## Sequential Detector Statistics for Speculative Bubbles

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## 1 Abstract

We propose a heteroskedasticity-robust locally best invariant (LBI) statistic to test the hypothesis of a unit root against the alternative of an explosive root associated with speculative bubbles. Compared to existing alternatives like Dickey-Fuller type tests, the proposed LBI test exhibits a standard limiting distribution and greater power, particularly in the empirically relevant scenario of a moderately explosive root. Further refinements, such as the point-optimal linear test, approach the power envelope remarkably closely. To detect bubbles with an unknown starting date, we consider sequential schemes based on forward and backward expanding windows. Among these, the stacked backward CUSUM procedure proposed by Otto and Breitung (2023) emerges as the most powerful detector for real-time monitoring. We then propose simple statistics for estimating the starting date of a bubble consistently. Finally, we illustrate our methods in two empirical examples.