Time-Varying Persistence in Real Oil Prices and its Determinant*

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December 5, 2017

Abstract

We provide empirical evidence for pronounced time-variation in the persistence of real oil prices. In particular, we find episodes of mild explosiveness next to periods with random walk and also mean-reverting behavior. We address the question whether dynamic persistence can be directly related to macro-financial variables, spot-futures spreads, spill-over effects from commodities and global real economic activity. Alongside these variables, we use a large data set of more than one-hundred fifty potential determinants featuring, for example, further oil-related variables (production and inventories) and key macroeconomic series for the G7 countries. By using model averaging techniques, we robustly account for the inherent model uncertainty when dealing with such many potential explanatory variables. As it turns out, the one and only significant measure to explain time-varying oil price persistence is the index of global economic activity by Kilian (2009). Other variables related to e.g. supply shocks or speculation are, however, insignificant. In line with recent findings, we argue that fundamentals rather than speculation were the drivers of the explosive oil price in the 2000s.

JEL-Codes: Q02, Q43, C22

Keywords: Oil prices · Fundamentals · Speculation · Explosiveness · Model averaging

*We would like to thank Lutz Kilian for constructive and helpful comments and the participants of the INREC 2017 in Essen for insightful discussions. Robinson Kruse gratefully acknowledges financial support from CREATES funded by the Danish National Research Foundation.

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1 Introduction

The oil price rally between 2003 and 2008 attracted the attention of media, policy makers and also academics. A popular view is that financialization of oil futures markets supported the rising oil prices at this time significantly.

"You have asked the question 'Are Institutional Investors contributing to food and energy price inflation?' And my unequivocal answer is 'YES.' In this testimony I will explain that Institutional Investors are one of, if not the primary, factors affecting commodities prices today." (Masters, 2008, before the Committee on Homeland Security and Governmental Affairs)

This so-called 'Masters Hypothesis' explains the spike in energy futures prices in 2007-08 as a result of long-only[1] index investments. While the majority of studies in the academic literature could not support the hypothesis that speculation was the main price driver during the episode between 2003 and 2008 (see, for a comprehensive overview of the literature, Fattouh, Kilian, and Mahadeva, 2013), there are some studies providing evidence in favour of a rational bubble in crude oil prices which collapsed in 2008 (see, for example, Phillips and Yu, 2011; Shi and Arora, 2012; Tsvetanov, Coakley, and Kellard, 2016). These results of the two strands of the literature – on the one hand empirical evidence against the 'Masters Hypothesis' and on the other hand empirical evidence supporting (speculative) bubbles – are contradictory in the sense that there might be speculation[2] without a rational bubble[3] in the market, but there could not be a rational bubble without speculation because a rational bubble is characterized by speculation.

In the context of this paper, the oil market contains a 'price bubble' if the explosive price decouples from its fundamental value and this bubble would be called 'speculative price bubble' (or 'rational bubble') if purchases of crude oil push the price beyond the intrinsic value because buyers anticipate rising oil prices. Hence, a rational price bubble could not exist without investors speculating on rising prices. Thus, if we find explosive behavior of oil prices which could be explained by speculation, it might be a speculative

[1] A representative investor who has a "long position" in an asset is the buyer of this asset and profits from increasing prices. Thus, a "long-only" strategy is characterized by the fact that an investor profits only in the case of positive performance of the price of the asset.

[2] 'Speculation' in the context of crude oil refers to a situation where someone buys oil for future rather than for current use (see Fattouh, Kilian, and Mahadeva, 2013; Kilian and Murphy, 2014).

[3] In general, a 'price bubble' describes a situation, where the price of an asset decouples from its fundamental (see, for example, Blanchard and Watson, 1982). A 'speculative price bubble' refers to a situation where investors push the price of an asset beyond its intrinsic value due to their expectations of rising prices.

bubble. However, if we could not explain explosiveness by speculation, we can conclude that the explosive behavior is not an expression of a speculative price bubble.

One of the first papers dealing with speculative bubbles in crude oil prices, is the study by Phillips and Yu (2011)[4]. The authors use a sequential right-sided unit root test which is based on estimated time-varying persistence. Beside other variables, Phillips and Yu (2011) apply this methodology to crude oil prices normalized by crude oil inventories as the fundamental value (similar to price-dividend or price-rent ratios in the stock or housing market literature). This might cause problems since (i) inventories might not capture the fundamental value of crude oil to a full extent and (ii) it imposes unrealistic restrictions on a cointegration vector between crude oil prices and inventories which might be unstable over time (see Section 2 for a more detailed discussion of these problems). Overall, relatively little attention is paid to possible underlying factors of time-varying persistence, including temporary explosive behavior.

We contribute to the literature by re-examining the persistence of oil prices. However, we do not rely on inventories or any other variable as fundamental value. Instead, we estimate the time-varying persistence of real Brent and WTI crude oil prices and draw inference about its relation to more than one-hundred fifty potential determinants like, for example, spot-futures spreads, world oil production, leading commodity prices or real economic activity. Compared to earlier studies (see, for example, Phillips and Yu, 2011; Shi and Arora, 2012; Brooks, Prokopczuk, and Wu, 2015; Caspi, Katzke, and Gupta, 2015), our approach has the advantage that we do not need to pre-specify a variable which captures the fundamental part of the oil price.[5] The analysis is carried out by model averaging techniques. The benefit from using model averaging instead of model selection is that model uncertainty is explicitly accounted for. Neglecting the model search can severely affect the subsequent inference. Our approach is robust to this uncertainty and also to first-stage estimation errors arising from estimating the dynamic persistence.

The remainder of this study is structured as follows: The next section provides a compact review of the relevant literature. Section 3 introduces the used data set and the econometric approach. Furthermore, Section 4 contains the empirical results and Sec-

[4] Also the paper by Miller and Ratti (2009), for example, considers oil price bubbles. However, these authors consider earlier periods and they focus more on the relationship between oil and stock markets and not on the results of oil price persistence based testing procedures.

[5] Pavlidis, Paya, and Peel (2017) suggest an alternative approach which exploits the fact that future and spot prices must converge in the absence of a bubble (see Pavlidis, Paya, and Peel (2016)). Their approach and ours have in common that it is not required to pre-specify a variable capturing the fundamental part of oil prices.

tion 5 discusses the results against the background of the current literature. Finally, Section 6 concludes. The appendix includes a list of selected variables with details, additional empirical results and elaborations on the *t*-statistics accounting for first-stage persistence estimation.

2 Literature Review

In this section we provide a compact review of the related literature regarding oil prices. In the first subsection we start with an overview about the traditional literature explaining oil prices by supply and demand shocks. Next, we form the connection to the literature which explicitly takes speculation into account. This strand of literature is strongly related to studies on speculative bubbles (or explosiveness) in oil prices which is separately discussed in the third subsection.

2.1 Supply and Demand Shocks

The traditional literature which argues that essentially all oil price shocks coincided with fundamental supply and demand shocks has its origin in articles by Hamilton (2003), Barsky and Kilian (2002, 2004) and Kilian (2008a,b, 2009). Kilian (2009) distinguishes three types of shocks: (i) oil supply shocks, (ii) aggregate demand shocks and (iii) oil-market specific demand shocks. While Hamilton (2003) and Kilian (2008a,b) study the role of supply shocks extensively, Kilian (2009) states that it is difficult to quantify demand shocks for basically two reasons: (i) at that time, there was no index available to capture the dynamics of global demand for all industrial commodities and (ii) expectations driving precautionary demand shocks are not observable.

Kilian (2008a) argues that supply shocks alone are insufficient to explain the major proportion of oil prices. Hence, Kilian (2009) proposes a measure of the dynamics in global real economic activity ('index of real economic activity') which affects the demand for all industrial commodities. Furthermore, Kilian (2009) employs a structural Vector AutoRegressive (VAR) model based on the 'real price of oil', the 'change in global crude oil production' and the 'index of real economic activity' to decompose the oil price. The author argues that the serially and reciprocally uncorrelated structural innovations might be interpreted as the oil supply shocks, aggregate demand shocks and the oil specific-demand shocks. Kilian (2009, p. 1053) finds evidence "*that oil price shocks historically have been driven mainly by a combination of global aggregate demand shocks and precautionary demand shocks.*"

Considering more recent data, Hamilton (2009) concludes that the oil price rally of 2007-08 was mainly driven by strong demand due to the boom in the global economy and stagnation in world oil production in 2005-07. However, Hamilton (2009, p. 234) notes that, "*One can thus tell a story of the oil price shock and subsequent collapse that is driven solely by fundamentals. But the speed and magnitude of the price collapse lead one to give serious consideration to the alternative hypothesis that this episode represents a speculative price bubble that popped.*"

2.2 Speculation

Kilian and Murphy (2014) investigate the role of speculation by employing a structural VAR model based on the 'real price of oil', the 'change in global crude oil production', the 'index of real economic activity' and the 'change in oil inventories above the ground' in order to identify three different kinds of shocks: (i) a flow oil supply shock, (ii) a flow oil demand shock and (iii) a speculative demand shock. Kilian and Murphy (2014) report that the real price of oil between 2003 and mid-2008 was not significantly driven by speculative demand or supply shocks rather by global oil demand shocks.

By using alternative proxies for global oil inventories, the result by Kilian and Murphy (2014) is basically confirmed by Kilian and Lee (2014, p. 85): *"Indeed, the view that an exogenous shift in the participation of financial investors in oil futures markets explains the surge in the real price of oil during 2003-08 can be ruled out on the basis of our results."* Both studies argue that the result that speculation was not the significant driver of the oil price surge does not mean that earlier (e.g., after the collapse of OPEC in 1986) or later episodes are not significantly influenced by speculative demand.

Juvenal and Petrella (2015) argue that small-scale VAR models do not capture enough information in order to identify the shocks. The authors employ a dynamic factor model for commodity prices as well as a battery of macroeconomic and financial variables from the G7 countries. The authors find that global demand plays the most import role between 2003 and mid-2008 and accounts for about 55% of the surge in the real price while speculation is the second important driver with 15% and less than 10% are attributed to oil supply shocks.

2.3 Bubbles or just mild Explosiveness?

Phillips and Yu (2011) apply the methodology by Phillips, Wu, and Yu (2011) to crude oil prices normalized by crude oil inventories and report the existence of a short bubble period in 2008. In order to use a test procedure which allows for more than one explosive period, Caspi, Katzke, and Gupta (2015) use the test by Phillips, Shi, and Yu (2015) and report several episodes of arising and collapsing explosiveness between 1949 and 2008 in the nominal oil price normalized by the inventory quantity.

Instead of using right-sided unit root tests, Shi and Arora (2012) and Brooks, Prokopczuk, and Wu (2015) apply regime switching methods. While Brooks, Prokopczuk, and Wu (2015) report some evidence in favour of bubbles in crude oil prices, Shi and Arora (2012) confirm the results by Phillips and Yu (2011) about a short bubble in oil prices in 2008. All these studies have in common that they need to pre-specify a measure which captures the fundamental of crude oil. However, following Gronwald (2016), this measure might not be observable and evidence in favour of a bubble might be caused by misspecified market fundamentals.

Brooks, Prokopczuk, and Wu (2015) use two methods to capture the fundamentals: The first stems from the present value model (also used by Shi and Arora (2012)) and the second relies on macroeconomic variables. The present value model which is based on convenience yields might be seen as a self assessment – rather than an objective assessment – of future market conditions and the second approach might neglect oil specific factors. Furthermore, Phillips and Yu (2011) and Caspi, Katzke, and Gupta (2015) normalize crude oil prices by crude oil inventories. This approach captures only the supply side and neglects important factors – like the global real economic activity – driving also the fundamental value of crude oil. Additional to this problematic point, the authors assume a cointegration vector of $(1, -1)$ between inventories and the oil price which is unlikely to hold over time (see Gronwald, 2016).

An approach to test for bubble behavior without taking an estimated or observed market fundamental directly into account, stems from the finding that spot and future prices are explosive in the presence of a bubble (see Diba and Grossman, 1988). Thus, Tsvetanov, Coakley, and Kellard (2016) apply tests for mildly explosive behavior to oil spot and future prices and report evidence in favor of a bubble between 2004 and 2008. However, Pavlidis, Paya, and Peel (2017) note that the shortcoming of this approach is the assumption that the fundamentals are non-explosive.

Thus, econometric approaches to identify bubble behavior suffer from the joint-hypothesis problem: *"What have we learned from bubble tests? This survey showed that bubble tests do not do a good job of differentiating between misspecified fundamentals and bubbles"* (see Gürkaynak, 2008, p. 182).

In a more recent paper by Pavlidis, Paya, and Peel (2017), the authors apply two methodologies proposed by Pavlidis, Paya, and Peel (2016) who utilize the fact that in the presence of a bubble, future spot prices and market expectations must diverge. Thus, the authors apply right-sided unit root tests to the difference between future spot oil prices and expected future spot prices from *Consensus Economics Inc.* and they test the unbiasedness hypothesis in the oil market. The latter approach exploits the fact that the Efficient Market Hypothesis implies that the unbiased predictor for the current spot price is the expected spot price in the absence of a bubble under rational expectations and risk neutrality. By using this approach bubble behavior can be analyzed without pre-specifying a market fundamental. The empirical results suggests that there is no evidence for a bubble in real oil prices.

3 Data and Methodology

First, we describe the data set in Section 3.1 and thereafter, we discuss estimation of dynamic persistence in Section 3.2. We apply an indirect inference approach which can cope with the bias arising from strong persistence and a finite (relatively small) number of observations. In Section 3.3, we explain a large set of potential variables driving the persistence of the oil price. We make use of a model averaging approach which deals with model uncertainty due to many potential explaining variables.

3.1 Data

We obtain data from the "Journal of Applied Econometrics Archive"[6] for the widely recognized article by Juvenal and Petrella (2015). This comprehensive data set includes many important variables regarding the oil market. Among these are world oil production, aggregate industrial production, inventories of oil and oil spot-future spreads. In addition, twenty leading real commodity prices (including metals, food and non-food) are covered. Next, key macroeconomic series for the G7 countries are included: real GDP, personal consumption, industrial production, (un)employment rates, employee earning indexes, consumer and producer price indexes, overnight and 10-year interest rates, money supply (M1 and M2), trade balances, stock market indexes, (real effective) exchanges rates and interest rate spreads (three months/ten years rate minus overnight rate). As a measure for global economic activity, the dry cargo shipping rate index developed by Kilian (2009) is considered as well. This index captures demand shifts for commodities which are driven by the global economic cycle as demand for transportation

[6] http://qed.econ.queensu.ca/jae/

is mainly driven world economic growth, see Kilian (2009). A full list of variables with additional information is available in Appendix A of Juvenal and Petrella (2015). A list of selected variables with details is included in the Appendix of this paper. Main sources are the International Financial Statistics database of the International Monetary Fund and the Organisation for Economic Cooperation and Development.

The WTI (Brent) oil price taken from the Federal Reserve Bank of St. Louise Data Base (FRED) in logs and deflated by the US CPI less food and energy. We augment the data set by the NBER based Recession Indicator for the United States, also taken from the FRED. Our sample comprises $K = 151$ variables and covers the important period from 1980Q1 to 2009Q4 yielding $T = 120$ quarterly observations. The reason for choosing this sample is that we want to focus on the potentially explosive regimes due to the price rally between 2003 and 2008. We use the first fifty observations (from 1980Q1 to 1992Q2) for rolling window estimation of persistence and the remaining seventy for model averaging regressions (from 1992Q3 to 2009Q4). All explanatory variables are suitably transformed to achieve stationarity, see Juvenal and Petrella (2015).

3.2 Indirect Inference Estimation

We start by considering a simple way of dynamic persistence estimation for the oil price z_t. An autoregressive (AR) model of order p is specified:

$$z_t = \mu + \rho z_{t-1} + \sum_{i=1}^{p-1} \tau_i \Delta z_{t-i} + v_t, \tag{1}$$

where ρ equals the sum of autoregressive coefficients. OLS estimation of ρ in the AR model (1) is heavily downward-biased in small samples and when the true value of ρ is in the vicinity of unity. In order to cope with the OLS bias, we apply an indirect inference estimator. Such an estimator offers bias-correction by comparing the average OLS estimate in relation to the true parameter value via simulation. In a representative empirical situation, the OLS estimate $\widehat{\rho}$ might result as 0.925, while the true value is $\rho = 1$. The indirect inference estimator corrects the OLS bias of -0.075 by adding this value to $\widehat{\rho}$ (subject to a minor simulation error). In this way, a bias-correction is achieved. In a companion paper, Kaufmann, Kruse, and Wegener (2017) compare a variety of different approaches to bias-correction in a large-scale Monte Carlo study. Their results demonstrate the usefulness of the indirect inference estimator over several other approaches, e.g. bootstrap and jackknife. The estimator is also robust against various kinds of mispecifications. Furthermore, the estimator shows excellent performance in terms of mean squared error (MSE) for highly persistent and possibly mildly explosive processes.

In some more detail, the indirect inference estimator $\widehat{\rho}^{II}$ (see Phillips et al., 2011) is given by

$$\widehat{\rho}^{II} = \arg\min_{\rho \in \Theta} \left\| \widehat{\rho} - \frac{1}{H} \sum_{h=1}^{H} \widehat{\rho}^{h}(\rho) \right\| ,$$

where Θ is a compact interval. For $h = 1, 2, ..., H$ the average OLS estimate is simulated for a given true value $\rho \in \Theta$, denoted as denoted as $\widehat{\rho}^{h}(\rho)$.[7] In theory, for $H \to \infty$ one obtains

$$\widehat{\rho}^{II} = \arg\min_{\rho \in \Theta} \| \widehat{\rho} - q(\rho) \| ,$$

where $q(\rho) = E\left(\widehat{\rho}^{h}(\rho)\right)$ is the so-called binding function.[8] The indirect inference estimator results by inversion of $q(\cdot)$ as

$$\widehat{\rho}^{II} = q^{-1}(\widehat{\rho}) .$$

Equation (1) is estimated by a rolling window scheme using w observations. This leads to a series of dynamic persistence labeled as $\widehat{\rho}_t$ and its bias-corrected counterpart $\widehat{\rho}_t^{II}$.

3.3 Model Averaging

In the following, we consider the $T \times K$-matrix of determinants X as described in Section 3.1. Our main interest lies in regressions of the type

$$y_t = \beta' x_t + u_t$$

with $y_t = \widehat{\rho}_t^{II}$ (cf. Section 3.2) and x_t being a subset of the full determinant set X_t. With K variables there are 2^K combinations when considering all possible subsets of determinants. In contrast to model selection, where a single model is selected and interpreted, all models contribute to the averaged parameter estimates. For the construction of a model averaging estimator all possible models are estimated and a smoothed weight is assigned to each model. The weight depends on the relative performance of the model in terms of an information criterion. Better performing models receive a higher weight than weaker ones and vice versa. A main benefit of model averaging in contrast to model

[7] For accuracy, H should be a large number and we set $H = 20{,}000$. In addition, we specify $\Theta = \{0.6, ..., 1.2\}$.

[8] The simulated binding function is available from the authors.

selection is that it incorporates the uncertainty inherent in the model selection process.

In the following, we consider the smoothed AIC frequentist model averaging approach which has been suggested by Buckland, Burnham, and Augustin (1997) and is further developed in contributions by Burnham and Anderson (2002) and Hjort and Claeskens (2003). As an advantage over Bayesian model averaging, frequentist approaches do not require the specification of prior distributions and are computationally simpler. Following the general model averaging approach, all possible sub-models (including the full model as well) of the reduced data set $\mathcal{X}$ are estimated. This leads to $M = 2^J - 1$ models.[9] For each estimated model $m = \{1,2,...,2^J - 1\}$, we compute information criteria (AIC; and BIC for comparison), i.e. $IC^m = \{AIC^m, BIC^m\}$.

Let $\nabla^m = IC^m - \min_m IC^m$ denote the difference in information between a model m and the best model according to a given criterion IC. For m being the best model, we have $\nabla^m = 0$, while for all others $\nabla^m > 0$ holds. ∇^m measures the information loss from fitting model m instead of the best model. The corresponding smoothed weight for model $m \in \{1,2,...,M\}$ is given by, see e.g. Buckland et al. (1997):

$$\omega^m = \frac{\exp(-\frac{1}{2}\nabla^m)}{\sum_{i=1}^{M}\exp(-\frac{1}{2}\nabla^i)} \in (0,1) \, , \qquad \sum_{m=1}^{M}\omega^m = 1 \, .$$

The model averaging (MA) estimator for the J-dimensional parameter vector is given by:

$$\widehat{\beta}_{MA} = \sum_{m=1}^{M}\omega^m\widehat{\beta}_0^m$$

with $\widehat{\beta}_0^m = (\widehat{\beta}^m, 0)'$ in the case of $m < M$ (due to zero-restrictions on a number of coefficients). Only for comparison, we also consider the best performing model m^* in terms of AIC and BIC:

$$\widehat{\beta}_{IC} = \widehat{\beta}^{m^*} \cdot 1(\nabla^{m^*} = 0) \, , \qquad m^* = \arg\min_m IC(m) \, ,$$

where $1(\cdot)$ defines the indicator function. We compute two different t-statistics for the elements of $\widehat{\beta}_{MA}$ and $\widehat{\beta}_{IC}$. The first one takes the model search into account (t_{rob}), while the second one ignores this issue (t_{naive}).[10] We report both types for illustration and for

[9]The model containing only an intercept is dropped due to standardization.

[10]Demetrescu, Kuzin, and Hassler (2011) demonstrate the pitfalls of post-model-selection testing in various situations. Typically, the empirical size of subsequent test statistics can be severely affected resulting in upward distortions. Therefore, it is important to take the effects of model search into

the purpose of judging impact of model uncertainty in our setting.[11] The variance of the k-th element of the estimated parameter vector ($\widehat{\beta}_{MA}$ or $\widehat{\beta}_{IC}$) is computed via

$$var(\widehat{\beta}^{(k)}) = \left[\sum_{m=1}^{M} \omega^{(m,k)} \sqrt{var(\widehat{\beta}^{(m,k)}|m) + (\widehat{\beta}^{(m,k)} - \widehat{\beta}^{(k)})^2} \right]^2$$

where $\omega^{(m,k)}$ denotes the model weights for the k-th parameter in model m and $var(\widehat{\beta}^{(m,k)}|m)$ is the variance of k-th parameter estimator in model m, respectively. As Burnham and Anderson (2002) point out, this estimator is conservative in the sense that it assumes perfect correlation between the estimates of different model and thereby can be interpreted as an upper bound for the variance. On the contrary, the second estimator ignores the model uncertainty component $(\widehat{\beta}^{(m,k)} - \widehat{\beta}^{(k)})^2$ and thereby leads to a lower variance per se and increases the absolute value of t-statistics spuriously. The differences in the two versions of the t-statistics provide an implicit measure of model uncertainty.

As K is large, the consideration of all these sub-models will be cumbersome and overly time-consuming. As we have even more variables than observations over time, we exclude some of the variables in X with *least* explanatory power. We follow Christiansen, Schmeling, and Schrimpf (2012) by computing a robust t-statistic for each single element of X via

$$y_t = \beta^{(k)} x_t^{(k)} + u_t^{(k)}\ , \qquad k = 1, 2, \ldots, K$$

where $x_t^{(k)}$ is the k-th element of X at time t.[12] The null hypothesis is $H_0 : \beta^{(k)} = 0$ and the corresponding t-statistics $t_{\beta^{(k)}}$ are robust towards the estimation error in y_t resulting from its estimation in the first step.[13] We construct the reduced set of determinants as follows:

$$\mathcal{X} = \left\{ x^{(k)} \ \middle|\ |t_{\beta^{(k)}}| > cv_{1-\alpha/2} \right\}, \qquad k = 1, 2, ..., K$$

meaning that only variables with a significant t-statistic are considered further on. The dimension of the reduced set of determinants is labeled as J and it is expected that $J < K$ and $J < T$. Once $\mathcal{X}$ is determined, some additional minor adjustments are applied. A variable is excluded from $\mathcal{X}$ if at least one of the following conditions is fulfilled: (i) the absolute value of the correlation between the particular variable and any other variable having a larger absolute t-value exceeds some positive threshold $\bar{c}$, (ii) it has too many missing values, i.e. at least $\widetilde{T}$ non-NA observations. The first restriction deals

account, see also the references therein.

[11]Both versions also account for persistence estimation uncertainty and serially correlated errors.

[12]Note that the intercept is omitted as data are standardized.

[13]Details are provided in Appendix 7.3.

with the potentially upcoming multi-collinearity problem in the subsequent multiple regression models. The second requirement ensures a balanced sample in the end for ease of comparison.

4 Empirical Results

We start by looking at the estimated time-varying persistence reported in Figures 1 and 2 for the WTI and the Brent prices, respectively. Our empirical findings from the application of the indirect inference estimator are as follows: (i) we find compelling evidence for time-variation and also mild explosive behavior; (ii) there is a clear need for bias-correction - the difference between the indirect inference and the standard OLS estimator is not just a linear level shift, but nonlinear with different strength of bias-correction for different levels of persistence; (iii) we observe different phases of unit root, stationary and mildly explosive behaviour; (iv) episodes of explosiveness resemble previous evidence in the bubble literature and (v) there is very little differences between the time-varying persistence WTI and Brent Oil prices.

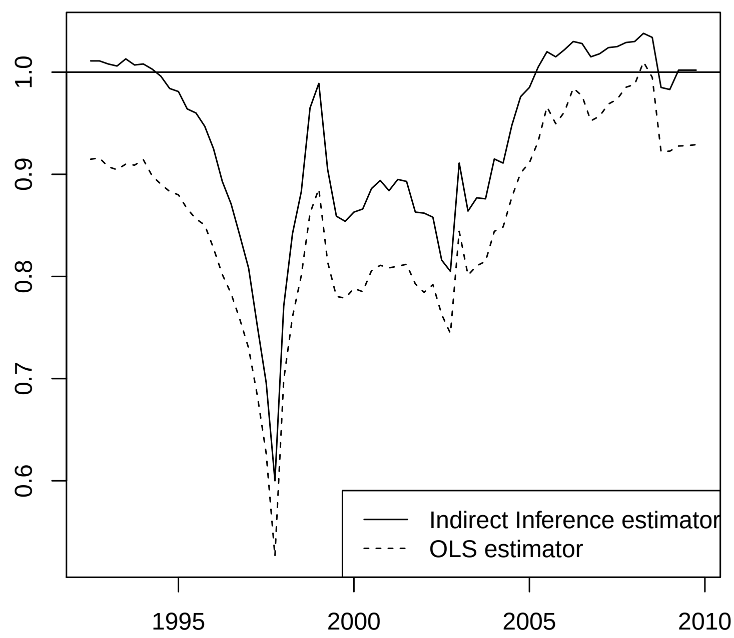

Figure 1: Dynamic persistence for the real price of crude oil (WTI) for sample from 1992Q3 to 2009Q4.

Table 1 collects some summary statistics on the indirect inference estimator. Real oil prices are highly persistent on average, but with considerable differences of minima and maxima. During the price rally, persistence peaks at 1.038 (WTI) and 1.040 (Brent) which indicates a sizable explosive component in the prices. Estimates of this magnitude are quite common in the related bubble literature.

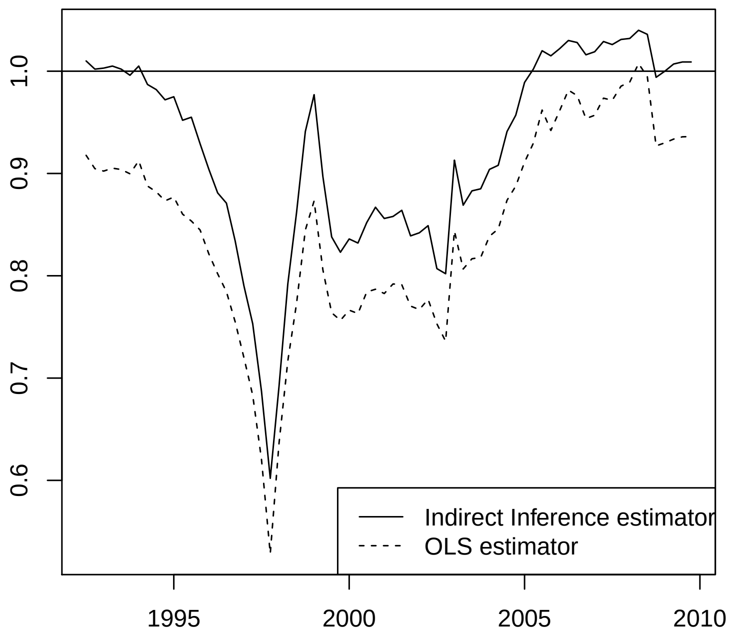

Figure 2: Dynamic persistence for the real price of crude oil (Brent) for sample from 1992Q3 to 2009Q4.

The core part of our empirical analysis is the model averaging approach which we compare to model selection as a robustness check. The initial data set containing more than one hundred fifty variables. Given the relatively large amount of regressors, especially in comparison to the sample size, we resort to necessary standard pretesting before we turn to the model averaging estimator (see e.g. Ludvigson and Ng (2009)). We consider

Table 1: Summary statistics for dynamic persistence (indirect inference)

Variable	Min	Median	Mean	Max
WTI	0.600	0.962	0.924	1.038
Brent	0.602	0.947	0.923	1.040

Table 2: Empirical findings – Model averaging, WTI

Variable	$\widehat{\beta}_{AIC}$	t_{rob}	t_{naive}	$\widehat{\beta}_{BIC}$	t_{rob}	t_{naive}
Employment Italy	-0.057	-1.298	-1.729	-0.053	-1.141	-1.790
Employment UK	-0.043	-0.809	-1.467	-0.032	-0.588	-1.139
Index of GEA	0.163	2.258	2.291	0.167	2.399	2.445
M1 Japan	-0.001	-0.436	-0.677	-0.001	-0.377	-0.793
Personal cons UK	-0.002	-0.368	-0.544	-0.001	-0.363	-0.662
REER UK	-0.007	-1.242	-1.456	-0.007	-1.251	-1.677
Spread 3m-OV Italy	0.003	0.364	0.548	0.002	0.244	0.348

Table 3: Empirical findings – Model selection, WTI

Variable	$\widehat{\beta}_{AIC}$	t_{rob}	t_{naive}	$\widehat{\beta}_{BIC}$	t_{rob}	t_{naive}
Employment Italy	-0.063	-1.430	-1.903	-0.075	-1.594	-2.556
Employment UK	-0.070	-1.161	-2.351	0	0	0
Index of GEA	0.170	2.355	2.399	0.176	2.519	2.588
M1 Japan	0	0	0	0	0	0
Personal cons UK	0	0	0	0	0	0
REER UK	-0.008	-1.329	-1.559	-0.009	-1.554	-2.08
Spread 3m-OV Italy	0	0	0	0	0	0

a regression with the respective variable and keep it if it turns out to be significant at the five percent level. We apply Andrews (1991) HAC standard errors and include a correction factor $\lambda \geq 1$ for the estimation error arising from the autoregression.[14] By doing so, we end up with a smaller set of variables such that an analytical evaluation of all possible models is computationally feasible for which the AIC and the BIC are computed. The information criteria balance between a good fit and the amount of parameters in the models. The exclusion of variables with least significance yields $J = 7$ (WTI) and $J = 12$ (Brent) remaining variables.[15] Below, we also consider a robustness check with an initial significance level of ten percent and thus larger sets of variables, i.e. $J = 13$ (WTI) and $J = 15$ (Brent).

These reduced set of regressors leads to the estimation of 127 (WTI) and 4,095 (Brent) different models. For each model, information criteria are computed and as a further results, a model weight between zero and one. The model averaging estimator is defined as weighted average of all estimated parameters in each single model by considering the

[14]See Table 6 in the Appendix for a list of selected variables alongside some statistics from the pretesting.
[15]We specify $\bar{c} = 0.7$ and $\widetilde{T} = 70$.

Table 4: Empirical findings – Model averaging, Brent

Variable	$\widehat{\beta}_{AIC}$	t_{rob}	t_{naive}	$\widehat{\beta}_{BIC}$	t_{rob}	t_{naive}
Employment Canada	-0.011	-0.417	-0.623	-0.007	-0.374	-0.762
Employment Italy	-0.059	-1.152	-1.686	-0.063	-1.264	-2.143
Employment UK	-0.044	-0.825	-1.579	-0.033	-0.605	-1.272
Index of GEA	0.191	2.346	2.366	0.196	2.488	2.512
M1 Japan	-0.001	-0.415	-0.633	-0.001	-0.294	-0.496
Personal cons Italy	-0.004	-0.439	-0.690	-0.002	-0.389	-0.848
Personal cons UK	-0.001	-0.303	-0.409	-0.001	-0.291	-0.491
Real GDP Italy	-0.001	-0.195	-0.245	-0.002	-0.299	-0.532
Real GDP US	0.000	-0.066	-0.073	-0.001	-0.197	-0.291
REER UK	-0.006	-1.115	-1.390	-0.006	-1.061	-1.529
Spread 3m-OV Italy	0.005	0.408	0.647	0.003	0.314	0.552
Unemployment Italy	0.024	0.512	0.856	0.018	0.459	0.998

Table 5: Empirical findings – Model selection, Brent

Variable	$\widehat{\beta}_{AIC}$	t_{rob}	t_{naive}	$\widehat{\beta}_{BIC}$	t_{rob}	t_{naive}
Employment Canada	0	0	0	0	0	0
Employment Italy	-0.079	-1.488	-2.288	-0.093	-1.776	-3.184
Employment UK	-0.078	-1.244	-2.783	0	0	0
Index of GEA	0.200	2.445	2.480	0.207	2.600	2.650
M1 Japan	0	0	0	0	0	0
Personal cons Italy	0	0	0	0	0	0
Personal cons UK	0	0	0	0	0	0
Real GDP Italy	0	0	0	0	0	0
Real GDP US	0	0	0	0	0	0
REER UK	-0.008	-1.307	-1.655	-0.009	-1.494	-2.215
Spread 3m-OV Italy	0	0	0	0	0	0
Unemployment Italy	0	0	0	0	0	0

AIC or BIC weights. Results are reported in Tables 2 and 4 for the WTI and the Brent price, respectively. In these two tables, we include the model averaging estimate for each candidate variable alongside the robust t-statistic accounting for model uncertainty (t_{rob}) and a naive t-statistic ignoring this effect (t_{naive}). A comparison of these two t-statistics reveals the impact of model search for each variable.

The results are clear-cut in the sense that only a single variable turns out to be significant: the index of global economic activity, developed and constructed by Kilian (2009).

It is the only regressor with a significant robust t-statistic in all considered versions of model averaging and even model selection (c.f. Tables 3 and 5). The robust t-statistics take value in the range from 2.258 (AIC model averaging, WTI) to 2.600 (BIC model selection, Brent) and are significant on the one percent level. The results are fully robust with respect to AIC and BIC, and also with regard to model averaging and model selection. The relationship between global economic activity and dynamic persistence in real oil prices is found to be positive. This positive relationship is plausible in the light of oil prices which are driven by macrocosmically induced demand and discussed in detail in Section 5. The exact interpretation of the estimated coefficients is not straightforward as the variable is an index. However, the sign of the estimated parameters is indicative for a significant positive relationship.

As a robustness check, we have experimented with a larger value of ten percent for the nominal significance α and thereby with a larger model set ($2^{13}-1 = 8,191$ (WTI) and $2^{15}-1 = 32,767$ (Brent)). Our main conclusions remain to hold and due to space considerations, we decide not to report them here, but in the Appendix (Tables 7 and 8). As a summary, we find the following results: while Gold (with an initial first-stage t-statistic of 1.767 (WTI) and 1.959 (Brent)) is included in the enlarged set of potential predictors entering the model averaging stage, it turns out to be insignificant in the final model averaging calculation accounting for the model uncertainty (with t-statistics equal to 0.691 (0.479) in the case of WTI with AIC (BIC) model averaging and 0.612 (0.404) for the Brent, respectively). Moreover, the oil related variables world oil production, spot-future spread and inventories are not even passing the first stage due to their inability to explain movements in the time-varying persistence of real oil prices. Their initial t-statistics equal for the WTI: -1.324, -0.808 and -0.601, respectively (very similar results are found for the Brent). This is similar to commodities which are typically traded by speculative investors, e.g. silver (0.521), soybeans (0.576), wheat (0.788) and rice (1.229). The given t-statistics indicate their irrelevance for the explanation of dynamic oil price persistence.

5 Discussion

We are interested regarding the question: What drives the price persistence of crude oil rather than the price itself? Explosive or stationary price behavior might be caused by rational bubble or bursting periods respectively. But, for the existence of a rational bubble, it is required that a speculative component is included in the prices[16] because

[16] Speculation refers to a situation where someone buys oil for future rather than for immediate use.

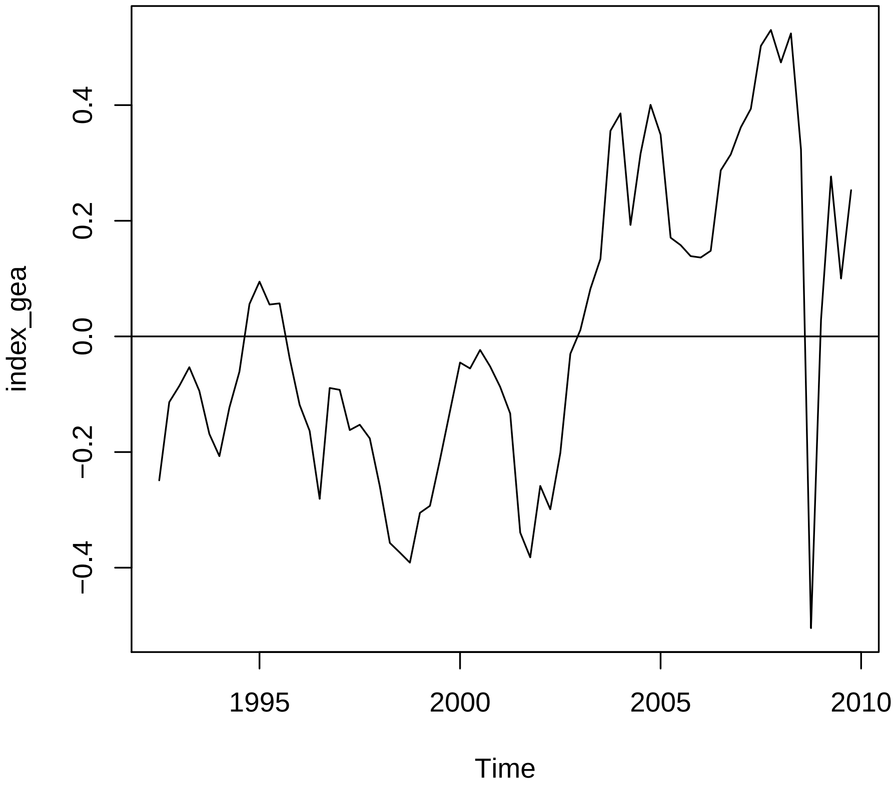

Figure 3: Index of global economic activity, see Kilian (2009).

a 'rational bubble' refers to a situation where buyers push the price of oil beyond the intrinsic value since these investors have expectations about rising oil prices. Thus, using the procedure of exclusion, if variables related to speculation – like the spot-futures spread – are not significant for the explosive persistence, we can conclude that the explosiveness has not been caused by a rational bubble.

Our results indicate that solely increases or decreases in the global demand for energy explain stationary, random walk and explosive behavior of oil prices. Just one variable – namely the global economic activity (index of GEA) by Kilian (2009) plotted by Figure 3 – is significant.[17] This index "*is based on dry cargo single voyage ocean freight rates and is explicitly designed to capture shifts in the demand for industrial commodities in global business markets*" (see Kilian, 2009, p. 1055).

[17]It is stationary by construction (see Kilian and Murphy, 2014). Thus, we do not have to apply a transformation of this index as presented by Table 6.

We find explosive regimes by considering the estimated persistence (see Figure 1 in the case of real WTI prices and Figure 2 in the case of real Brent prices) between 2005 and 2008 in oil prices which is in line with the majority of studies employing persistence-based methodology (see, for example, Caspi, Katzke, and Gupta, 2015; Gronwald, 2016). Just by eyeballing, the estimated persistence and the trajectory of the index of GEA, it stands to reason that these variables are related. The explosive regime between 2005 and 2008 comes along with a strong increase in the global demand for industrial commodities and its dip is coincident with a decrease of the persistence.[18]

To be cautious regarding the interpretation of this result as non-speculation – while speculation does not drive the persistence, it might effect the price itself because there is no rational bubble without speculation but there might be speculation in the market without a bubble. However, speculation which does not lead to bubble behavior has, of course, different economic implications which are not in the scope of this paper. Nonetheless, we find an explosive regime (see Figure 1 and Figure 2), which could not be explained by variables related to speculation. Thus, our results indicate that this explosive regime is not related to a rational bubble which is in line with Pavlidis, Paya, and Peel (2017).

While the price itself could also be driven by supply shocks, the persistence appears not to be driven by oil supply. However, Kilian and Murphy (2014) argue that the peak oil hypothesis[19] might be rejected. Additionally, we would not conclude in favour of speculative bubbles if oil supply would have been a variable effecting the persistence of oil prices significantly. Hence this is not the case, we conclude that supply shocks were not the main driver for the explosive regime. Thus, our results are in line with Kilian and Murphy (2014) concerning supply shocks.

Compared to other studies, like for example, Phillips and Yu (2011) or Caspi, Katzke, and Gupta (2015), our approach has the advantage that we do not need to pre-specify a fundamental for crude oil prices in order to investigate whether an explosive regime is related to a rational bubble or not. Hence, in this particular sense, our approach is comparable to Pavlidis, Paya, and Peel (2017) who also overcome the pre-specification of the market fundamental for crude oil to analyze bubble behavior. However, contrary to Pavlidis, Paya, and Peel (2017), our approach is data-driven and permits to capture

[18] Etienne, Irwin, and Garcia (2014) find that explosive regimes in grain futures prices are positively connected to the index of global economic activity as well.

[19] The oil peak hypothesis supposes a peak in the oil production in 2006 and, as the result, increasing oil prices.

a large number of potential drivers of the persistence for crude oil prices.

6 Conclusions

In this article we tackle the question whether time-varying persistence in real oil prices (including mildly explosive periods) can be linked to macro-financial variables from G7 countries, oil-related measures and commodity markets. The employed econometric techniques range from indirect inference bias-corrected estimation (see Phillips, Wu, and Yu (2011)) to frequentist model averaging (see Burnham and Anderson (2002)). We account for estimation and model uncertainty in the different steps of our analysis. Our major finding is that the index of global economic activity (c.f. Kilian (2009)) is the only driver of persistence in real oil prices. Our interpretation is that dynamic persistence is solely driven by global energy demand. All other considered variables from a comprehensive data set by Juvenal and Petrella (2015) turn out to be insignificant. As financial variables like the spot-futures spread do not play a role in explaining the time-varying persistence, it is likely that the temporary explosive behavior is not related to (rational) financial bubbles, thereby supporting the notion of Gronwald (2016). Our results are found to be robust in several dimensions. While we consider an extensive collection of data, there is still the risk of neglecting a significant variables which might also explain the persistence of oil prices. Thus, our conclusions are based on the comprehensiveness of the data compilation.

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7 Appendix

7.1 Details on selected variables

Table 6: Details on selected variables (data specifics and pretesting results)

Variable	Source	Unit	Transformation	$\widehat{\beta}^{(k)}$	$\widetilde{t}_{\beta^{(k)}}$	$\widehat{\lambda}^{(k)}$
Employment Canada	OECD EO	%	Δ	-0.239	-2.017	1.308
Employment Italy	OECD EO	%	Δ	-0.329	-2.498	1.401
Employment UK	OECD EO	%	Δ	-0.347	-3.244	1.763
Gold price	IFS	Deflated by US CPI	$\Delta \log$	0.365	1.959	1.167
Index of GEA	Kilian (2009)	Index	—	0.513	3.032	1.210
M1 Japan	OECD MEI	Deflated by CPI	$\Delta \log$	-0.397	-2.548	1.256
M2 Canada	OECD MEI	Deflated by CPI	$\Delta \log$	0.387	1.702	1.106
Personal cons Italy	IFS	Bil. EUR	$\Delta \log$	-0.284	-2.086	1.365
Personal cons UK	IFS	Bil. GBP	$\Delta \log$	-0.331	-3.000	1.687
Real GDP Canada	OECD	Mil. CAD	$\Delta \log$	-0.299	-1.831	1.228
Real GDP Italy	OECD	Mil. EUR	$\Delta \log$	-0.269	-2.073	1.420
Real GDP US	OECD	Mil. USD	$\Delta \log$	-0.251	-2.053	1.496
REER UK	JP Morgan	Index	$\Delta \log$	-0.323	-1.996	1.235
Spread 3m-OV Italy	IFS	%	—	0.368	2.263	1.231
Unemployment Italy	OECD EO	%	Δ	0.237	1.980	1.352

7.2 Additional Empirical Results

Table 7: Empirical findings – Model averaging, WTI, $\alpha = 10\%$ for $\mathcal{X}$

Variable	$\widehat{\beta}_{AIC}$	t_{rob}	t_{naive}	$\widehat{\beta}_{BIC}$	t_{rob}	t_{naive}
Employment Canada	-0.011	-0.410	-0.606	-0.007	-0.361	-0.702
Employment Italy	-0.043	-0.999	-1.617	-0.041	-0.958	-2.193
Employment UK	-0.037	-0.777	-1.549	-0.028	-0.615	-1.700
Gold price	0.002	0.691	1.010	0.001	0.479	0.812
Index of GEA	0.142	2.065	2.252	0.154	2.332	2.569
M1 Japan	-0.002	-0.561	-1.020	-0.001	-0.394	-0.779
Personal cons Italy	-0.003	-0.358	-0.513	-0.002	-0.310	-0.517
Personal cons UK	-0.001	-0.313	-0.431	-0.001	-0.255	-0.385
Real GDP Italy	0.000	0.005	0.005	-0.001	-0.220	-0.338
Real GDP US	0.000	-0.029	-0.031	-0.001	-0.142	-0.187
REER UK	-0.006	-1.207	-1.568	-0.006	-1.180	-1.858
Spread 3m-OV Italy	0.005	0.418	0.679	0.003	0.363	0.745
Unemployment Italy	0.015	0.424	0.663	0.013	0.430	0.953

Table 8: Empirical findings – Model averaging, Brent, $\alpha = 10\%$ for $\mathcal{X}$

Variable	$\widehat{\beta}_{AIC}$	t_{rob}	t_{naive}	$\widehat{\beta}_{BIC}$	t_{rob}	t_{naive}
Employment Canada	-0.006	-0.334	-0.480	-0.002	-0.225	-0.344
Employment Italy	-0.078	-1.782	-2.299	-0.079	-1.835	-2.627
Employment UK	-0.042	-0.830	-1.639	-0.022	-0.514	-1.127
Gold price	0.001	0.612	0.929	0.001	0.404	0.670
Index of GEA	0.186	2.861	2.964	0.189	2.778	2.851
M1 Japan	-0.001	-0.454	-0.728	-0.001	-0.338	-0.626
M2 Canada	0.023	1.402	1.567	0.022	1.352	1.679
Personal cons Italy	0.000	-0.067	-0.072	0.000	-0.141	-0.182
Personal cons UK	-0.002	-0.348	-0.495	-0.001	-0.253	-0.403
Real GDP Canada	0.017	0.702	1.171	0.006	0.395	0.763
Real GDP Italy	-0.001	-0.174	-0.212	-0.001	-0.132	-0.165
Real GDP US	0.003	0.365	0.543	0.001	0.256	0.445
REER UK	-0.004	-0.975	-1.456	-0.003	-0.794	-1.726
Spread 3m-OV Italy	0.001	0.108	0.125	0.001	0.188	0.274
Unemployment Italy	0.016	0.440	0.713	0.012	0.371	0.678

7.3 Robust t-statistics when the regressand is estimated

Estimation of the regression model $\rho_t = \beta^{(k)} x_t^{(k)} + u_t^{(k)}$ is infeasible as ρ_t is unobserved. Instead, we have $\widehat{\rho_t} = \rho_t + \epsilon_t$, with ϵ_t being the estimation error with $var(\epsilon_t) = \sigma_\epsilon^2$ denote its variance. A feasible regression is (see Dumont, Rayp, Thas, and Willemé, 2005)

$$\widehat{\rho_t} = \beta^{(k)} x_t^{(k)} + (u_t^{(k)} + \epsilon_t).$$

Neglecting the fact that ρ_t is estimated leads to an upward-bias in absolute t-statistics. Under the assumption that $u_t^{(k)}$ and ϵ_t are independent of each other, a correction factor for the t-statistics can be constructed along the lines of Dumont et al. (2005):

$$\lambda^{(k)} = \frac{\sigma_\epsilon^2 + \sigma_{u^{(k)}}^2}{\sigma_{u^{(k)}}^2} \geq 1.$$

The corresponding robust t-statistic for testing $H_0 : \beta^{(k)} = 0$ is then given by

$$\widetilde{t}_{\beta^{(k)}} = \frac{\widehat{\beta}^{(k)}}{\sqrt{\widehat{\lambda}^{(k)} / \sum_t x_t^{(k)} x_t^{(k)}}}.$$

As ρ_t is estimated in a rolling window fashion, a sequence of estimated variances for $\widehat{\rho_t}$ is obtained. We use the median of the sequence to measure the overall estimation uncertainty. Another issue is the widely acknowledged problem of heteroscedasticity and autocorrelation in the residuals. We additionally employ Newey-West HAC standard errors following the suggestions made in Andrews (1991) in the computation of $\widetilde{t}_{\beta^{(k)}}$.