

# Regularized Wishart Stochastic Volatility for High-Dimensional Covariance Forecasting

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## Abstract

We extend Uhlig's (1994, 1997) Wishart stochastic volatility (WSV) model by introducing a regularized state transition for the precision matrix that shrinks the covariance forecasts toward a prior reference matrix. This regularization ensures stationarity of the return process and stabilizes the eigenvalues of the covariance forecasts, while preserving closed-form expressions for filtering, prediction, and likelihood evaluation. We provide conditions for stationarity and for the existence of second- and fourth-order moments, show that the model admits a multivariate GARCH representation, and derive bounds that illustrate how regularization prevents degeneracy and excessive dispersion in the eigenvalues of the covariance matrix forecasts. To account for regime shifts in the correlation structure, we further embed a time-varying directional forgetting scheme into the regularized model, allowing the forgetting rate to differ over time and across directions in the return space. In a high-dimensional application with up to 1,000 assets, the regularized models deliver significantly more accurate covariance forecasts than the baseline WSV and perform well compared to several benchmark models.

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