanagement*, ed. by J. Kleeberg and H. Rehkugler, Uhlenbruch, 893–919.

Krishan, C., R. Petkova, and P. Ritchken (2009): "Correlation risk," *Journal of Empirical Finance*, 16(3), 353–367.

Markowitz, H. (1952): "Portfolio selection," *Journal of Finance*, 7(1), 77–91.

Merton, R. (1980): "On estimating the expected return on the market: an exploratory investigation," *Journal of Financial Economics*, 8(4), 323–361.

Ploberger, W., W. Krämer, and K. Kontrus (1989): "A new test for structural stability in the linear regression model," *Journal of Econometrics*, 40(2), 307–318.

Vostrikova, L. (1981): "Detecting 'disorder' in multidimensional random processes," *Soviet Mathematics Doklady*, 24, 55–59.

Wied, D., M. Arnold, N. Bissantz, and D. Ziggel (2012a): "A new fluctuation test for constant variances with application to finance," *Metrika*, 75(8), 1111–1127.

Wied, D. and P. Galeano (2013): "Monitoring correlation change in a sequence of random variables," *Journal of Statistical Planning and Inference*, 143(1), 186–196.

Wied, D., W. Krämer, and H. Dehling (2012b): "Testing for a change in correlation at an unknown point in time using an extended functional delta method," *Econometric Theory*, 28(3), 570–589.

Zhou, X. and G. Yin (2003): "Markowitz's mean-variance portfolio selection with regime switching: A Continuous-Time Model," *SIAM Journal on Control and Optimization*, 42(4), 1466–1482.

Ziggel, D. and D. Wied (2012): "Trading strategies based on new fluctuation tests," *IFTA-Journal (Journal of the International Federation of Technical Analysts)*, 2012 Edition, 17–20.

Interval Re-Balancing	Sharpe Ratio	Return	Volatility	Turnover (R)	Turnover (A)
21	0.3620 (0.3604)	8.08% (8.05%)	19.29%	0.03	5.66
63	0.3633 (0.3622)	8.07% (8.05%)	19.20%	0.05	3.24
252	0.3638 (0.3633)	8.04% (8.03%)	19.08%	0.10	1.62

Table 1: *Results for the equally weighted portfolios including 18 sector subindices based on the STOXX EUROPE 600. Values in brackets include transaction costs.*

# Data	Interval Re-Opt.	Limits	Sharpe Ratio	Return	Volatility	To. (R)	To. (A)
250	21	$\|a_i\| < 1$	0.5819 (0.5388)	9.02% (8.43%)	13.61%	0.61	118.53
250	21	$0 < a_i < 1$	0.5128 (0.5014)	8.56% (8.40%)	14.55%	0.17	33.27
250	63	$\|a_i\| < 1$	0.6283 (0.6013)	9.46% (9.10%)	13.30%	1.14	73.18
250	63	$0 < a_i < 1$	0.4935 (0.4860)	8.29% (8.18%)	14.57%	0.34	21.99
250	252	$\|a_i\| < 1$	0.5338 (0.5196)	8.51% (8.32%)	13.89%	2.44	38.95
250	252	$0 < a_i < 1$	0.4754 (0.4715)	8.18% (8.12%)	14.88%	0.81	13.03
500	21	$\|a_i\| < 1$	0.6800 (0.6549)	10.24% (9.90%)	13.44%	0.35	68.29
500	21	$0 < a_i < 1$	0.5211 (0.5141)	8.68% (8.58%)	14.55%	0.10	20.26
500	63	$\|a_i\| < 1$	0.7014 (0.6852)	10.52% (10.30%)	13.43%	0.69	43.85
500	63	$0 < a_i < 1$	0.5158 (0.5113)	8.66% (8.59%)	14.65%	0.22	14.07
500	252	$\|a_i\| < 1$	0.6074 (0.5985)	9.66% (9.53%)	14.09%	1.61	25.77
500	252	$0 < a_i < 1$	0.5142 (0.5110)	8.75% (8.70%)	14.88%	0.59	9.36
1000	21	$\|a_i\| < 1$	0.7295 (0.7144)	11.01% (10.80%)	13.58%	0.22	42.12
1000	21	$0 < a_i < 1$	0.5584 (0.5542)	9.35% (9.29%)	14.77%	0.07	13.08
1000	63	$\|a_i\| < 1$	0.7292 (0.7197)	11.12% (10.99%)	13.74%	0.42	26.63
1000	63	$0 < a_i < 1$	0.5582 (0.5553)	9.41% (9.36%)	14.88%	0.15	9.47
1000	252	$\|a_i\| < 1$	0.6149 (0.6093)	9.98% (9.89%)	14.43%	1.10	17.58
1000	252	$0 < a_i < 1$	0.5618 (0.5597)	9.58% (9.55%)	15.09%	0.44	7.04

Table 2: *Results for optimizations using the empirical covariance matrix including 18 sector subindices based on the STOXX EUROPE 600. Values in brackets include transaction costs.*

α	Interval Re-Opt.	Limits	Sharpe Ratio	Return	Volatility	To. (R)	To. (A)
95%	21	$\|a_i\| < 1$	0.6901 (0.6790)	10.77% (10.62%)	14.02%	0.16	30.25
95%	21	$0 < a_i < 1$	0.6658 (0.6628)	11.15% (11.10%)	15.09%	0.05	9.79
95%	63	$\|a_i\| < 1$	0.6750 (0.6681)	10.89% (10.79%)	14.51%	0.30	19.26
95%	63	$0 < a_i < 1$	0.6578 (0.6555)	11.08% (11.04%)	15.17%	0.11	7.32
95%	252	$\|a_i\| < 1$	0.6063 (0.6023)	10.60% (10.54%)	15.67%	0.78	12.52
95%	252	$0 < a_i < 1$	0.6433 (0.6416)	10.99% (10.96%)	15.37%	0.36	5.69
99%	21	$\|a_i\| < 1$	0.7021 (0.6920)	10.97% (10.82%)	14.05%	0.15	29.75
99%	21	$0 < a_i < 1$	0.6747 (0.6714)	11.30% (11.25%)	15.12%	0.05	9.71
99%	63	$\|a_i\| < 1$	0.6936 (0.6865)	11.06% (10.96%)	14.37%	0.30	19.30
99%	63	$0 < a_i < 1$	0.6665 (0.6643)	11.20% (11.16%)	15.15%	0.11	7.24
99%	252	$\|a_i\| < 1$	0.6355 (0.6313)	11.01% (10.95%)	15.60%	0.77	12.33
99%	252	$0 < a_i < 1$	0.6393 (0.6372)	10.96% (10.93%)	15.43%	0.35	5.61

Table 3: *Results for optimizations using the test of Aue et al. (2009) including 18 sector subindices based on the STOXX EUROPE 600. Values in brackets include transaction costs.*

α	Interval Re-Opt.	Limits	Sharpe Ratio		Return		Volatility	To. (R)	To. (A)
95%	21	$\|a_i\| < 1$	0.5813	(0.5142)	11.34%	(10.16%)	17.61%	1.23	239.12
95%	21	$0 < a_i < 1$	0.3882	(0.3545)	7.28%	(6.74%)	15.92%	0.56	108.29
95%	63	$\|a_i\| < 1$	0.5958	(0.5682)	12.33%	(11.81%)	18.85%	1.64	104.82
95%	63	$0 < a_i < 1$	0.6144	(0.5953)	10.70%	(10.40%)	15.62%	0.95	60.86
95%	252	$\|a_i\| < 1$	0.3728	(0.3629)	8.73%	(8.53%)	20.48%	2.50	40.06
95%	252	$0 < a_i < 1$	0.3418	(0.3359)	6.72%	(6.62%)	16.44%	1.23	19.61
99%	21	$\|a_i\| < 1$	0.3539	(0.3149)	7.18%	(6.51%)	17.17%	0.70	135.93
99%	21	$0 < a_i < 1$	0.4675	(0.4569)	8.82%	(8.64%)	16.51%	0.18	35.67
99%	63	$\|a_i\| < 1$	0.4615	(0.4452)	8.96%	(8.68%)	17.03%	0.88	56.22
99%	63	$0 < a_i < 1$	0.4900	(0.4831)	9.23%	(9.11%)	16.59%	0.36	23.22
99%	252	$\|a_i\| < 1$	0.4075	(0.4029)	8.33%	(8.24%)	17.73%	1.10	17.63
99%	252	$0 < a_i < 1$	0.5138	(0.5098)	9.12%	(9.06%)	15.62%	0.71	11.35

Table 4: *Results for optimizations using the tests of Wied et al. (2012b) and Wied et al. (2012a) including 18 sector subindices based on the STOXX EUROPE 600. Values in brackets include transaction costs.*

Interval Re-Balancing	Sharpe Ratio		Return		Volatility	Turnover (R)	Turnover (A)
21	0.5520	(0.5496)	11.50%	(11.45%)	18.84%	0.04	18.49
63	0.5475	(0.5464)	11.42%	(11.37%)	18.84%	0.07	10.59
252	0.5366	(0.5362)	11.14%	(11.13%)	18.70%	0.14	4.90

Table 5: *Results for the equally weighted portfolios including 18 stocks listed on the DAX 30. Values in brackets include transaction costs.*

α	Interval Re-Opt.	Limits	Sharpe Ratio		Return		Volatility	To. (R)	To. (A)
250	21	$\|a_i\| < 1$	0.6489	(0.6205)	11.39%	(10.94%)	15.85%	0.42	181.30
250	21	$0 < a_i < 1$	0.6567	(0.6435)	11.83%	(11.62%)	16.34%	0.20	85.58
250	63	$\|a_i\| < 1$	0.5888	(0.5722)	11.03%	(10.75%)	16.87%	0.77	110.70
250	63	$0 < a_i < 1$	0.6235	(0.6154)	11.56%	(11.43%)	16.78%	0.37	53.32
250	252	$\|a_i\| < 1$	0.6413	(0.6334)	12.29%	(12.15%)	17.45%	1.56	54.48
250	252	$0 < a_i < 1$	0.6489	(0.6446)	12.36%	(12.28%)	17.35%	0.86	30.11
500	21	$\|a_i\| < 1$	0.6178	(0.6016)	10.89%	(10.63%)	15.84%	0.24	103.90
500	21	$0 < a_i < 1$	0.6345	(0.6264)	11.47%	(11.34%)	16.34%	0.13	53.53
500	63	$\|a_i\| < 1$	0.6229	(0.6124)	11.29%	(11.13%)	16.37%	0.46	65.59
500	63	$0 < a_i < 1$	0.6453	(0.6399)	11.75%	(11.66%)	16.51%	0.24	34.26
500	252	$\|a_i\| < 1$	0.6578	(0.6527)	12.37%	(12.28%)	17.13%	1.04	35.86
500	252	$0 < a_i < 1$	0.6669	(0.6641)	12.51%	(12.46%)	17.10%	0.61	21.33
1000	21	$\|a_i\| < 1$	0.6553	(0.6457)	11.58%	(11.42%)	15.99%	0.14	62.10
1000	21	$0 < a_i < 1$	0.6611	(0.6559)	11.93%	(11.84%)	16.38%	0.08	34.46
1000	63	$\|a_i\| < 1$	0.6519	(0.6460)	11.65%	(11.55%)	16.18%	0.27	39.08
1000	63	$0 < a_i < 1$	0.6568	(0.6538)	11.91%	(11.86%)	16.45%	0.15	21.81
1000	252	$\|a_i\| < 1$	0.6861	(0.6824)	12.50%	(12.44%)	16.62%	0.67	23.52
1000	252	$0 < a_i < 1$	0.6783	(0.6765)	12.54%	(12.51%)	16.86%	0.40	13.93

Table 6: *Results for optimizations using the empirical covariance matrix including 18 stocks listed on the DAX 30. Values in brackets include transaction costs.*

α	Interval Re-Opt.	Limits	Sharpe Ratio		Return		Volatility	To. (R)	To. (A)
95%	21	$\|a_i\| < 1$	0.6493	(0.6439)	11.74%	(11.65%)	16.38%	0.09	36.71
95%	21	$0 < a_i < 1$	0.6588	(0.6553)	11.99%	(11.93%)	16.53%	0.06	23.60
95%	63	$\|a_i\| < 1$	0.6602	(0.6565)	11.93%	(11.87%)	16.41%	0.16	22.81
95%	63	$0 < a_i < 1$	0.6637	(0.6515)	12.10%	(12.06%)	16.57%	0.10	14.45
95%	252	$\|a_i\| < 1$	0.6868	(0.6851)	12.72%	(12.69%)	16.91%	0.39	13.66
95%	252	$0 < a_i < 1$	0.6820	(0.6807)	12.65%	(12.63%)	16.93%	0.26	9.10
99%	21	$\|a_i\| < 1$	0.6552	(0.6496)	11.85%	(11.76%)	16.41%	0.08	35.88
99%	21	$0 < a_i < 1$	0.6623	(0.6585)	12.05%	(11.99%)	16.54%	0.05	23.14
99%	63	$\|a_i\| < 1$	0.6632	(0.6600)	11.98%	(11.92%)	16.40%	0.16	22.32
99%	63	$0 < a_i < 1$	0.6636	(0.6614)	12.08%	(12.04%)	16.54%	0.10	14.21
99%	252	$\|a_i\| < 1$	0.6952	(0.6937)	12.80%	(12.77%)	16.82%	0.37	12.90
99%	252	$0 < a_i < 1$	0.6846	(0.6836)	12.64%	(12.62%)	16.85%	0.25	8.67

Table 7: *Results for optimizations using the test of Aue et al. (2009) including 18 stocks listed on the DAX 30. Values in brackets include transaction costs.*

α	Interval Re-Opt.	Limits	Sharpe Ratio		Return		Volatility	To. (R)	To. (A)
95%	21	$\|a_i\| < 1$	0.4929	(0.4420)	10.04%	(9.12%)	18.14%	0.86	367.74
95%	21	$0 < a_i < 1$	0.5020	(0.4847)	10.52%	(10.19%)	18.76%	0.30	130.15
95%	63	$\|a_i\| < 1$	0.4418	(0.4186)	11.74%	(11.18%)	24.09%	1.55	221.80
95%	63	$0 < a_i < 1$	0.5463	(0.5324)	10.47%	(10.24%)	17.16%	0.65	93.48
95%	252	$\|a_i\| < 1$	0.5128	(0.5033)	14.21%	(13.97%)	25.56%	2.76	96.66
95%	252	$0 < a_i < 1$	0.6264	(0.6212)	12.28%	(12.19%)	17.86%	1.07	37.59
99%	21	$\|a_i\| < 1$	0.4593	(0.4358)	11.21%	(10.69%)	22.01%	0.48	206.76
99%	21	$0 < a_i < 1$	0.4115	(0.4046)	9.52%	(9.38%)	20.46%	0.13	56.86
99%	63	$\|a_i\| < 1$	0.4456	(0.4281)	10.56%	(10.19%)	21.23%	1.03	147.80
99%	63	$0 < a_i < 1$	0.6377	(0.6314)	11.94%	(11.83%)	17.00%	0.29	42.02
99%	252	$\|a_i\| < 1$	0.5524	(0.5464)	12.51%	(12.38%)	20.65%	1.45	50.83
99%	252	$0 < a_i < 1$	0.6529	(0.6496)	12.50%	(12.44%)	17.46%	0.67	23.38

Table 8: *Results for optimizations using the tests of Wied et al. (2012b) and Wied et al. (2012a) including 18 stocks listed on the DAX 30. Values in brackets include transaction costs.*

α	Interval Re-Opt.	Limits	Sharpe Ratio		Return		Volatility	To. (R)	To. (A)
95%	21	$\|a_i\| < 1$	0.7886	(0.7744)	11.94%	(11.75%)	13.75%	0.20	39.41
95%	21	$0 < a_i < 1$	0.6103	(0.6065)	10.16%	(10.10%)	14.84%	0.06	11.37
95%	63	$\|a_i\| < 1$	0.8080	(0.7986)	12.34%	(12.21%)	13.91%	0.42	26.63
95%	63	$0 < a_i < 1$	0.6083	(0.6054)	10.19%	(10.15%)	14.95%	0.14	8.65
95%	252	$\|a_i\| < 1$	0.6501	(0.6440)	10.61%	(10.52%)	14.63%	1.11	17.81
95%	252	$0 < a_i < 1$	0.5799	(0.5776)	9.87%	(9.84%)	15.13%	0.44	6.96
99%	21	$\|a_i\| < 1$	0.7724	(0.7591)	11.63%	(11.45%)	13.63%	0.19	36.62
99%	21	$0 < a_i < 1$	0.6035	(0.5996)	10.07%	(10.01%)	14.86%	0.06	11.55
99%	63	$\|a_i\| < 1$	0.7655	(0.7570)	11.76%	(11.64%)	13.93%	0.38	24.06
99%	63	$0 < a_i < 1$	0.6084	(0.6055)	10.21%	(10.16%)	14.97%	0.14	8.75
99%	252	$\|a_i\| < 1$	0.6611	(0.6557)	11.05%	(10.97%)	15.05%	1.03	16.51
99%	252	$0 < a_i < 1$	0.6146	(0.6126)	10.53%	(10.50%)	15.34%	0.40	6.46

Table 9: *Results for optimizations using the test of Aue et al. (2009) in combination with the modified critical values including 18 sector subindices based on the STOXX EUROPE 600. Values in brackets include transaction costs.*

$d =$	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
Number of Rejections	10	8	7	7	9	13	12	9	8	7	12	3	7	9	9	11	13	7

Table 10: Number of rejections of the null hypothesis of constant volatility for each asset for an optimization using the test of Wied et al. (2011) including 18 stocks listed on the DAX 30 under the option setup of 1% significance level and a test interval of 21 days.

$d =$	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
1	0	5	6	6	8	6	6	6	2	5	6	6	8	5	7	10	11	7
2		0	4	5	8	8	6	10	5	3	5	6	8	5	6	9	7	7
3			0	10	5	9	7	10	4	2	4	5	3	6	8	11	9	7
4				0	6	11	7	8	9	5	7	6	7	4	5	7	7	5
5					0	11	10	6	2	2	4	5	7	5	7	15	9	10
6						0	7	7	6	6	5	9	10	5	6	10	7	11
7							0	8	5	4	5	8	6	6	10	11	8	8
8								0	4	3	4	6	6	3	8	13	7	10
9									0	2	4	6	6	3	4	6	6	4
10										0	3	5	7	1	3	5	4	4
11											0	4	3	4	5	8	7	8
12												0	5	2	9	9	6	6
13													0	3	8	7	8	7
14														0	3	7	7	7
15															0	15	10	9
16																0	13	9
17																	0	8
18																		0

Table 11: Number of rejections of the null hypothesis of constant correlation for each bivariate combination of the assets for an optimization using the test of Wied et al. (2012) including 18 stocks listed on the DAX 30 under the option setup of 1% significance level and a test interval of 21 days.