Explosive Behaviour and Long Memory with an application to European Bond Yield Spreads*

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Abstract

This article deals with the interplay of explosive behaviour and long memory. We conduct Monte Carlo simulations and study the finite-sample properties of the popular unit root test by Phillips, Wu, and Yu (2011) against explosive alternatives. This test exhibits severe upward size distortions under the presence of strongly autocorrelated residuals. We propose the usage of a set of adjusted critical values which leads to a size-controlled test with increased power. As a complement, we consider the Lagrange Multiplier test against long memory by Tanaka (1999). We study European government bond yield spreads during the financial crisis.

JEL classification: C12, C22, G01.

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1. Introduction

In recent years the academic literature on explosive time series has received more and more attention. One of the major reasons for this development is the economic interpretation of temporary explosive roots. Explosiveness in stock prices, price-dividend and price-earnings ratios might be caused by financial bubbles (see, for example, Diba and Grossman, 1988; Anderson, Brooks, and Katsaris, 2010; Homm and Breitung, 2012). But also other asset class prices – like commodity, housing and energy prices – might contain explosive roots due to speculation (or even bubbles) (see, for example, Shi and Arora, 2012; Figuerola-Ferretti, Gilbert, and McCrorie, 2015; Engsted, Hviid, and Pedersen, 2016). Furthermore, explosiveness in macroeconomic variables like Debt/GDP ratios (see, for example Yoon, 2011, 2012), inflation (see, for example Casella, 1989) and government bond yield spreads (see Wegener, Kruse, and Basse, 2017) indicates economically relevant interpretations like for example unsustainable debt policies, hyperinflation or sovereign debt crises. Explosive behaviour in economic and financial variables is typically a temporary rather than a permanent feature. However, as the empirical evidence has demonstrated explosive episodes might last even for more than a decade.

During the same period, methods from the field of long memory time series gained increased popularity. The main feature of long memory is a long range dependence in the sense that the impact of past shocks does not die out exponentially, but at a slower hyperbolic rate. Thereby, distant shocks may still play a role although with a slowly diminishing impact. Stock market volatilities (and others) have been characterized as long memory time series by many empirical studies (see, for example Ding, Granger, and Engle, 1993; Bollerslev and Mikkelsen, 1996; Sibbertsen, 2004). Evidence supporting the hypothesis of long memory behaviour has also been documented for commodity and energy spot and future returns and volatilities (see, for example Haldrup and Nielsen, 2006; Baillie, Han, Myers, and Song, 2007; Chkili, Hammoudeh, and Nguyen, 2014). Furthermore, many macroeconomic time series like exchange rates (see, for example Cheung, 1993), inflation (see, for example Hassler and Wolters, 1995) and government bond yield spreads (see, for example Sibbertsen, Wegener, and Basse, 2014) show strong dependent behaviour in the form of a long memory process. An ongoing discussion largely triggered by the influential study by Diebold and Inoue (2001) relates long memory to structural changes. Long memory might be caused by many different sources and neglected structural breaks is one of those. Given the fact that many structural change processes lead to long memory effects, one might argue that a long memory model with a single parameter is a simple but effective approximation to potentially complicated and unknown regime changes.

As the main contribution, this paper links both strands of the literature in the context of an explosive autoregressive model with long memory innovations. Motivated by the limit theory for mildly explosive models by Magdalinos (2012), we decouple the consistent estimation of the explosive autoregressive parameter and the long memory index of the residuals. Thus, we are able to test the unit root hypothesis against an explosive alternative in the presence of long range dependencies by adjusting the critical values of the test by Phillips, Wu, and Yu (2011). The standard test does not allow for long memory innovations and we quantify severe upward size distortions leading to spurious rejections of the unit root hypothesis in favor of the explosive alternative. As a solution, we propose adjusted critical values for the Phillips, Wu, and Yu (2011) test motivated by the fact that the limit distribution depends on the (single) long memory parameter d and thereby allow for long memory innovations in the autoregressive model. The resulting test controls its size and even gains in power.

In a related paper, Pedersen and Montes Schütte (2017) find that strongly autocorrelated, but still short memory, residuals lead to severe problems for the test by Phillips, Wu, and Yu (2011). The rejection rates under the true null hypothesis are way too large in their simulations, too – hence, the evidence for explosiveness respectively bubbles might be spurious. The authors propose a bootstrap method to cope with the serial correlation. A similar, but more complicated, long memory boostrap technique would be applicable in our context, but we do not pursue this avenue. Instead, we propose an adjustment of critical values which works well in practice as indicated by our simulations.

We are particularly interested in finite-sample properties of all methods in this context, because explosive behaviour seems to be a temporal phenomenon. In addition to considering the test for explosive behaviour under long memory, we also consider the reversed situation. Specifically, we investigate the properties of a Lagrange Multiplier test against long memory for the residuals of an explosive autoregression. Similar to the unit root test, the test against long memory also needs sets of adjusted critical values, here accounting for the strength of explosiveness. In both situations, the critical values exhibit a clear monotonic shape.

As a contribution to the empirical literature and to demonstrate the applicability of our suggested testing procedures, we apply the extended methods to European government bond yield spreads. We use French interest rates against the benchmark from Germany as an example of relatively small spreads and Greece against Germany as an example of relatively large spreads. The motivation of this example is rooted in the literature: While some studies find evidence for long memory in interest rate differentials (see, for example, Baum and Barkoulas, 2006; Sibbertsen, Wegener, and Basse, 2014), a recent contribution by Wegener, Kruse, and Basse (2017) reports evidence for explosive spreads during the financial and the sovereign debt crisis. In order to bring both strands together, we allow for strong dependent innovations within an right-tailed unit root test applied to the spreads.

The paper is structured as follows: Section 2 introduces the autoregressive model with a unit or explosive root and innovations characterized by long memory behaviour. Moreover, in Section 3 we introduce the testing procedure for unit root behaviour against explosiveness under strong dependent residuals. In Section 4, we modify the test by Tanaka (1999) in order to apply this test to residuals of unit root or explosive models. The results of a Monte Carlo simulation are reported and discussed in Section 5. Here, we evaluate both – the performance of the unit root test under strong dependencies and the behaviour of the test against strong dependent residuals under unit and explosive roots. These methods are applied to European government bond yield spreads in Section 6 and the last section concludes.

2. Model Specification

In the following, we introduce a simple persistent time series model which allows for mild explosiveness (or unit root behaviour) with strongly dependent innovations as originally considered in Magdalinos (2012). Let y_t be a stochastic process in discrete time of the form

$$y_t = \mu + \rho y_{t-1} + u_t \tag{1}$$

with $t = 0, 1, ..., T$, u_t as the innovation sequence, $\rho = 1 + \frac{c}{k_T}$ with $c \geq 0$ and $y_0 = 0$. $(k_T)_{T\geq 1}$ is a sequence which increases to infinity such that $k_T = o(T)$ when $T \to \infty$. The process in (1) exhibits a unit root (viz. $c = 0$ and $\rho = 1$) or a mildly explosive root (viz. $c > 0$ and $\rho > 1$). As long as the autoregressive coefficient is local-to-unity (such that, $\rho \to 1$ as $T \to \infty$), the process is said to be mildly explosive. Asymptotic theory for mildly explosive autoregressive models has been developed by Phillips and Magdalinos (2007).

We assume that the innovation sequence u_t follows a long memory process. Such a process can be characterized by a single parameter, often labeled as d with $0 < d < 1/2$ via the behaviour of the spectral density at the origin. Hence, a times series with spectral density $f(\lambda)$, with $\lambda \in [-\pi, \pi]$, shows long memory with long memory parameter d if

$$f(\lambda) \sim L_f |\lambda|^{-2d} \tag{2}$$

for $d \in (0, 1/2)$ as $\lambda \to 0$. $L_f(\cdot)$ is slowly varying at the origin. Short memory autoregressive moving average (ARMA) models do not share this feature as they depend on possibly many parameters. More specifically, if u_t follows fractional white noise as a prototypical example, we have

$$(1 - L)^{d_u} u_t = \epsilon_t \tag{3}$$

with $E(\epsilon_t) = 0$, $\sup_t E(\epsilon_t^2) < \infty$ and $\epsilon_t = 0$ for $t < 0$; L denotes the lag operator and $(1-L)^{d_u}$ is called the fractional filter. Fractionally differencing the innovation process with d_u leads to a white noise process ϵ_t, whereby $d_u \in (0, 1/2)$ is the memory parameter of the innovations. In general, the fractional white noise process might be replaced by a more general ARFIMA(p, d, q) process allowing for additional short-run components. A typical procedure, which we adopt in the following, is to prewhiten the residuals (i.e. removing the short-run ARMA component) in the first place and to work with the fractional white noise assumption.

The given model implies two main interesting restrictions to test empirically: (i) first and foremost, a test of the unit root hypothesis against explosiveness, i.e. $H_0 : \rho = 1$ against $H_1 : \rho > 1$, is relevant in many financial applications; (ii) second, a residual-based test for long memory, i.e. $H_0 : d = 0$ (weak dependence) against $H_1 : d > 0$ (strong dependence) gives insights into the dynamics of the innovations. In the following two sections, we discuss some popular tests and evaluate their finite-sample properties via simulations.

3. Testing against Explosiveness under Strong Dependence

Phillips, Wu, and Yu (2011) use a t-type test procedure to test the hypothesis

$$\mathcal{H}_0 : \rho = 1 \text{ (unit root behaviour)} \quad \text{against} \quad \mathcal{H}_1 : \rho > 1 \text{ (explosive behaviour)}$$

and apply the test statistic

$$t_\rho = \frac{\hat{\rho} - 1}{\hat{\sigma}_{\hat{\rho}}} \tag{4}$$

based on the auxiliary regression

$$y_t = \mu + \rho y_{t-1} + u_t \tag{5}$$

where $\hat{\sigma}_\rho$ is the standard deviation of $\hat{\rho}$. Sowell (1990) proves that under the null hypothesis of this test (viz. $\rho = 1$),

$$t_\rho \Rightarrow \frac{\int_0^1 \widetilde{W}_{d_u} dW_{d_u}}{\left(\int_0^1 \widetilde{W}_{d_u}^2\right)^{1/2}} \text{ if } d_u \in [0, 0.5) \tag{6}$$

as $T \to \infty$ with W_{d_u} ($\widetilde{W}_{d_u}$) as the (demeaned) fractional Brownian motion as defined by Mandelbrot and Van Ness (1968) depending on the degree of integration of the innovations d_u. Instead of considering a complicated sieve-type bootstrap method for long memory models (see e.g. Pedersen and Montes Schütte (2017)), we exploit the simple fact that the critical values depend on d_u. Hence, we propose to estimate model (5) by OLS and to estimate the persistence parameter d_u consistently by maximum likelihood entailing the usual rate $\sqrt{T}$.

Under the alternative, Magdalinos (2012) shows that the asymptotic behaviour of the sample moments $\sum_{t=1}^{T} y_{t-1}^2$ and $\sum_{t=1}^{T} y_{t-1} u_t$ are affected by a stationary long memory innovation sequence. However, this effect is canceled out in least squares regression theory. This result enables us to estimate ρ consistently by OLS and to analyze the persistence of the residuals d_u also under the alternative. Thus, we are able to construct a right-tailed unit root test which allows a stationary long memory process as the innovation sequence by employing the standard t-value (see Equation (4)) as the test statistic and d_u-adjusted critical values.

Phillips, Wu, and Yu (2011) propose also a full sample test which uses the supremum of a sequence of t-statistics to test against explosiveness without known start- and endpoint of the explosive regime. This procedure is labeled as the Supremum Augmented Dickey Fuller ($SADF$) test. However, we focus on the t-type test as a subsample or robustness test procedure. Thus, if the $SADF$ statistic indicates explosiveness in an empirical application, we propose to estimate d_u from the residuals $\hat{u}_t$ during the indicated explosive regime (see, for methods to estimate consistently the start and endpoint of explosive behaviour, Phillips, Wu, and Yu, 2011; Phillips, Shi, and Yu, 2015a,b) and to apply the robust critical values $cv_{\alpha,T}(d_u = \hat{d}_u)$ instead of the standard set $cv_{\alpha,T}(d_u = 0)$.

We simulate quantiles of these limiting distribution of t_ρ depending on d_u with $10,000$ Monte Carlo repetitions and with d_u in the interval of $[0, 0.5)$ by steps of 0.01 and estimate response curves of the form

$$cv_{\alpha,T}(d_u) = \sum_{i=0}^{s} \beta_i d_u^i \tag{7}$$

for $T = 250$ and $T = 500$. The maximal polynomial order is denoted by s. Figure 1 shows the simulated quantiles (y-axis) depending on d_u (x-axis) and the fitted response curves. The resulting estimated polynomial response curves are available from the authors upon request. The excellent fit is reflected by an $R^2 \geq 0.999$. The displayed curves of critical values show a clear monotonic relationship between the magnitude of critical values and the strength of memory in the innovations. Stronger dependence require larger critical values in order to avoid spurious over-rejections.

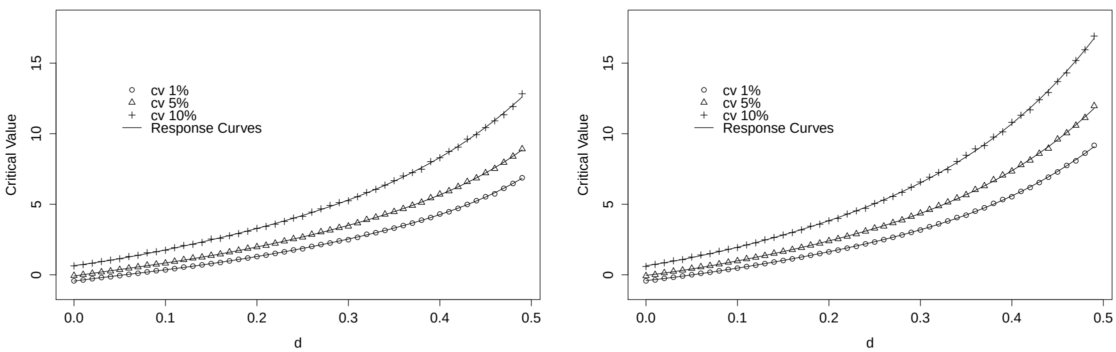

Fig. 1. Simulated quantiles of t_ρ with $T = 250$ (left) and $T = 500$ (right).

4. Testing against Strong Dependent Residuals under Explosive and Unit Root Behaviour

Tanaka (1999) suggests a Lagrange multiplier (LM) test for long memory (see also Robinson (1991)) in the following framework

$$(1 - L)^{d+\theta} u_t = \epsilon_t.$$

Testing for long memory with parameter d amounts to testing for the restriction $\theta = 0$. The LM test assumes normality of the innovations ϵ_t and is given by

$$\tau = \frac{1}{\sqrt{T}} \sum_{t=2}^{T} \epsilon_t \epsilon_{t-1}^* \tag{8}$$

with $\epsilon^*_{t-1} = \sum_{j=1}^{t-1} \frac{\epsilon_{t-j}}{j}$. Under the validity of the null hypothesis and when ϵ_t is i.i.d., τ is asymptotically normally distributed. Breitung and Hassler (2002) suggest a version of the LM test based on the auxiliary regression

$$\epsilon_t = \phi \epsilon^*_{t-1} + e_t. \tag{9}$$

In its classical form, the authors retain limiting normality of the t_ϕ-statistic which is used to test for short memory, i.e. $H_0 : \phi = 0$, against fractional alternatives, i.e. $H_1 : \phi < 0$, in the time domain. When ϵ_t follows a stationary and invertible ARMA process, the test can be applied after prewhitening (i.e. removing the ARMA component). In this setting we consider prewhitened residuals $\tilde{u}_t$ using the following procedure (see also Qu (2011)): First of all, we fit an ARFIMA(p, d, q) model to the residuals via maximum likelihood

$$\Phi(L)(1 - L)^{\hat{d}_u} \hat{u}_t = \Psi(L)\epsilon_t. \tag{10}$$

All roots of the AR and MA polynomials, i.e. $\Phi(L)$ and $\Psi(L)$, are assumed to lie outside the unit circle and ϵ_t is independent and identically distributed with $E(\epsilon_t) = 0$, $\sup_t E(\epsilon_t^2) < \infty$ and $\epsilon_t = 0$ for $t < 0$. In contrast to the fractional noise process in Equation (3), this specification contains short-run dynamics – an autoregressive and a moving average part. We determine the lag order of the autoregressive part $p \in \{0, 1\}$ and of the moving average part $q \in \{0, 1\}$ by the Bayesian Information Criterion (BIC). With a_p as the autocorrelation coefficient and b_q as the moving average coefficient of $\hat{u}_t$, we subsequently construct $\tilde{u}_t = (1 - a_p B)(1 + b_q B)^{-1} \hat{u}_t$. In order to ensure that all autoregressive short-run dynamics are well captured, we augment the auxillary test regression Equation (9) by the first lag of ϵ_t.

As a consequence of an ARMA component, the variance of the limiting normal distribution depends on the parameters of the ARMA process in a nonlinear fashion. A closed form solution is availalbe for the AR(1) case for instance and careful inspection shows that the dependence on the AR parameter is approximately linear in the local-to-unity region.[1] As we work with uni root and mildly explosive AR processes instead, no closed form solution exists. However, the simulated response curves in Figure 2 suggest a clear linear pattern for the mildly explosive case and thereby mirrors the stationary counterpart.[2] Therefore, using $10,000$ Monte Carlo repetitions, we simulate the limiting distribution of t_ϕ and we estimate corresponding (linear) response curves, see Figure 2.[3]

[1]See Equation (52) in Tanaka (1999, p.564).

[2]When investigating residuals from an explosive autoregression it is important to note that Phillips and Magdalinos (2007) show that $\hat{\rho}$ has a $k_T \rho^T$ convergence rate for $\rho > 1$ – thus, $\hat{\rho}$ converges with a faster rate than in the unit root case. This is important for the test statistic of this test as it depends strongly on $\hat{\rho}$.

[3]Critical values are simulated on the basis of an estimated autoregressive parameter rather than the

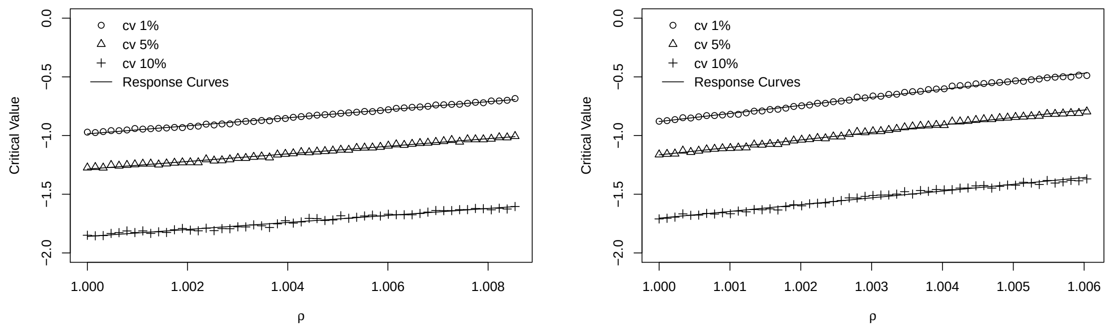

Fig. 2. Simulated quantiles of $t_{\hat{\phi}}$ with $T = 250$ (left) and $T = 500$ (right).

5. Monte Carlo Simulations

In this section, we evaluate our adjustments by means of a Monte Carlo simulation study. As outlined in the previous section, we work with prewhitening of the residuals u_t. The prewhitened residuals $\tilde{u}_t$ are used to "rebuild" the series $\tilde{y}_t = \hat{\rho}\tilde{y}_{t-1} + \tilde{u}_t$ with $\tilde{y}_0 = 0$ with approximately white noise innovations and the ARFIMA model is fitted via maximum likelihood. As a consequence, lag augmentation of the Dickey-Fuller test regression is not necessary. The test against long memory is directly applied to prewhitened residuals $\tilde{u}_t$. In the subsequent simulations, innovations ϵ_t are drawn from the standard normal distribution and u_t is therefore a (fractional) white noise process.

Table 1: Size ($c = 0$) of the test against explosiveness (5% level).

	$T = 250$			$T = 500$		
c	$d_u = 0$	$d_u = 0.2$	$d_u = 0.4$	$d_u = 0$	$d_u = 0.2$	$d_u = 0.4$
0.000	0.0450	0.3156	0.4670	0.0480	0.3360	0.4763

First of all, we consider the size of the t_ρ-statistic in a situation where we do not use adjusted, but original critical values which are valid only when $d_u = 0$. Table 1 shows the results of this experiment using $10,000$ Monte Carlo replications. As expected, we consider severe upward size distortions for $d_u > 0$. The stronger the long memory effect, the more spurious rejections in favor of the explosive alternative we observe. Increasing the sample size further amplifies this problem slightly. However, for the case of short memory ($d_u = 0$), the test is able to provide size control as expected. But, even a relatively small extent of

unknown true value. For small and moderate sample sizes, the OLS estimates are downward biased. This would lead to nonlinear response curves. However, by using already estimated counterparts, this effect cancels out.

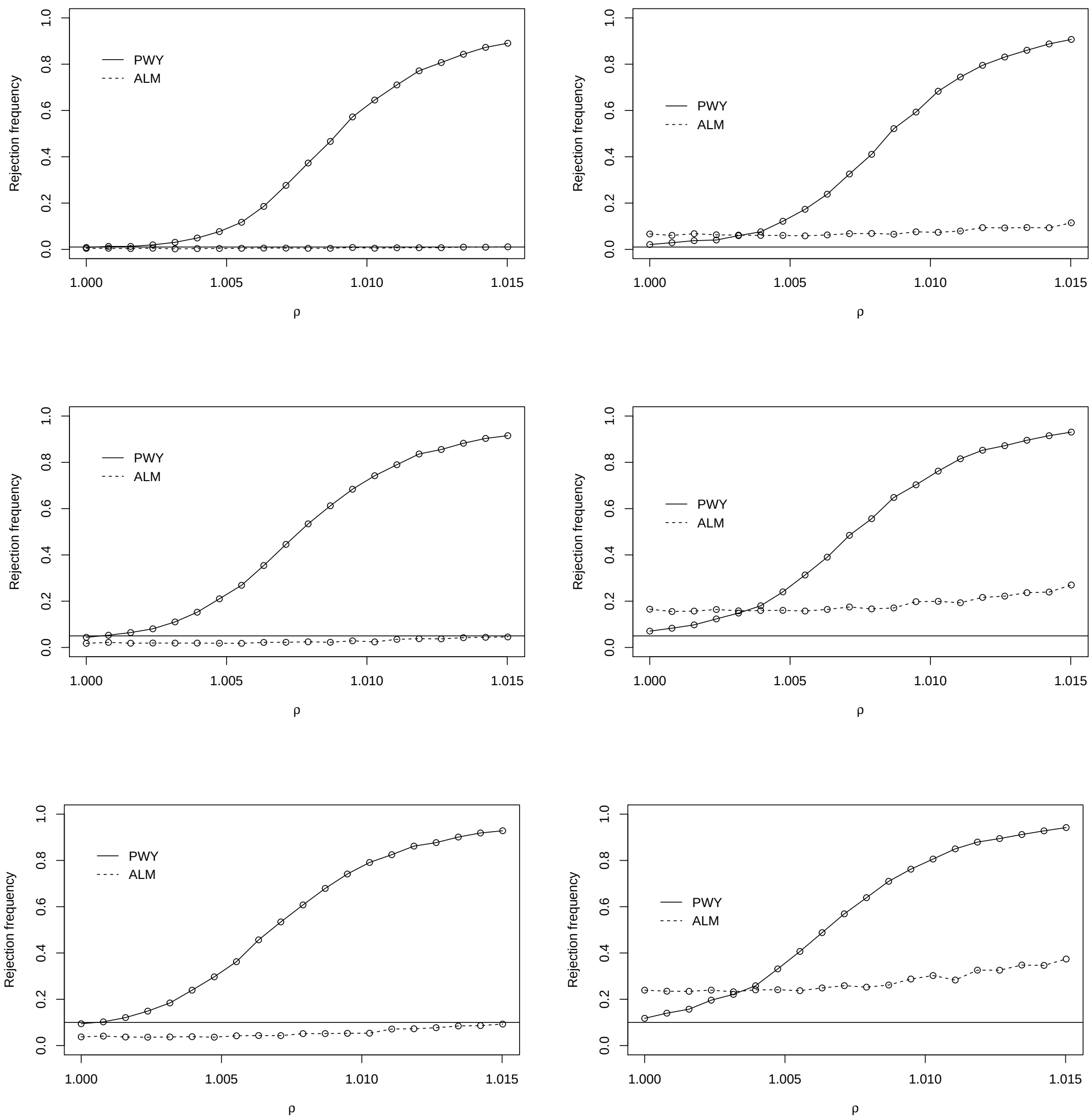

Fig. 3. These figures show the power curves of the adjusted test against strong dependent residuals (dashed lines) and the test against explosiveness (solid lines). The upper figures show the results for 1%, the figure in the middle for 5% and the figure below for 10% nominal size (marked by the horizontal line). The left panel refers to the case with $d_u = 0$ and the right panel to the case with $d_u = 0.1$.

long memory is sufficient to generate practical problems for the test.

The Monte Carlo simulation results for both methods – the test against long range dependencies in the residuals and the test against explosiveness – are given by Figure 3 and by Figure 4. First, the test against explosiveness adjusted for strong dependent innovations

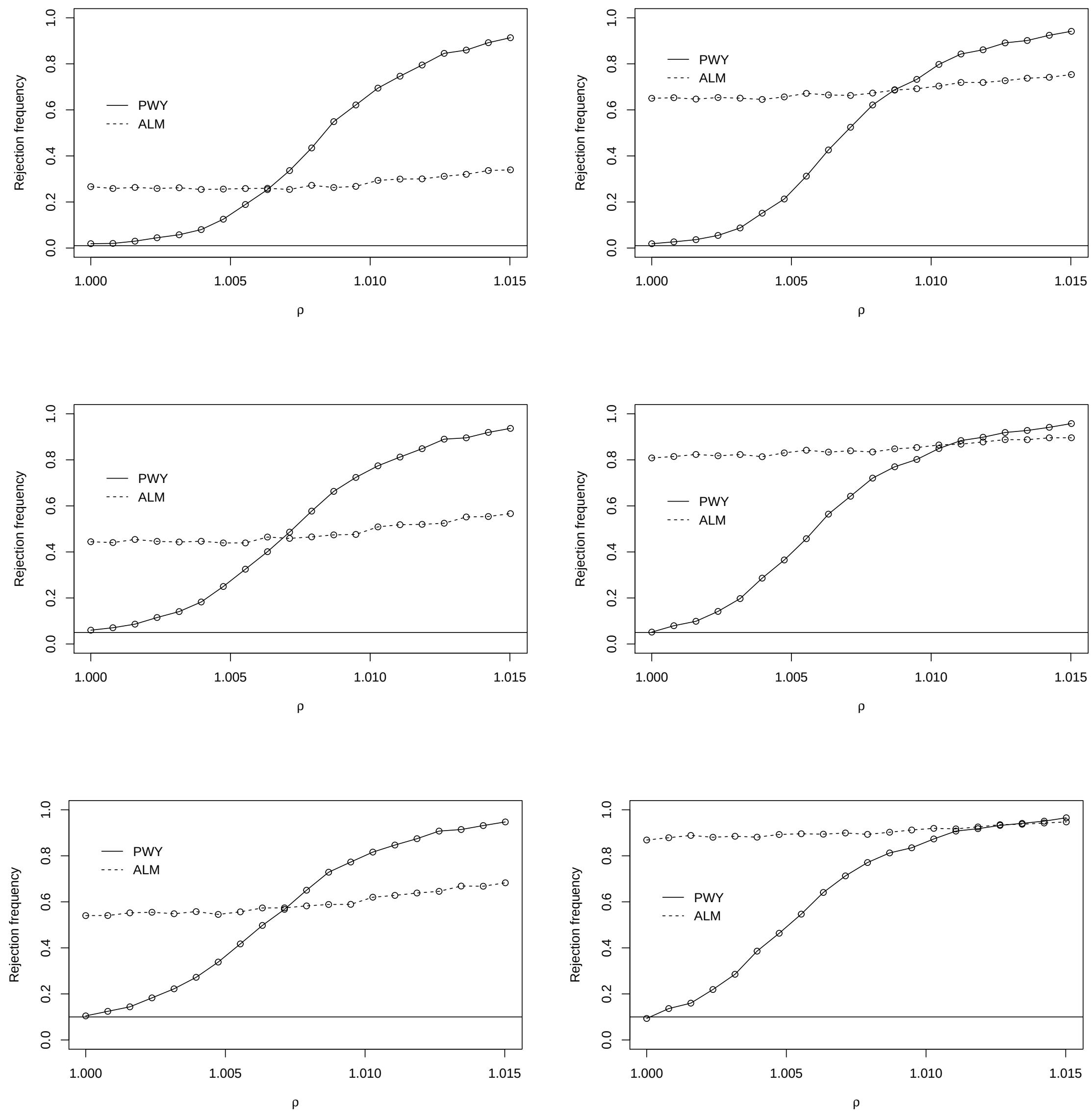

Fig. 4. These figures show the power curves of the adjusted test against strong dependent residuals (dashed lines) and the test against explosiveness (solid lines). The upper figures show the results for 1%, the figure in the middle for 5% and the figure below for 10% nominal size (marked by the horizontal line). The left panel refers to the case with $d_u = 0.2$ and the right panel to the case with $d_u = 0.4$.

controls its size and shows some power gains. In the case of $T = 250$, under the null hypothesis (viz. $c = 0$) the rejection frequency is 4.5% for $d_u = 0$ and 5.3% for $d_u = 0.4$. Under the alternative – for $c = 0.108$ – the rejection frequency is 41.60% for $d_u = 0$ and 62.55% for $d_u = 0.4$. For $T = 500$, under the null hypothesis the rejection frequency is 4.8% for $d_u = 0$ and 5.1% for $d_u = 0.4$. Under the alternative – for $c = 0.108$ – we receive a power of 69.2% for $d_u = 0$ and 81.3% for $d_u = 0.4$. The solid curves in both figures show increases with an increasing explosive ρ – as to be expected. Second, the test against strong dependent residuals also works in practice. The test is slightly conservative around the unit root but holds its size in the explosive region. This might be caused by a bias of the OLS estimator in the unit root region. However, the test against long memory residuals is well sized in the explosive region, which is most important for empirical applications we have in mind in this work. Furthermore, this procedure has strong increasing power with increasing d_u reflecting its consistency.

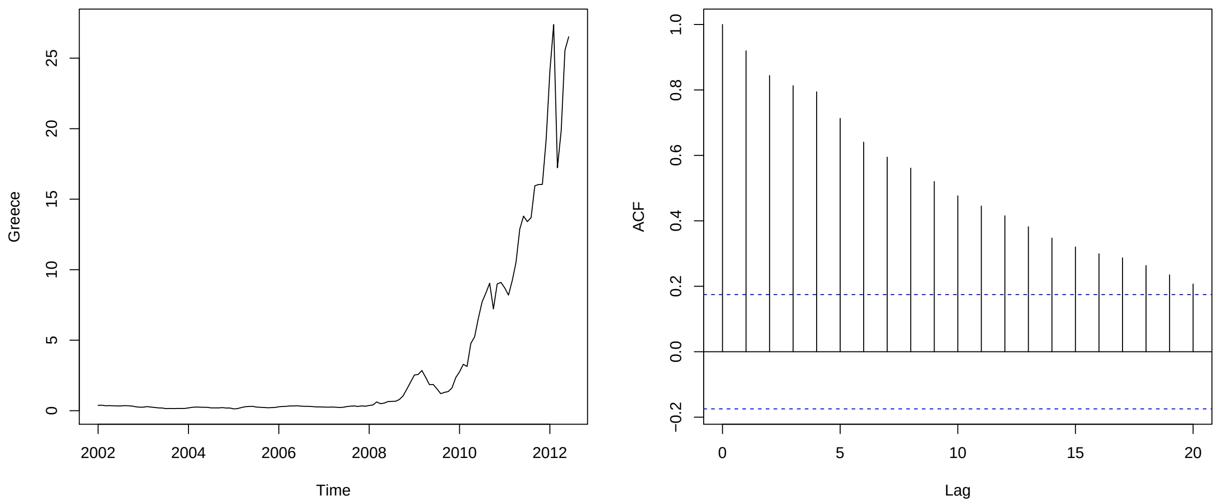

Fig. 5. Greek government bond yield spread (left) and its estimated ACF (right).

6. European Government Bond Yield Spreads

As an empirical illustration, we consider the government bond yield spreads between Greece and France, relative to Germany. We consider a sample of monthly observations from January 2002 to June 2012 with $T = 126$. The maturity of the bonds is ten years and the data is obtained from the publicly available FRED data base. Our sample starts in January, 2002 in order to exclude the effects of the Euro introduction and the repercussions of the bursting dot-com bubble. For the determination of the end date in June, 2012, it is important to note that our approach is a full-sample test which can be, for example, used as a robustness check for a researcher who has found an explosive regime. Here, we follow Homm and Breitung (2012) who argue convincingly that, "[...] in applications, one typically has a clear indication for the end of the explosive regime, just from visual inspection of the time series" and "If the observations after the peak are included, the chance that the bubble tests detect the bubble will be very low (see Section 4.3)." (see Homm and Breitung, 2012, p. 219 and p. 223). Therefore, we let the sample end in June 2012. Furthermore, we exemplify the methods with two rather contrary countries, namely Greece and France. Both countries had quite different experiences in the recent past.

6.1. Greece

We start by showing the time series and its estimated autocorrelation function (up to twenty monthly lags) in Figure 5. It can be clearly seen that the spread was relatively low and stable in the beginning, but changed its behaviour after 2010. The corresponding autocorrelation function shows a hyperbolic decay which is an indication of long memory effects in the series. Overall, the series is very strongly autocorrelated.

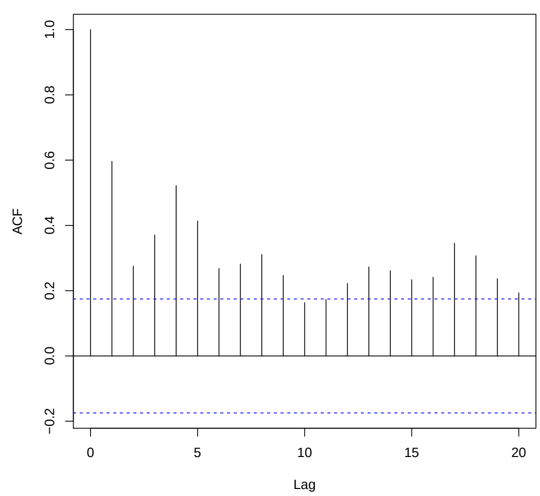

Fig. 6. ACF of residuals from estimated autoregression (Greece).

The estimated autoregression is $\hat{y}_t = -0.304 + 1.036 y_{t-1}$ suggesting a clear explosive behaviour. Testing the unit root hypothesis against the explosive alternative leads to a test statistic of 43.80 which is to be compared against critical value of 7.66 (for the five percent level).[4] Hence, a clear rejection emerges. The residuals of this explosive autoregression are

[4]We use response curves simulated for $T = 250$. We expect the slight mismatch in the sample sizes to be negligible for the analysis.

now investigated towards potential long memory. First, we plot the estimated autocorrelation function in Figure 6 and detect significant autocorrelation even at distant lags. The estimated memory parameter is $\hat{d} = 0.462$ which is still in the stationary region, but indicates strong dependence. The selected short-run model is an MA(1). The prewhitened residual series is then tested for short memory against long memory. The test statistic equals -3.204 and should be compared to the following adjusted critical value (at the five percent level): -0.119. Hence, we find a clear signal from the second test as well leading to the conclusion that the Greek government bond yield spread exhibits both time series features: explosiveness and long memory in the innovations.

While the common empirical literature on the (un)covered interest rate parity (see, for example, Baum and Barkoulas, 2006) expects *stationary* behaviour of the spreads, we find *explosiveness* of government bond yield spreads between Greece and Germany even under strong dependent residuals. Some studies document divergence of interest rates in the European Monetary Union during the financial crisis (see, for example, Ludwig, 2014; Sibbertsen, Wegener, and Basse, 2014) by using left-tailed unit root or cointegration tests. However, Wegener, Kruse, and Basse (2017) use a right-tailed unit root test and document evidence in favour of explosiveness – also for the Greek-German spread. This might be explained by safe-haven effects on the one side and fast increasing risk factors on the other side. While Germany as the benchmark has very low interest rates, Greece has very high yields due to increasing sovereign credit risk. The explosiveness of the spread might be caused by a combination of safe-haven and increasing risk effects.

6.2. France

For France, we observe a rather distinct behaviour of the series, see Figure 7. The spread remains at quite low levels and does not react as the Greek spread towards the end of the sample. The corresponding autocorrelation function shows somewhat less persistence although still on high levels.

We find the following result for the estimated autoregression: $\hat{y}_t = 0.003 + 0.949 y_{t-1}$ which does not support the notion of explosiveness at all. A formal test of the unit root hypothesis against the explosive alternative yields a non-rejection (-1.357 as test statistic versus -0.08 for the critical value). The estimated memory component in the residuals is very close to zero ($\hat{d} = -0.037$) which explains that the used critical value of -0.08 is similar to the original one (up to rounding). Thus, the unit root hypothesis is supported. An inspection

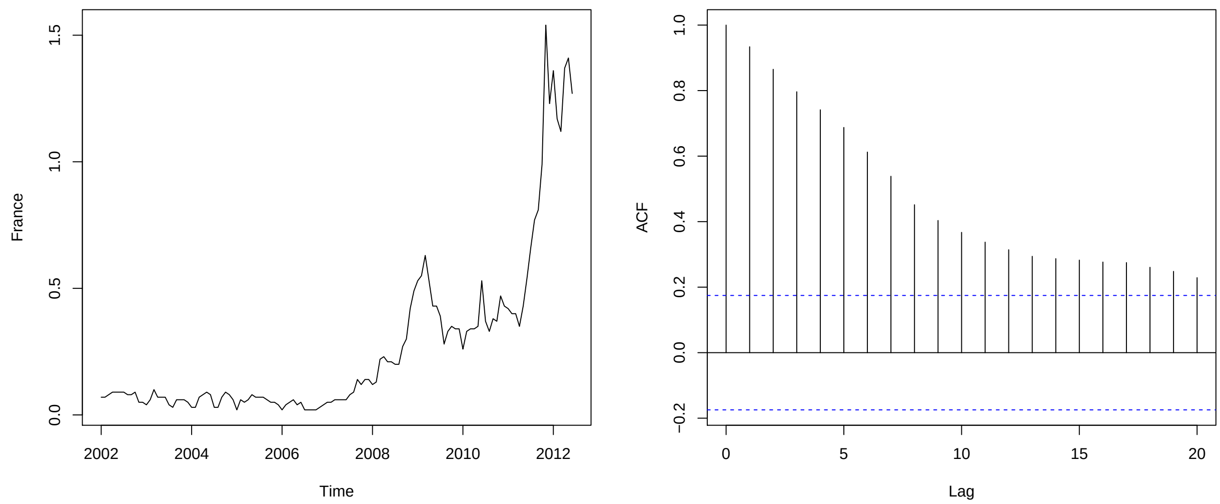

Fig. 7. French government bond yield spread (left) and its estimated ACF (right).

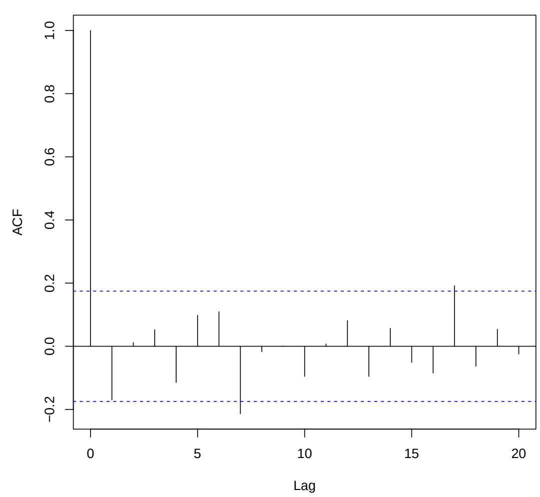

Fig. 8. ACF of residuals from estimated autoregression (France).

of the residuals ACF further confirms the absence of strong dependence. The application of the test against long memory does not contradict this interpretation. As a conclusion, the French government bond yield spread is neither explosive not strongly dependent (see Figure 8 for the ACF of residuals from the estimated autoregression), but better described as a random walk.

This result is quite surprising in the sense that the common literature assumes cointegration among government bond yields as mentioned in the foregone subsection – this might in particular hold in the case of the French-German spread since both interest rates are often treated as risk free. However, crises in Europe might have also stopped convergence among these government bond yields – for example, due to safe-haven effects. Furthermore, this finding corresponds with the empirical results by Frömmel and Kruse (2015) before the advent of the Euro and with Sibbertsen, Wegener, and Basse (2014) for the time during the financial crisis.

7. Conclusions

In this article, we investigate two popular features of time series together in a simple autoregressive model. This model is highly persistent and contains a unit root or even explosive behaviour. In addition, its innovations are strongly dependent in the sense of a long memory model. First, we identify severe upward size distortions of a widespread right-tailed unit root test and suggest simulated critical values which lead to improved properties of the test in finite samples. Spurious rejections are alleviated and even power increases are possible under strong dependence. As a complimentary test in this model, we also consider a Lagrange Multiplier statistic against long memory. Similarly to the unit root test, adjusted critical values are needed for a well-functioning test in terms of size. Our simulation study reveals that the modifications perform well.

Furthermore, we apply the methodologies to interest rate differentials between France and Germany and Greece and Germany. While we find unit root behaviour of the French-German spread and no evidence for strong dependent residuals, the case of the Greek-German spread appears to be more interesting. Here, we find both time series properties in the focus of this paper – long memory as well explosive behaviour. This empirical finding – the rejection of the null hypothesis of a unit root test in favour of an explosive alternative in the presence of strong dependent residuals – motivates the importance and advantages of the suggested procedures.

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