On the applicability of a new non-parametric distribution free dependence measure test*

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October 8, 2018

Abstract

We are investigating a cumulative sum (CUSUM) type test for constant distribution free dependence measures, considering pairwise averaged Spearman's rho and quantile dependencies in an equidependence setting, first proposed in "Testing for Structural Breaks in Factor Copula Models" by Manner, Stark, and Wied (2017). The asymptotic null distribution is not known in closed form and therefore estimated by an i.i.d. bootstrap procedure. We extend previous simulation studies by analyzing size and power properties, using different dependence measure settings. To decide wheter two estimated break point locations, scaled to the uniform interval, using different dependence vector settings, belong to the same break event we use a heuristic procedure. We apply the test for different dependence settings to historical data of ten large financial firms during the last financial crisis from 2002 to the mid of 2013. The results suggest that the test is suitable for financial data applications and it is advantageous to consider different settings of the dependence vector.

JEL Classification: C12, C14

Keywords: Dependence measures testing, Spearman's rho, quantile dependence, portfolio optimization

*Research is supported by Deutsche Forschungsgemeinschaft (DFG grant "Strukturbrüche und Zeitvariation in hochdimensionalen Abhängigkeitsstrukturen"). The computations were parallelized and performed using CHEOPS at the Regional Computing Center of the University of Cologne (RRZK).

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1. INTRODUCTION

Detecting structural breaks in statistics or statistical models is a wide research topic. Early works can be found in Page (1954), Page (1955) and Page (1959) who investigated quality control problems. Chow (1960), Brown, Durbin, and Evans (1975) and Krämer, Alt, and Ploberger (1988) among others detect structural breaks in various statistical models, where prominent examples of change point analysis are the detection of instabilities in mean and variance (e.g. Horvath, Kokoszka, and Steinebach (1999) and Aue, Hörmann, and Reimherr (2009) among others). These results have been extended to more complex regression models for example in Andrews (1993) and Bai and Perron (1998).

A current research topic is analyzing dependence changes in financial data, such as, stock returns. During the last financial crisis starting in 2007 one could observe that dependence and volatility between financial market variables rapidly increase, which resulted in inaccurate estimation and prediction of different risk figures (Bissantz, Steinorth, and Ziggel (2011)). Therefore, one can not expect these risk figures to be constant over time (e.g. Longin and Solnik (1995)). In portfolio management the portfolio manager is interested in decreasing the risk of losses by dividing the fortune in different investment possibilities. Such an effect is called the diversification effect. The increase of dependence measures, e.g., correlation of assets within a portfolio during financial crisis (Sanchetta and Satchell (2007)), therefore takes bad effects on the efficiency of the portfolio itself. This motivates the usage of structural break tests for dependence measures, where a detected break point indicates that the chosen portfolio might not be optimal anymore and a different investment strategy should be chosen. In particular, several tests for constant dependencies have recently been developed, see, e.g., Bücher and Ruppert (2013) for the case of the copula, Dehling, Vogel, Wendler, and Wied (2017) for the case of Kendall's tau, Wied, Krämer, and Dehling (2012) for the case of correlation and Wied, Dehling, van Kampen, and Vogel (2013) for the case of Spearman's rho. The main motivation for such tests is that dependencies usually increase in times of crises. Therefore, they can be applied to detect and quantify contagion between different financial markets or to construct optimal portfolios in portfolio management.

In this paper, we investigate a test which compares different dependence measure vectors jointly estimated using the whole sample information to successively estimated counterparts, where the dependence vectors consists of Spearman's rank correlation and quantile dependencies. The test was first proposed in "Testing for structural breaks in factor copula models" by Manner et al. (2017), where the authors focused on the parameter test for detecting structural breaks in factor copula models. We now pay more attention to the non-parametric dependence measure vector based test. The test is constructed to analyze the hypothesis of no dependence change in a pre-specified vector of dependence measures. We consider residual data from a pre-estimated data model, e.g. ARCH and GARCH models such that the test is of non-parametric nature once we determined the residuals.

The asymptotic distribution of the test statistic is mainly obtained in Lemma 7 in Manner et al. (2017) and is followed by a combination of the asymptotic behavior of the sequential copula process (cf. Bücher, Kojadinovic, Rohmer, and Segers (2014)) and results from Bücher and Kojadinovic (2016) such as Remillard (2017) to give a convergence result for the usage of residual data determined by a pre-estimated data model. For this reason it is important to use distribution free dependence measures which can be expressed by terms on the copula. Due to the fact that the asymptotic distribution is not known in closed form we have to estimate the critical values by an i.i.d. bootstrap procedure. We extend previous made simulation studies in Manner et al. (2017) by analyzing size and power properties of the test for different skewed and fat tailed distributions for different settings of the used dependence measure vector. We also use an heuristic procedure to be able to make a statement for equality of two estimated break point locations, scaled to the uniform interval, using different dependence settings. Here, the (pivot) confidence intervals for both break point estimates have to be determined using a (percentile) bootstrap procedure and we consider two estimated break points as equal if they both lie in the intersection of the two confidence intervals. Finally, we use the test in a real-data application on daily returns of ten large financial firms during the last financial crisis, in which we use the test on the whole period and in a rolling window of a fixed window size.

The rest of the paper is structured as follows. Section 2 shows the test statistic, its asymptotic distribution and the general testing problem. Here, the results from Manner et al. (2017) are shown specially for the dependence measure test. Results from the Monte Carlo simulations and the confidence interval procedure can be found in Section 3. Section 4 presents our empirical application and Section 5 concludes the paper. The bootstrap procedure for determining critical values and some figures are included in the appendix.

2. TESTING FOR CONSTANCY IN DISTRIBUTION FREE DEPENDENCE MEASURES

In this section we describe the non parametric distribution free dependence measure test proposed in Manner et al. (2017).

2.1. Data generating process and dependence measures

We assume that our data $\boldsymbol{Y} := [\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_T]$ (e.g. log returns) of sample size T and cross-sectional dimension N, with $\boldsymbol{Y}_t := [Y_{1t}, \ldots, Y_{Nt}]'$, can be described by a data model for every time step $t = 1, \ldots, T$ of the form

$$[Y_{1t}, \ldots, Y_{Nt}]' = \boldsymbol{Y}_t = \boldsymbol{\mu}_t(\boldsymbol{\phi}_0) + \boldsymbol{\sigma}_t(\boldsymbol{\phi}_0)\boldsymbol{\eta}_t, \tag{2.1}$$

with conditional mean $\boldsymbol{\mu}_t(\boldsymbol{\phi}_0) := [\mu_{1t}(\boldsymbol{\phi}_0), \ldots, \mu_{Nt}(\boldsymbol{\phi}_0)]'$, conditional variance $\boldsymbol{\sigma}_t(\boldsymbol{\phi}_0) := \mathsf{diag}\{\sigma_{1t}(\boldsymbol{\phi}_0), \ldots, \sigma_{Nt}(\boldsymbol{\phi}_0)\}$ and residual $\boldsymbol{\eta}_t := [\eta_{1t}, \ldots, \eta_{Nt}]'$. The residuals follow an unknown joint distribution $\boldsymbol{F}_{\boldsymbol{\eta}} = C(F_1(\boldsymbol{\eta}^1), \ldots, F_N(\boldsymbol{\eta}^N))$, with marginal distributions F_i and a copula function $C(\cdot)$, linking the marginal distributions to the joint distribution $\boldsymbol{F}_{\boldsymbol{\eta}}$. Here

$\boldsymbol{\eta} := [\boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_T]$ and $\boldsymbol{\eta}^i$ for $i = 1, \ldots, N$ are row vectors of $\boldsymbol{\eta}$. We assume that the copula captures the dependence of our real data $\boldsymbol{Y}$. We can $\sqrt{T}$ consistently estimate the data parameter vector $\boldsymbol{\phi}_0 := [\boldsymbol{\phi}_0^1, \ldots, \boldsymbol{\phi}_0^N]$, where this property is fulfilled by many time series models, e.g. ARMA and GARCH models and the estimator is denoted as $\hat{\boldsymbol{\phi}} := [\hat{\boldsymbol{\phi}}^1, \ldots, \hat{\boldsymbol{\phi}}^N]$. Note that $\boldsymbol{\phi}_0^i$ is the data parameter vector for the time sequence $i = 1, \ldots, N$ and $\hat{\boldsymbol{\phi}}^i$ the belonging estimator. The marginal distributions of the residuals $F_i(.)$ for $i = 1, \ldots, N$ are estimated by the empirical distribution function $\hat{F}_i$. In the following we consider estimated residual data $\{\hat{\boldsymbol{\eta}}_t := \boldsymbol{\sigma}_t^{-1}(\hat{\boldsymbol{\phi}})[\boldsymbol{Y}_t - \boldsymbol{\mu}_t(\hat{\boldsymbol{\phi}})]\}_{t=1}^T$, which we get from the inversed pre-estimated data model in (2.1), where all estimators of the defined components are denoted with hats.

Mainly, the pre-proposed test by Manner et al. (2017) compares sequential estimated dependence measures, stacked into a vector, to the full sample estimated analogue. The sequential dependence measure vectors $\hat{m}_{sT}$ of dimension $k \times 1$, where $s \in (0, 1]$ is a location parameter, consists of averaged pairwise dependence measure vectors $\hat{m}_{sT}^{ij}$, computed from the residuals $\{\hat{\boldsymbol{\eta}}_t\}_{t=1}^{\lfloor sT \rfloor}$, using data for time points $t = 1, \ldots, sT$. For the dependence measures of the pair $(\boldsymbol{\eta}^i, \boldsymbol{\eta}^j)$ copula based dependence measures that do not depend on the marginal distribution of the data are used. As in the work of Oh and Patton (2013), Spearman's rank correlation ρ^{ij} and quantile dependence λ_q^{ij} are considered. These can be defined for the bi-variate case in terms of the copula as

$$\rho^{ij} := 12 \int_0^1 \int_0^1 C_{ij}(u, v) du dv - 3$$

$$\lambda_q^{ij} := \begin{cases} P[F_i(\boldsymbol{\eta}^i) \leq q | F_j(\boldsymbol{\eta}^j) \leq q] = \frac{C_{ij}(q,q)}{q}, & q \in (0, 0.5] \\ P[F_i(\boldsymbol{\eta}^i) > q | F_j(\boldsymbol{\eta}^j) > q] = \frac{1-2q+C_{ij}(q,q)}{1-q}, & q \in (0.5, 1), \end{cases}$$

with probability transform $u = F_i(\boldsymbol{\eta}^i)$, $v = F_j(\boldsymbol{\eta}^j)$ and $C_{ij}(u, v)$ is the copula of the pair

$(\boldsymbol{\eta}^i, \boldsymbol{\eta}^j)$. The sample counterparts based on recursive samples are defined as

$$\hat{\rho}^{ij} := \frac{12}{\lfloor sT \rfloor} \sum_{t=1}^{\lfloor sT \rfloor} \hat{F}_i^s(\hat{\eta}_{it}) \hat{F}_j^s(\hat{\eta}_{jt}) - 3$$

$$\hat{\lambda}_q^{ij} := \begin{cases} \frac{\hat{C}_{ij}^s(q,q)}{q}, & q \in (0, 0.5] \\ \frac{1-2q+\hat{C}_{ij}^s(q,q)}{1-q}, & q \in (0.5, 1) \end{cases},$$

with sequential empirical bi-variate copula $\hat{C}_{ij}^s(u,v) := \frac{1}{\lfloor sT \rfloor} \sum_{t=1}^{\lfloor sT \rfloor} \mathbb{1}\{\hat{F}_i^s(\hat{\eta}_{it}) \leq u, \hat{F}_j^s(\hat{\eta}_{jt}) \leq v\}$ and sequential empirical distribution function $\hat{F}_i^s(y) := \frac{1}{\lfloor sT \rfloor} \sum_{t=1}^{\lfloor sT \rfloor} \mathbb{1}\{\hat{\eta}_{it} \leq y\}$, where data for time points $t = 1, \ldots, sT$ is considered. The Spearman's rank correlation coefficient is a global measure of the dependence structure, where the quantile dependencies measure the dependence structure in specific regions, e.g the lower and upper tails of a distribution. Note the pairwise dependence measures can be averaged for pairs that are assumed to be in the same industry block as in the case of equidependence or block equidependence models for further details see Oh and Patton (2017).

2.2. Testing problem

The null hypothesis which is considered is a constant dependence measure vector against the alternative of a single breakpoint at an unknown point in time,

$$H_0 : m_1 = m_2 = \cdots = m_T \qquad H_1 : m_t \neq m_{t+1} \text{ for some } t = \{1, \ldots, T-1\}.$$

The CUSUM-type test statistic is based on the maximum difference between the recursive estimates and the full sample estimate of the dependence measure vector. Formally, it is defined as

$$M := \sup_{s\in[\varepsilon,1]} M_{sT} := \sup_{s\in[\varepsilon,1]} s^2 T(\hat{m}_{sT} - \hat{m}_T)'(\hat{m}_{sT} - \hat{m}_T) \qquad (2.2)$$

$$\simeq \max_{\lfloor \varepsilon T \rfloor \leq t \leq T} \left(\frac{t}{T}\right)^2 T(\hat{m}_t - \hat{m}_T)'(\hat{m}_t - \hat{m}_T),$$

where $\hat{m}_{sT}$ is the recursive dependence measure vector defined above that uses the information up to time point $t = \lfloor sT \rfloor$ and $\hat{m}_T$ the full sample dependence measure vector analogue such as $\varepsilon > 0$ a trimming parameter. In the paper of Manner et al. (2017) it is noted that ε has to be chosen strictly greater than zero and in finite sample cases it should be chosen in away that we have enough data information to receive reasonable dependence measure vector estimates. For the finite sample case Manner et al. (2017) propose $\varepsilon = 0.2$ in the context of the copula parameter test for a better comparison of the two proposed tests in this paper. Note that it is sufficient to use $\varepsilon = 0.1$ for the non parametric dependence measure test in the later considered simulation study and empirical application. The test rejects the null hypothesis of a constant dependence measure vector, if the sequential estimated dependence vectors fluctuate to much over time, which is measured by $\sup_{s\in[\varepsilon,1]} s^2 T(\hat{m}_{sT} - \hat{m}_T)'(\hat{m}_{sT} - \hat{m}_T)$. The pre-factor $s^2 T$ puts less weight on deviations at the beginning of the observed period, due to the fact that $\hat{m}_{sT}$ fluctuates more for smaller sequential sample sizes sT. As mentioned, the statistic is of non-parametric nature and one can consider an appropriate subset of dependence measure settings and test for, e.g. breaks in the lower/upper quantile dependencies, Spearman's rank correlation, separately lower or upper quantile dependencies such as a combination of both, rank correlation and quantile dependencies. An analysis of different selected dependence measure settings, where residual data is simulated from different fat-tailed and skewed copula distribution models can be found in the simulation section.

The analytical results for the asymptotic distribution of the test statistic M can be found in Corollary 2 in Manner et al. (2017). Here the asymptotic results can be obtained by using two main assumptions (Assumption 1 and 2 in Manner et al. (2017)) where these ensure that the estimated rank correlation and quantile dependencies converge to their respective population counterparts, using residual data from a pre-estimated data model. Then under the null hypothesis $H_0 : m_1 = m_2 = \cdots = m_T$ and if Assumptions 1-2 hold, it follows

$$M = \sup_{s\in[\varepsilon,1]} s^2T(\hat{m}_{sT} - \hat{m}_T)'(\hat{m}_{sT} - \hat{m}_T) \xrightarrow{d} \sup_{s\in[\varepsilon,1]} (A(s) - sA(1))'(A(s) - sA(1)), \tag{2.3}$$

as $T \to \infty$, where $A(s)$ is defined in the proof of Lemma 7 in Manner et al. (2017). The proof of the limit result in equation (2.3) follows by a steady transformation of the result in Manner et al. (2017) Lemma 7. Under consideration of the quantiles of the limiting distribution, we receive an asymptotic α-test, with $\alpha \in (0, 1)$. We reject the null if

$$M > q_{1-\alpha}, \tag{2.4}$$

where $q_{1-\alpha}$ is the $(1 - \alpha)$-quantile of $\sup_{s\in[\varepsilon,1]} (A(s) - sA(1))'(A(s) - sA(1))$. If we reject the null hypothesis we speak of a structural break. The estimation of the change point location, once we detected a structural break, is embedded in calculating the test statistic and is given by $\hat{k} := \lfloor \hat{s}T \rfloor$, where $\hat{s}$ is the maximum point of the quadratic left side of (2.3), i.e.

$$\hat{s} = \operatorname*{argmax}_{s\in[\varepsilon,1]} s^2T(\hat{m}_{sT} - \hat{m}_T)'(\hat{m}_{sT} - \hat{m}_T). \tag{2.5}$$

The distribution term $A(s)$ in the asymptotic distribution of the test statistic is in general not known in closed form and depends on the underlying sample. For this reason critical values can not be computed or simulated directly. To overcome this issue a bootstrap procedure similar to the one in Manner et al. (2017) is used, which can be found in the appendix.

3. SIMULATIONS

In this section we want to analyze size and power properties of the dependence measure vector based test for different dependence measure settings in Monte Carlo simulations. We consider the following dependence measure settings

$$\hat{m}_1^{ij} = (\hat{\rho}^{ij}\ \hat{\lambda}_{0.05}^{ij}\ \hat{\lambda}_{0.1}^{ij}\ \hat{\lambda}_{0.9}^{ij}\ \hat{\lambda}_{0.95}^{ij})'$$
$$\hat{m}_2^{ij} = (\hat{\lambda}_{0.05}^{ij}\ \hat{\lambda}_{0.1}^{ij}\ \hat{\lambda}_{0.9}^{ij}\ \hat{\lambda}_{0.95}^{ij})'$$
$$\hat{m}_3^{ij} = (\hat{\lambda}_{0.9}^{ij}\ \hat{\lambda}_{0.95}^{ij})'$$
$$\hat{m}_4^{ij} = (\hat{\lambda}_{0.05}^{ij}\ \hat{\lambda}_{0.1}^{ij}\ )'$$
$$\hat{m}_5^{ij} = \hat{\rho}^{ij},$$

where we average all pairwise dependence measures in an equidependence way, i.e. $\hat{m} = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{m}^{ij}$, due to $\hat{m}^{ij} = \hat{m}^{ji}$ for $i \neq j$ and $\hat{m}^{ii} = 1$.

For our investigations, we consider a level of 5% and repeat the test 301 times for every scenario. Due to the fact that we are mainly interested in comparing the different dependence settings, we fix the data size and cross sectional dimension to $T = 1000$ and $N = 10$. An analysis for different combinations of T and N in the case of $\hat{m}_1^{ij} = (\hat{\rho}^{ij}\ \hat{\lambda}_{0.05}^{ij}\ \hat{\lambda}_{0.1}^{ij}\ \hat{\lambda}_{0.9}^{ij}\ \hat{\lambda}_{0.95}^{ij})'$ can be found in Manner et al. (2017). For a better comparison of the break point location estimates in empirical applications, determined by different dependence measure settings within the testing procedure, we use an heuristic procedure. With this we are able to make a statement whether two estimated break point locations $\hat{s}_a$ and $\hat{s}_b$, where $a \neq b$, belong to the same class of break points, choosing the dependence measure settings $\hat{m}_a^{ij}$ and $\hat{m}_b^{ij}$, with $a, b \in \{1, 2, 3, 4, 5\}$. Note that we consider the break point location estimator defined in equation (2.5), which is a scalar in the uniform interval $(0, 1]$. We determine (pivot) confidence intervals $\hat{K}_a := [2\hat{s}_a - \hat{c}^a_{1-\frac{\alpha}{2}}, 2\hat{s}_a - \hat{c}^a_{\frac{\alpha}{2}}]$ and $\hat{K}_b := [2\hat{s}_b - \hat{c}^b_{1-\frac{\alpha}{2}}, 2\hat{s}_b - \hat{c}^b_{\frac{\alpha}{2}}]$, where $\hat{c}^a_{(\cdot)}$ and $\hat{c}^b_{(\cdot)}$ are estimated quantiles of the bootstrap distribution of $\hat{s}_a$ and $\hat{s}_b$ which can be determined by using the following (percentile) bootstrap procedure.

We consider the residual sample $\{\boldsymbol{\eta}_t\}_{t=1}^{T}$ in which we detected two break point locations $\hat{s}_a$ and $\hat{s}_b$, using the dependence measure setting $\hat{m}_a^{ij}$ and $\hat{m}_b^{ij}$.

i) Split the sample in $\{\hat{\boldsymbol{\eta}}_t\}_{t=1}^{\hat{s}_a T}$ and $\{\hat{\boldsymbol{\eta}}_t\}_{t=\hat{s}_a T+1}^{T}$ for setting $\hat{m}_a^{ij}$ such as $\{\hat{\boldsymbol{\eta}}_t\}_{t=1}^{\hat{s}_b T}$ and $\{\hat{\boldsymbol{\eta}}_t\}_{t=\hat{s}_b T+1}^{T}$ for setting $\hat{m}_b^{ij}$.

ii) Sample separately with replacement from $\{\hat{\boldsymbol{\eta}}_t\}_{t=1}^{\hat{s}_a T}$ and $\{\hat{\boldsymbol{\eta}}_t\}_{t=\hat{s}_a T+1}^{T}$ such as $\{\hat{\boldsymbol{\eta}}_t\}_{t=1}^{\hat{s}_b T}$ and $\{\hat{\boldsymbol{\eta}}_t\}_{t=\hat{s}_b T+1}^{T}$ to obtain B bootstrap samples $\{\hat{\boldsymbol{\eta}}_{t,a}^{(p)}\}_{t=1}^{T}$ and $\{\hat{\boldsymbol{\eta}}_{t,b}^{(p)}\}_{t=1}^{T}$, for $p = 1, \ldots, B$.

iii) Estimate the break point location $\hat{s}_a^{(p)}$ for each bootstrap sample $\{\hat{\boldsymbol{\eta}}_{t,a}^{(p)}\}_{t=1}^{T}$ and $\hat{s}_b^{(p)}$ for each bootstrap sample $\{\hat{\boldsymbol{\eta}}_{t,b}^{(p)}\}_{t=1}^{T}$for $p = 1, \ldots, B$, using (2.5).

iv) Determine the critical values $\hat{c}_{\frac{\alpha}{2}}^{a}, \hat{c}_{\frac{\alpha}{2}}^{b}$ and $\hat{c}_{1-\frac{\alpha}{2}}^{a}, \hat{c}_{1-\frac{\alpha}{2}}^{b}$ with

$$\frac{1}{B}\sum_{p=1}^{B} \mathbb{1}\{\hat{s}_{(\cdot)}^{(p)} < c\} \stackrel{!}{=} \frac{\alpha}{2} \qquad \text{and} \qquad \frac{1}{B}\sum_{p=1}^{B} \mathbb{1}\{\hat{s}_{(\cdot)}^{(p)} > c\} \stackrel{!}{=} \frac{\alpha}{2},$$

where $\alpha \in (0, 1)$.

We say that two estimated break point locations $\hat{s}_a$ and $\hat{s}_b$ can be considered equal if both lie in the intersection of the two determined confidence intervals, i.e.

$$\hat{s}_a, \hat{s}_b \in \hat{K}_a \cap \hat{K}_b. \tag{3.1}$$

Note, this procedure is only plausible if we consider the same testing period for both dependence settings $\hat{m}_a^{ij}$ and $\hat{m}_b^{ij}$. Therefore, in the empirical application the procedure can only be applied for a break comparison in the full sample testing and can not be used in the sequential testing procedure, due to the fact that similar break point locations may belong to different tested periods. Further note that, the pivot confidence interval argues that the estimation error between the estimated break point $\hat{s}$ and the true break point s_0 is approximately the same as the difference between the estimated break location $\hat{s}$ and the bootstrap break estimate $\hat{s}^{(p)}$, i.e.

$$\begin{aligned}1-\alpha &\approx P(\hat{c}_{\frac{\alpha}{2}} \leq \hat{s}^{(p)} \leq \hat{c}_{1-\frac{\alpha}{2}}) \\ &= P(\hat{k} - \hat{c}_{1-\frac{\alpha}{2}} \leq \hat{s} - \hat{s}^{(p)} \leq \hat{s} - \hat{c}_{\frac{\alpha}{2}}) \\ &\approx P(\hat{s} - \hat{c}_{1-\frac{\alpha}{2}} \leq s_0 - \hat{s} \leq \hat{s} - \hat{c}_{\frac{\alpha}{2}}) \\ &= P(2\hat{s} - \hat{c}_{1-\frac{\alpha}{2}} \leq s_0 \leq 2\hat{s} - \hat{c}_{\frac{\alpha}{2}}).\end{aligned}$$

The construction of the confidence intervals is for example similar to the one in Hušková and Kirch (2008). For the simulations we directly simulate residual data from different skewed and fat tailed Copulas $C(\cdot)$. We first consider that our residuals $\boldsymbol{\eta}_t$ to be jointly distributed with a simple one factor copula model following (Oh and Patton, 2013) and (Oh and Patton, 2017), where the copula is implied by the following factor structure

$$\boldsymbol{\eta}_t = [\eta_{1t}, \ldots, \eta_{Nt}]' = \boldsymbol{\beta}_t Z + \boldsymbol{q} \sim C(\cdot, (\boldsymbol{\beta}_t, \nu, \lambda)), \tag{3.2}$$

with $\boldsymbol{\beta}_t = (\beta_t, \ldots, \beta_t)'$ a parameter vector of size N, $Z \sim \mathsf{Skew\ t}\,(\nu^{-1}, \lambda)$[1] and $\boldsymbol{q} = [q_{1t}, \ldots, q_{Nt}]'$ with $q_{it} \overset{i.i.d.}{\sim} \mathsf{t}\,(\nu^{-1})$ for $i = 1, \ldots, N$ and $t = 1, \ldots, T$. We fix $\nu^{-1} = 0.25$ and vary $\lambda = \{-0.5, 0, 0.5\}$, such that our model is parametrized by the single factor loading $\theta = \beta_t$. For the power analysis we generate data with a break point at $\frac{T}{2}$ for different sample sizes, where the data is simulated with $\theta = 1$ for the first $\frac{T}{2}$ data points, denoted by θ_0, whereas after the break we increase the parameter to $\theta = \{1.1, 1.2, 1.3, 1.4, 1.5\}$, denoted by θ_1. The results for the factor copula model (3.2) using $\lambda = \{-0.5, 0, 0.5\}$ can be seen in Table 1. Table 1 reveals that $\hat{m}_5^{ij} = \hat{\rho}^{ij}$ gains overall the highest power, directly followed by the setting $\hat{m}_1^{ij}$ considering Spearman's rho, lower and upper quantile dependence measures. The cases where only upper $\hat{m}_3^{ij}$ or lower quantiles $\hat{m}_4^{ij}$ are considered suffer from poor power properties compared to the other dependence settings. Considering both upper and lower quantile

[1] As in Oh and Patton (2017) this refers to the skewed t-distribution by Hansen (1994).

Table 1: Size and power Factor Copula

$T=1000, N=10$	$\theta_0=1$	$\theta_1=1.1$	$\theta_1=1.2$	$\theta_1=1.3$	$\theta_1=1.4$	$\theta_1=1.5$
			$\lambda=-0.5$			
$\hat{m}_1^{ij}$	0.0465	0.1628	0.3887	0.6013	0.8007	0.9269
$\hat{m}_2^{ij}$	0.0498	0.1462	0.3189	0.5249	0.6944	0.8704
$\hat{m}_3^{ij}$	0.0365	0.0897	0.2259	0.4784	0.7043	0.8571
$\hat{m}_4^{ij}$	0.0498	0.1329	0.2724	0.4286	0.5781	0.7176
$\hat{m}_5^{ij}$	0.0532	0.2558	0.6645	0.9435	0.9934	1.0000
			$\lambda=0$			
$\hat{m}_1^{ij}$	0.0532	0.1993	0.4485	0.7010	0.9003	0.9767
$\hat{m}_2^{ij}$	0.0532	0.1927	0.3787	0.6213	0.8538	0.9358
$\hat{m}_3^{ij}$	0.0432	0.1229	0.2625	0.4385	0.6478	0.8206
$\hat{m}_4^{ij}$	0.0565	0.1495	0.2857	0.4485	0.6146	0.7641
$\hat{m}_5^{ij}$	0.0598	0.2791	0.7176	0.9668	0.9967	1.0000
			$\lambda=0.5$			
$\hat{m}_1^{ij}$	0.0565	0.1661	0.3322	0.5781	0.8538	0.9635
$\hat{m}_2^{ij}$	0.0764	0.1495	0.2890	0.4917	0.7342	0.9203
$\hat{m}_3^{ij}$	0.0731	0.1395	0.2658	0.3854	0.5781	0.7741
$\hat{m}_4^{ij}$	0.0332	0.1096	0.2558	0.4651	0.6611	0.8538
$\hat{m}_5^{ij}$	0.0498	0.2658	0.7043	0.9502	1.0000	1.0000

Note: Table 1 reports the rejection rate for $\theta_0 = 1.0$ and $\theta_1 = \{1.1, 1.2, 1.3, 1.4, 1.5\}$ for different dependence measure combinations and $\lambda = \{-0.5, 0, 0.5\}$ in the DGP (3.2).

dependencies in $\hat{m}_2^{ij}$ has better power properties than the separated cases. For a clearer comparison of the different dependence vector settings, especially for the cases where only lower or upper quantile dependence measures are used, we consider two more data generating processes. First we consider residual data generated by a Clayton copula, where we vary the parameter from $\theta_0 = 2.0$ to $\theta_1 = \{2.2, 2.4, 2.6, 2.8, 3.0\}$. Second we consider residual data generated from a Gumbel copula, where we vary $\theta_0 = 2.0$ to $\theta_1 = \{3.0, 4.0, 5.0, 6.0, 7.0\}$. In both cases again θ_1 denotes the parameter value after the break at $\frac{T}{2}$ and θ_0 the parameter value before the break. Due to the heavy tailed behavior of the Clayton (strong lower quantile dependence) and Gumbel Copula (strong upper quantile dependence), see Figure 6.5 and Figure 6.6 in the appendix, the dependence structure in the lower (Clayton) and upper (Gumbel) cases just changes slightly by varying the parameter values after the break. This yields poor power properties of the test by only using lower quantile dependencies $\hat{m}_4^{ij}$, where the Clayton Copula is used as the data generating process (DGP) (cf. Table 2).

Table 2: Size and power Clayton Copula

$T = 1000, N = 10$	$\theta_0 = 2.0$	$\theta_1 = 2.2$	$\theta_1 = 2.4$	$\theta_1 = 2.6$	$\theta_1 = 2.8$	$\theta_1 = 3.0$
$\hat{m}_1^{ij}$	0.0332	0.6777	0.9734	1.0000	1.0000	1.0000
$\hat{m}_2^{ij}$	0.0332	0.5847	0.9169	0.9834	1.0000	1.0000
$\hat{m}_3^{ij}$	0.0532	0.4186	0.9169	0.9967	1.0000	1.0000
$\hat{m}_4^{ij}$	0.0365	0.5050	0.7375	0.8372	0.9834	0.9874
$\hat{m}_5^{ij}$	0.0498	0.9867	1.0000	1.0000	1.0000	1.0000

Note: Table 2 reports the rejection rate for $\theta_0 = 2.0$ and $\theta_1 = \{2.2, 2.4, 2.6, 2.8, 3.0\}$ for different dependence measure combinations, where the DGP is a Clayton Copula with parameter α (more mass on lower tail).

On the other hand, only allowing for upper quantile dependencies $\hat{m}_3^{ij}$, where the Gumbel Copula is used as the DGP, the test suffers also from poor power properties (cf. Table 3).

Similar results, only using upper or lower quantile dependencies can be seen for the factor copula model in the case of $\lambda = \{-0.5, 0.5\}$, see Table 1. A combination of lower and upper quantile dependence measures ($\hat{m}_2^{ij}$) again yields therefore better power properties. Yet, the dependence vector settings $\hat{m}_1^{ij}$ and $\hat{m}_5^{ij}$, where Spearman's rank correlation is included, imply again better power properties of the test. This can be explained due to the fact that quantile dependencies suffer from a poor data amount in the tails and a larger data size is required to gain better power properties, compared to the usage of Spearman's rho, where the final rank correlation coefficient is computed out of the whole data information and is a global dependence measure. Before taking a look at the simulation results one would expect that the usage of more dependence measures within the dependence measure vector increases the power of the test (2.4). However, it is not the case for the setting $\hat{m}_1^{ij}$. It seems that the dependence vector compounded of a mixture of quantile dependencies and rank correlation inherits the bad power properties of the quantile dependencies and the better performance, compared to the settings $\hat{m}_2^{ij}, \hat{m}_3^{ij}$ and $\hat{m}_4^{ij}$, is mainly driven by the usage of the rank correlation coefficient.

However, the usage of various dependence settings may give us different break point estimates, which can be seen in the empirical application, c.f. section 4. In times of clearer structural break, i.e. periods which can be assigned to events that effected the financial market, one can be more sure whether the detected break is plausible if several dependence settings result in the same break event. To test for equality of two found break points use the confidence interval procedure explained above.

In the following we include a small simulation study for the confidence interval procedure using the dependence measure settings $\hat{m}_1^{ij}$ and $\hat{m}_3^{ij}$, where the break estimates of these settings are later compared in the full sample testing in the empirical application. We simulate a residual data set, using the DGP in (3.2) with $\lambda = -0.5$, where we constructed a break

Table 3: Size and power Gumbel Copula

$T = 1000, N = 10$	$\theta_0 = 2.0$	$\theta_1 = 3.0$	$\theta_1 = 4.0$	$\theta_1 = 5.0$	$\theta_1 = 6.0$	$\theta_1 = 7.0$
$\hat{m}_1^{ij}$	0.0399	0.1628	0.4352	0.8671	0.9668	1.0000
$\hat{m}_2^{ij}$	0.0365	0.1329	0.3654	0.7874	0.9003	0.9834
$\hat{m}_3^{ij}$	0.0399	0.1229	0.2492	0.4618	0.5648	0.6678
$\hat{m}_4^{ij}$	0.0565	0.3522	0.6445	0.8571	0.9402	0.9734
$\hat{m}_5^{ij}$	0.0532	0.5282	0.9435	1.0000	1.0000	1.0000

Note: Table 3 reports the rejection rate for $\theta_0 = 2.0$ and $\theta_1 = \{3.0, 4.0, 5.0, 6.0, 7.0\}$ for different dependence measure combinations, where the DGP is a Gumbel Copula with parameter α (more mass on upper tail).

at $\frac{T}{2}$, i.e. $s_0 = 0.5$. We fix the cross sectional dimension to $N = 10$ and vary the sample size $T = \{500, 1000, 1500\}$ and the break size by $\theta_1 = \{1.5, 2.0, 2.5\}$. For all simulations we use $B = 500$ bootstrap replications. We present the coverage probability $P(0.5 \in \hat{K}_1)$, the coverage probability $P(0.5 \in \hat{K}_3)$ and the probability that the constructed break at $s_0 = 0.5$ lies in the intersection of $\hat{K}_1 \cap \hat{K}_3$, i.e. $P(0.5 \in \hat{K}_1 \cap \hat{K}_3)$, using again 301 Monte-Carlo simulations and a confidence level of 5 percent. Table 4 reveals that the coverage probability of $\hat{K}_1$ and $\hat{K}_3$ tends to $1 - \alpha = 0.95$ for increasing sample size and break size. The probability that the actual break at $s_0 = 0.5$ lies in the interval $\hat{K}_1 \cap \hat{K}_3$ tends to $(1 - \alpha)^2$.

Next we simulate two residual data sets, using again the DGP in (3.2), where we constructed break points, lying nearby at $s_0^{(1)} = 0.429$ in the first set and a break at $s_0^{(2)} = 0.5$ in the second set. For all simulations we consider the same scenarios as before and we present the coverage probability $P(0.429 \in \hat{K}_1)$, the coverage probability $P(0.5 \in \hat{K}_3)$ and the probability that both constructed breaks 0.429 and 0.5 lie in the intersection of $\hat{K}_1 \cap \hat{K}_3$, i.e. $P(0.429, 0.5 \in \hat{K}_1 \cap \hat{K}_3)$. The results can be seen in Table 5.

Here, the coverage probability of $\hat{K}_1$ for a break at 0.429 and $\hat{K}_3$ for a break at 0.5 tends to

Table 4: Coverage probability same break points

	$\Big(P(0.5 \in \hat{K}_1)\ P(0.5 \in \hat{K}_3)\ P(0.5 \in \hat{K}_1 \cap \hat{K}_3)\Big)$		
$B = 500, N = 10$	$T = 500$	$T = 1000$	$T = 1500$
$\theta_1 = 1.5$	(0.80 0.78 0.64)	(0.93 0.87 0.81)	(0.94 0.90 0.84)
$\theta_1 = 2.0$	(0.91 0.88 0.80)	(0.95 0.93 0.89)	(0.95 0.92 0.89)
$\theta_1 = 2.5$	(0.93 0.94 0.88)	(0.93 0.94 0.88)	(0.95 0.95 0.91)

Note: Table 4 reports the coverage probability of 301 simulated confidence intervals $\hat{K}_1$ and $\hat{K}_3$ for a break constructed at 0.5 such as the coverage probability of $0.5 \in \hat{K}_1 \cap \hat{K}_3$ where the factor copula model (3.2) is used as the DGP and $\alpha = 0.05$.

Table 5: Coverage probability different break points

	$\Big(P(0.429 \in \hat{K}_1)\ P(0.5 \in \hat{K}_3)\ P(0.429, 0.5 \in \hat{K}_1 \cap \hat{K}_3)\Big)$		
$B = 500, N = 10$	$T = 500$	$T = 1000$	$T = 1500$
$\theta_1 = 1.5$	(0.81 0.78 0.46)	(0.91 0.87 0.52)	(0.92 0.90 0.31)
$\theta_1 = 2.0$	(0.93 0.88 0.21)	(0.95 0.93 0.01)	(0.96 0.92 0.00)
$\theta_1 = 2.5$	(0.95 0.94 0.01)	(0.95 0.94 0.00)	(0.95 0.95 0.00)

Note: Table 5 reports the coverage probability of 301 simulated confidence intervals $\hat{K}_1$ and $\hat{K}_3$ for a break constructed at 0.5 and 0.429 such as the coverage probability of $0.429, 0.5 \in \hat{K}_1 \cap \hat{K}_3$ where the factor copula model (3.2) is used as the DGP and $\alpha = 0.05$.

$1 - \alpha = 0.95$ for increasing sample size and break size, where the results of $P(0.5 \in \hat{K}_3)$ are obviously the same as in Table 4. On the other hand, the probability that the two break points 0.429 and 0.5 lie in the interval $\hat{K}_1 \cap \hat{K}_3$ tends to zero for increasing sample size and break steps, where the convergence is much faster in the break step θ_1. Note that for example a break step of $\theta_1 = 1.5$ implies a rank correlation change before and after the break of 0.17, where a break change of $\theta_1 = 2.0$ implies a rank correlation change of 0.25. Thus we conclude, that our procedure to test for equality of two estimated break point locations results in a reasonable sized and powered testing procedure, if the break steps and the sample size are high enough.

4. APPLICATION

To analyze the applicability of the dependence measure vector based test we apply the test to a financial data set and determine structural breaks in the dependence measure vector. We are interested in the similarity and diversity of the estimated break point locations using different dependence measure settings analyzed in the simulation section 3. For a better comparison of similar break dates we use the confidence interval procedure presented and analyzed in Section 3. We use daily stock log-return data between 29.01.2002 to 01.07.2013 of ten large firms, namely Citigroup, HSBC Holdings ($), UBS-R, Barclays, BNP Paribas, HSBC Holdings (ORD), Mitsubishi, Royal Bank, Credit Agricole and Bank of America. This implies a sample size of $T = 2980$ and cross sectional dimension $N = 10$. For all data points the closing price for the calculation of the log-returns are used. By taking a look at the time evolution of the log-returns of the portfolio in Figure 4.1, one can immediately see the strong fluctuations between 2002-2003, 2007-2008 and 2011-2012 in nearly all single assets, indicating a joint behavior during these periods within the portfolio. To get a better understanding of the joint dependence behavior we calculated the pairwise averaged Spearman's rank correlation coefficient in a rolling window of size 150, the result can be seen in Figure 4.2.

The strongest joint fluctuations and increase of the correlation coefficient appear between the time span of the heights of the last financial crisis in the time span between the beginning of 2007 and the end of 2008. The other strong fluctuations and increase of the correlation can be explained by the downturn in stock prices in stock exchanges across the United States, Canada, Asia and Europe in October 2002 and the euro crisis peek in 2011.

Due to the fact that we consider residual data, we first have to estimate a data model for

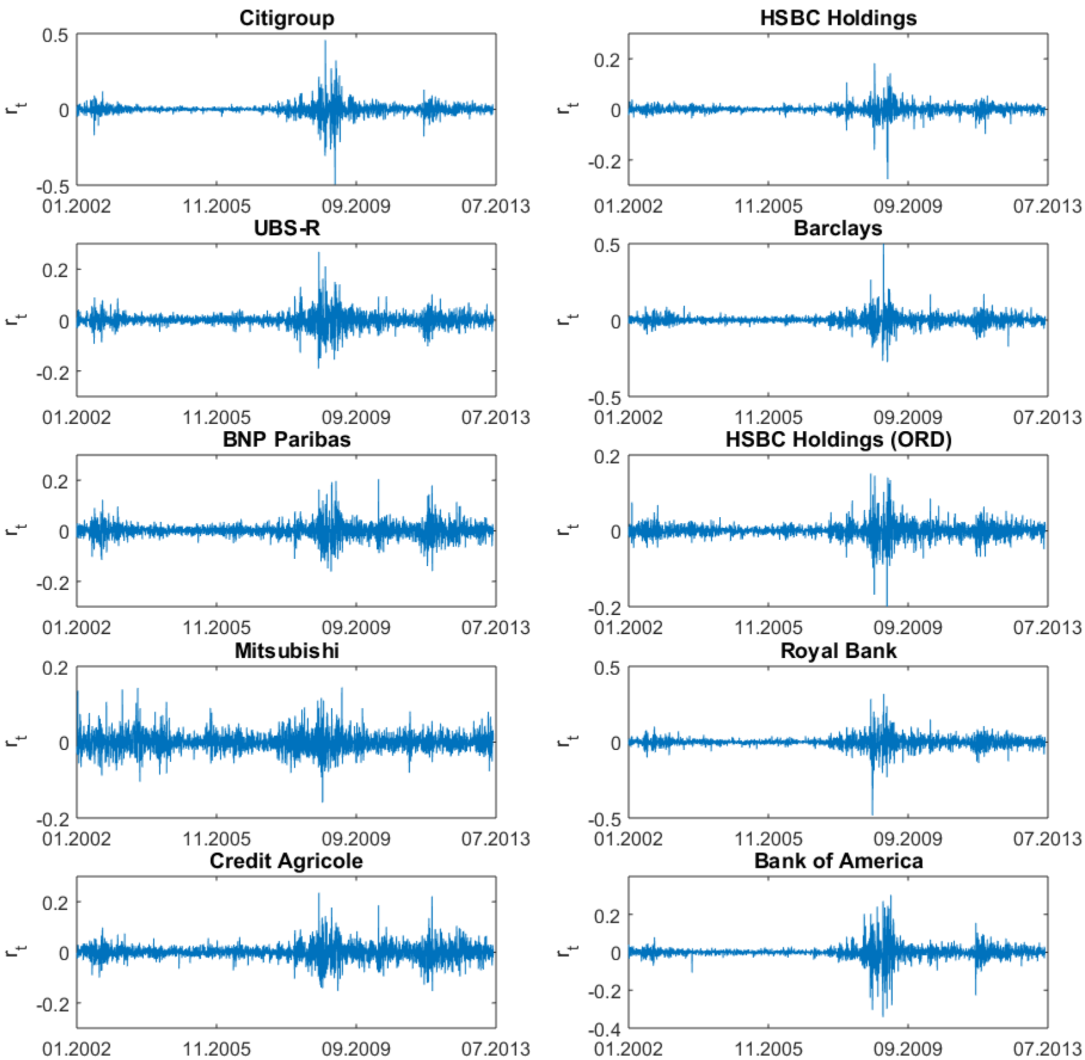

Figure 4.1: Daily log returns of our considered portfolio between 29.01.2002 to 01.07.2013.

each return series $i = 1, \ldots, N$. Therefore we use an AR(1)-GARCH(1,1) model to model the conditional mean and variance

$$r_{i,t} = \alpha_i + \beta_i r_{i,t-1} + \sigma_{i,t}\eta_{it},$$

$$\sigma_{it}^2 = \gamma_{i0} + \gamma_{i1}\sigma_{i,t-1}^2 + \gamma_{i2}\sigma_{i,t-1}^2\eta_{i,t-1}^2,$$

for $t = 1, \ldots, 2980$.

By using the data model parameter estimates we are able to compute the standardized residuals by inverting the model. Note that the marginal distributions of the residuals are estimated using the empirical CDF. We apply the test, considering the five dependence vector settings from the simulation section, to the residual data. We consider two approaches:

1) We apply the test to the pre-determined residual data considering information between 1 and T.

2) We apply the test in a rolling window setting where we consider periods of size 400. If a breakpoint is detected in the period $[t_1, (t_1 - 1) + 400]$ we estimate the break point location $\hat{k}$ and $[t_1 + 1, t_1 + 400]$ is the next considered period, where $t_1 = \hat{k}$. If no break point is detected consider the next time step $t_1 + 1$. We start the procedure by setting $t_1 = 1$ and terminate the procedure if $t_1 + 400 > T$. The data model is re-estimated for every considered period.

The break detection results for approach 1) can be found in Table 6 and Figure 4.2 where the results for approach 2) can be found in Table 7 and Figure 4.3. First we take a look

Table 6: List of found breakpoints approach 1)

year	$\hat{m}_1^{ij}$	$\hat{m}_2^{ij}$	$\hat{m}_3^{ij}$	$\hat{m}_4^{ij}$	$\hat{m}_5^{ij}$	avg
2007	09.07.2007	09.07.2007	08.08.2007	09.07.2007	09.07.2007	17.07.2007

Note: Table 6 reports the found break points for the five dependence vector setting with in the test using approach 1) such as the averaged break point location.

at the results for approach 1), here nearly all dependence settings found the same break at the 09.07.2007 (transformed to the uniform interval $\hat{s}_1 = 0.476$) except the setting choosing

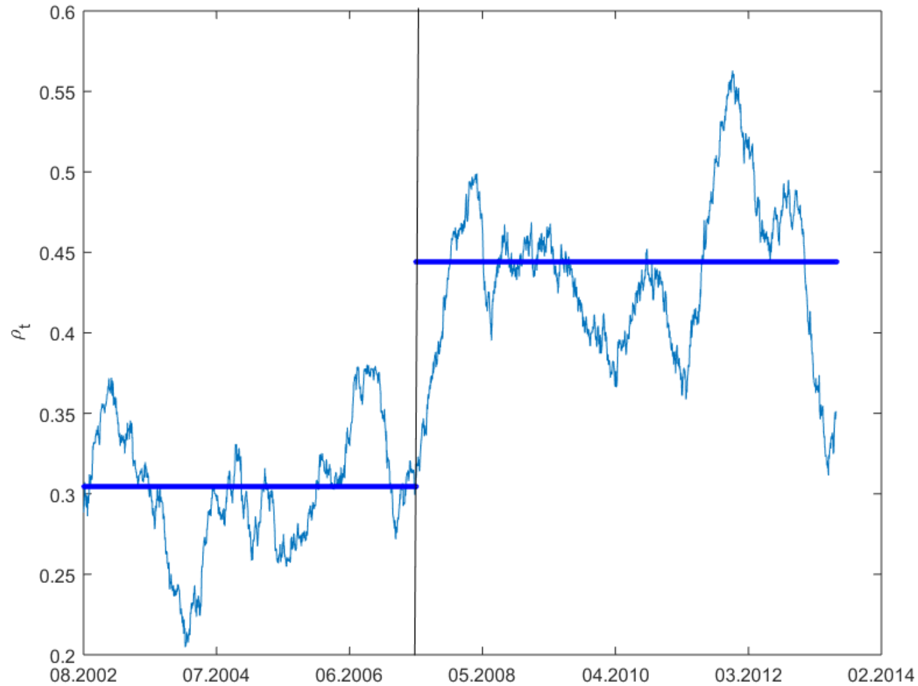

Figure 4.2: Pairwise averaged Spearman's rank correlation coefficient in a rolling window of size 150 with averaged found break point location (black line) at 17.07.2007 using approach 1) such as the estimated rank correlation coefficient from break to break (thick blue line).

the upper quantiles, here the break is found close by at the 08.08.2007 (transformed to the uniform interval $\hat{s}_3 = 0.484$). It seems obvious that the different estimated break point also belongs to the other four break events, which can be explained by one height of the last financial crisis. Nevertheless we check the suspicion using the confidence interval procedure from section 3 with the settings $\hat{m}_1^{ij}$ and $\hat{m}_3^{ij}$. We find that the two estimated break point locations $\hat{s}_1 = 0.476$ and $\hat{s}_3 = 0.484$ lie in the intersection of both confidence intervals and thus conclude that the estimated break point locations belong to the same break event. As we will see by using approach 2), there might be more break point locations, but approach 1) gives us the most significant break in our data set. By taking a look at figure 4.2, where we plotted the pairwise averaged Spearman's rank correlation coefficient in a rolling window of size 150, we see a strong increase of the rank correlation coefficient after the break (indicated by the black line) from 0.31 up to 0.44 (indicated by the thick blue line), where the overall maximum jump is even higher between 0.27 and 0.49.

Table 7: List of found breakpoints approach 2)

year	$\hat{m}_1^{ij}$	$\hat{m}_2^{ij}$	$\hat{m}_3^{ij}$	$\hat{m}_4^{ij}$	$\hat{m}_5^{ij}$	avg
2002/2003	30.12.2002	08.01.2003	20.12.2002		23.12.2002	27.12.2002
2004	19.02.2004	26.02.2004		05.03.2004	04.03.2004	01.03.2004
2005						
2006				11.05.2006		11.05.2006
2007	24.07.2007	11.07.2007	17.07.2007	16.02.2007	09.07.2007	15.06.2007
2008	16.07.2008	08.08.2008	16.07.2008	17.07.2008		23.07.2008
2009						
2010	21.04.2010	15.06.2010	29.04.2010	10.06.2010		19.05.2010
2011	28.06.2011	28.06.2011	21.09.2011	14.06.2011	20.05.2011	05.07.2011
2012				14.08.2012		15.08.2012

Note: Table 7 reports the found break points for the five dependence vector setting with in the test using approach 2) such as the averaged break point locations.

Next we take a look at the results for approach 2), where we tested the whole period sequentially in a rolling window of size 400, where we found some more breaks. Note that an obvious issue with this procedure is its multiple testing nature, in particular one should adapt the confidence levels accordingly and be aware of this when interpreting testing results. By taking a look at the breakpoints (cf. Table 7) most detections can be explained by well known financial market crashes from the last twenty years. First as already mentioned the downturn in stock prices in stock exchanges across the United States, Canada, Asia and Europe in October 2002, the start of the Iraqi war 2003/2004, start of the last financial crisis in 2007 as well as the bankruptcy of Lehman Brother's in 2008 and last the Euro crisis starting at the end of 2009 with its height in 2011. The break point estimates of the dependence settings $\hat{m}_1^{ij}, \hat{m}_2^{ij}$ and $\hat{m}_3^{ij}$ seem to be really closely related and belong to the above mentioned events, where the break event in 2004 seems not to be significant for the upper quantile setting. In contrast to the simulation study, where the setting $\hat{m}_5^{ij}$ gains the highest power over all other settings, the test with the setting $\hat{m}_5^{ij}$ detects only four significant break dates at 2002, 2004, 2007 and 2011, which are overall the most significant. A slightly different break result is given by the lower quantile setting $\hat{m}_4^{ij}$, where the tested periods in 2002/2003 seem not to be significant. On the other hand, breaks are detected in the mid of 2006 and 2012.

One advantage of the use of different dependence settings is that we are able to conclude that a detected break point is in a way more significant than another, if it is detected by more than one setting. The break events in 2002, 2004, 2007, 2008, 2010 and 2011 are such break points and the breaks in 2002, 2004, 2008 and 2010 are detected by four settings where the break events in 2007 and 2011 are detected by even five settings, corresponding to the most significant breaks and can be explained by the pre-mentioned well known financial market crashes. The detected break point in 2007 is also in line with the detected break event using approach 1), corresponding to the highest dependence change in the considered testing period. On the other hand we also get a different break picture as in the case of using the setting $\hat{m}_4^{ij}$, motivating the usage of flexible dependence measure settings for a clearer and wider interpretation of the results. Due to the fact that the found break dates correspond over all to the same break events and being really close to each other in the most cases, we average the break dates over all settings (cf. last column in Table 7).

We again plot the pairwise averaged Spearman's rank correlation coefficient in a rolling window and mark the averaged break point estimates together with the rank correlation estimates from break to break. Here five similar averaged break dates marked with the black line, four averaged detected break dates marked with the red line and single detected break points corresponding to the green line. The horizontal blue lines correspond to the values of the estimated rank correlation coefficient from break to break, see figure 4.3. Noticeable

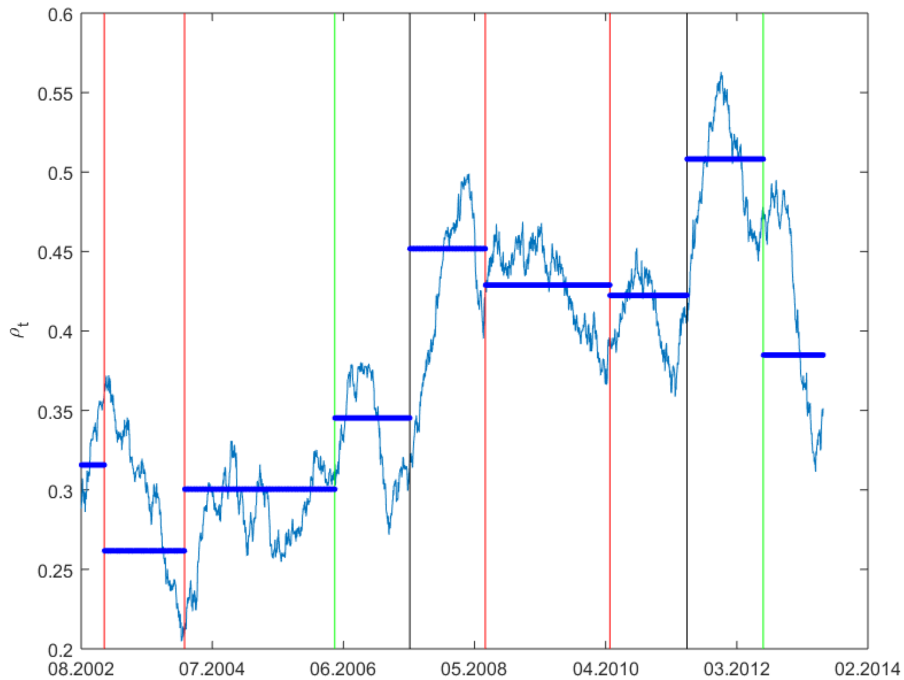

Figure 4.3: Pairwise averaged Spearman's rank correlation coefficient in a rolling window of size 150 and averaged break point locations (averaged over five similar detected break points (black line), averaged over four similar detected break points (red line) and single detected break points (green line)) using approach 2) such as the estimated rank correlation coefficient from break to break (thick blue line).

are the high jumps of the rank correlation in the periods of the most significant breaks in 2007 and 2011, where the correlation jumps from 0.35 to 0.45 (2007) and 0.42 to 0.51 (2011) considering break to break estimates. Note that the overall increase, considering the rolling window rank correlation estimates, is even higher. Furthermore the upper quantile dependence measures increase strongly here (cf. Figure 4.4).

In general, nearly all detected break events correspond in an increase of the considered dependence measures (cf. Figure 4.4), expect the first detected break in 2002/2003, where the peak in this period of nearly all dependencies is reached. Also in the time span after the

Figure 4.4: Dependence measure estimates (Spearman's rank correlation and $(0.05, 0.1, 0.9, 0.95)$-quantile dependencies) from break to break.

last financial crisis, we see a decrease of the rank correlation and the 0.9-quantile dependence. However, we see an overall increase of the dependencies within the portfolio. From a portfolio manager point of view the increase of the dependencies within our considered portfolio is disadvantageous and one is rather interested to decrease the dependencies by changing the portfolio to lower the risk of losses, which is known as the diversification effect.

5. CONCLUSION

We investigated a test for constant distribution free dependence measures, considering pairwise averaged Spearman's rho and quantile dependencies in equidependence settings, first proposed by Manner et al. (2017). The asymptotic null distribution is not known in closed form and therefore estimated by an i.i.d. bootstrap procedure. A size and power anaylsis, using different dependence measure settings for different simulated fat and skewed distributed data, is considered. Here the best power properties are gained by considering solely Spearman's rank correlation and a combination of Spearman's rank correlation and quantile dependencies, where the simple setting using only the rank correlation coefficient works best. The settings using only upper or lower quantile dependencies suffer from poor power properties. Further we found that the use of upper quantile dependencies results in better power properties at present strongly left skewed data compared to lower quantile dependencies and on the other way around lower quantile dependencies result in better power properties by considering right skewed data compared to the usage of upper quantile dependencies. Considering jointly lower and upper quantile dependencies always results in better power properties as the separate considerations. The test is also applied to a real data application to show the usefulness of the flexibility by the choice of different dependence measure settings. We use historical data of ten large companies during the last financial crisis from 2002 to the mid of 2013. One advantage of the use of different dependence settings is that we are able to conclude that a detected break point is in a way more significant than another, if it is detected by more than one setting. On the other hand we also get a different break picture, motivating the usage of flexible dependence measure settings and the combination of rank correlation and quantile dependencies. Further we use an heuristic procedure to be able to make a statement for equality of two estimated break point locations, transformed to the uniform interval, using different dependence vector settings.

6. APPENDIX

6.1. Bootstrap distribution

The bootstrap distribution of the test statistic M is obtained by calculating B versions of the process $\frac{t}{T}\sqrt{T}\left(\hat{m}_t^{(b)} - \hat{m}_T^{(b)}\right)$, which can be calculated fast and directly from the data, where $t = sT$. The following steps are used:

i) Sample with replacement from the standardized residuals $\{\hat{\eta}_i\}_{i=1}^{T}$ to obtain a B bootstrap samples $\{\hat{\eta}_i^{(b)}\}_{i=1}^{T}$, for $b = 1, \dots, B$.

ii) Use $\{\hat{\eta}_i^{(b)}\}_{i=1}^{t}$ to compute $\hat{m}_t^{(b)}$ for $b = 1, \dots, B$ and $t = \varepsilon T, \dots, T$ and $\{\hat{\eta}_i\}_{i=1}^{T}$ to obtain $\hat{m}_T$.

iii) Calculate the bootstrap analogue of the limiting distribution in equation (2.3)

$$K^{(b)} := \underset{t\in\{\varepsilon T,\dots,T\}}{max} \left(A^{(b)}\left(\frac{t}{T}\right) - \frac{t}{T}A^{(b)}(1)\right)' \left(A^{(b)}\left(\frac{t}{T}\right) - \frac{t}{T}A^{(b)}(1)\right),$$

with $A^{(b)}\left(\frac{t}{T}\right) := \frac{t}{T}\sqrt{T}\left(\hat{m}_t^{(b)} - \hat{m}_T\right)$.

iv) Compute B versions of $K^{(b)}$ and determine the critical value K such that

$$\frac{1}{B}\sum_{b=1}^{B} \mathbb{1}\{K^{(b)} > K\} = \alpha,$$

for a confidence level $\alpha \in (0, 1)$, e.g. $\alpha = 0.05$.

Manner et al. (2017) give an intuition for the validity of the bootstrap.

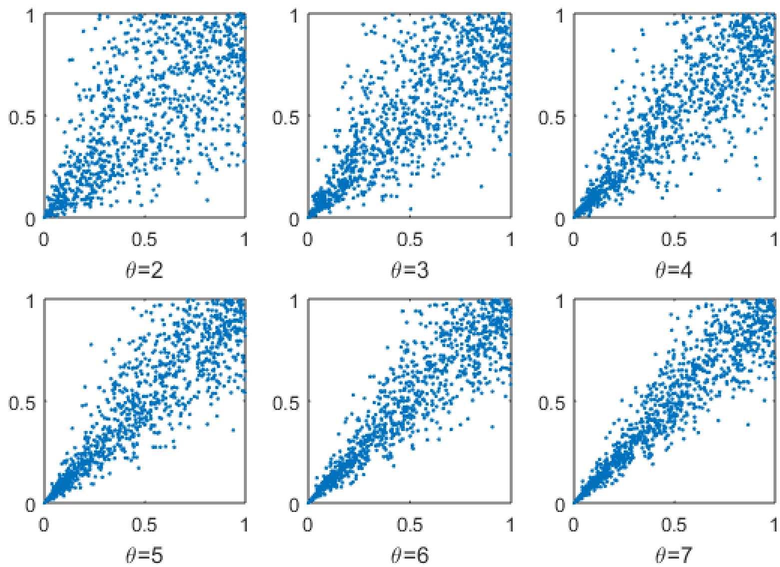

Figure 6.5: Simulated points (u, v), using a Clayton copula with $T = 1000$ and θ.

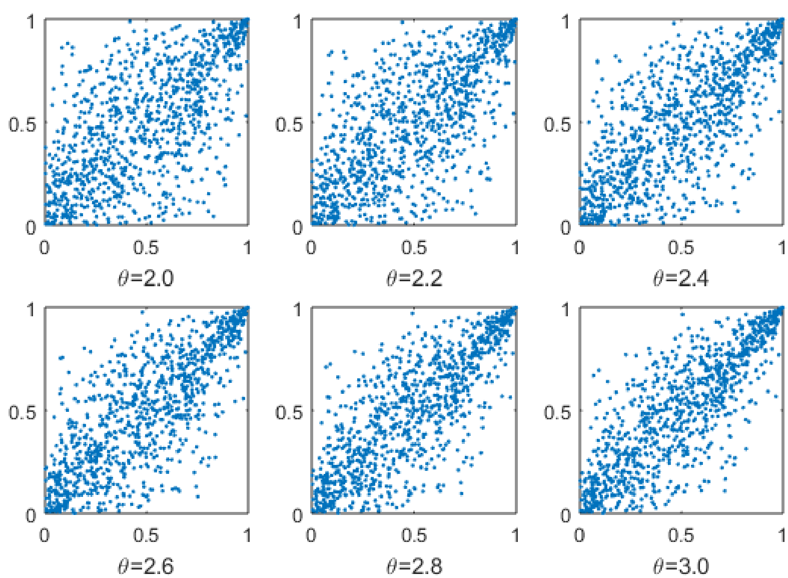

Figure 6.6: Simulated points (u, v), using a Gumbel copula with $T = 1000$ and θ.

7. COMPLIANCE WITH ETHICAL STANDARDS

Funding: This study was funded by DFG - Deutsche Forschungsgemeinschaft (grant number MA 7225/1-1).

The author declares that he has no conflict of interest.

This article does not contain any studies with human participants or animals performed by any of the authors.

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